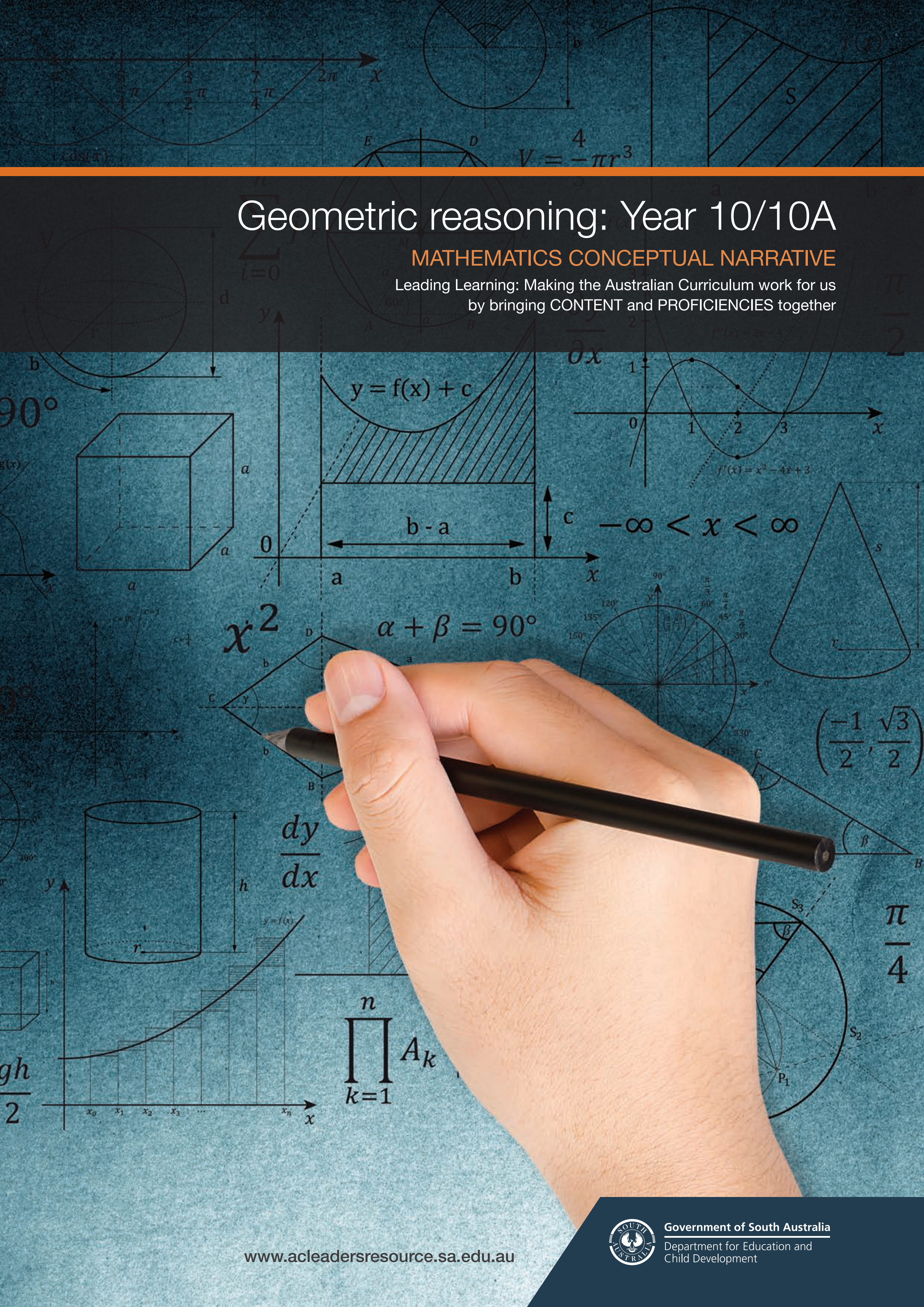


Geometric reasoning: Year 10/10A

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together

www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Geometric reasoning' from Foundation to Year 10A** (see page 23)— we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with geometric reasoning:

- Estimate/measure/compare/identify/classify using geometrical reasoning
- Investigate and generalise using geometrical reasoning
- Solve and prove using geometrical reasoning.

What the Australian Curriculum says about ‘geometric reasoning’

Content descriptions

Strand | Measurement and geometry.

Sub-strand | Geometric reasoning.

Year 10 ◆ | ACMMG243
Students formulate proofs involving congruent triangles and angle properties.

Year 10 ◆ | ACMMG244
Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes.

Year 10A ◆ | ACMMG272
Students prove and apply angle and chord properties of circles.

Year level descriptions

Year 10 ◆ | Students formulating geometric proofs involving congruence and similarity.

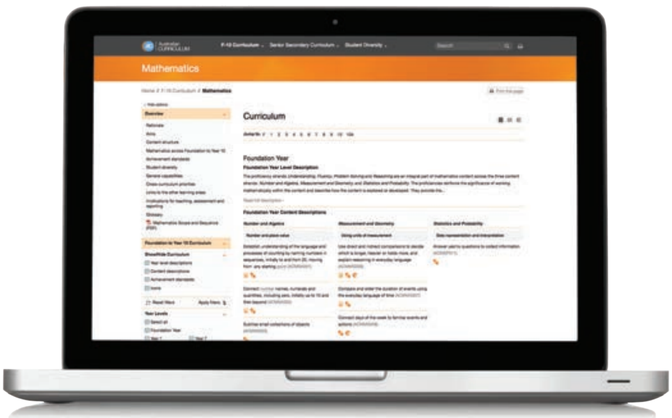
Achievement standards

Year 10 ◆ | Students apply deductive reasoning to proofs and numerical exercises involving plane shapes.

Year 10 ◆ | Students use triangle and angle properties to prove congruence and similarity.

Numeracy continuum

End of Year 10 ◆ ◆ | Students visualise, describe and analyse the way shapes and objects are combined and positioned in the environment for different purposes (Using spatial reasoning: Visualise 2D shapes and 3D objects).



Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Working with ‘geometric reasoning’

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 10/10A geometric reasoning

In Year 9 students continue to make observations about angles and side lengths of triangles and they use observations together with an understanding of enlargement and ratio to explain similarity. Students establish the conditions for similarity in triangles and they compare these observations to the conditions required for congruence of triangles.

In Year 10 students establish the difference between demonstration and proof. Students apply their understanding of congruence and angle properties to deduce properties of other geometric figures. They also use congruence and similarity to calculate unknown angles, even when there are multiple steps involved in reaching the solution.

In Year 10A students extend their geometric reasoning to prove and apply angle and chord properties of circles.

- **Notice that it is *only* in the numeracy continuum that we see an acknowledgement of the need for students to *visualise* shapes.** Visualisation is an important skill for young mathematicians to be developing especially when they are formulating geometric arguments. The numeracy continuum also states the need to *describe and apply* their understanding of the features and properties of 2D shapes and 3D objects. This could be easily overlooked when only reading the content strands and achievement standards.
- Although the AC Mathematics states that Year 8 students were expected to ***develop*** the conditions for congruent triangles and ***use*** congruent triangles to establish properties of quadrilaterals, it is not explicitly stated that students need to be able to communicate their reasoning using the protocols of formal proof. As Year 10 students are expected to ***formulate proofs*** and ***apply logical reasoning*** using congruency, similarity and angle properties, it is essential that the ideas developed in Year 8 are revisited and the protocols of recording formal proofs is explicitly modelled and scaffolded.
- **Practice on its own is not enough.** Often we think students will get better at formal proof if we just give them enough practice, but we need to be more deliberate and account for it in our learning design. Rather than giving students a large number of proofs to do, use some different strategies, ask students to give feedback to a fictitious student’s attempt at proof, reconstruct a ‘jigsaw’ of a formal written solution, do an audio tape of a spoken proof etc.
- **At this stage of development, students often have difficulty distinguishing between:**
  - **the information you can use as truth**
  - **things that look like they are true**
  - **what you are trying to prove.**

  As teachers we should support students to develop a habit of clarifying these three things before they start to formulate an argument or proof for the general case by asking the following questions:
  - *What do you know is true?* (Only what you are given, or what you can deduce.) *How could you show that on the diagram?*
  - *What do you think might be true but you don’t know for sure?*
  - *What do you want to prove?*

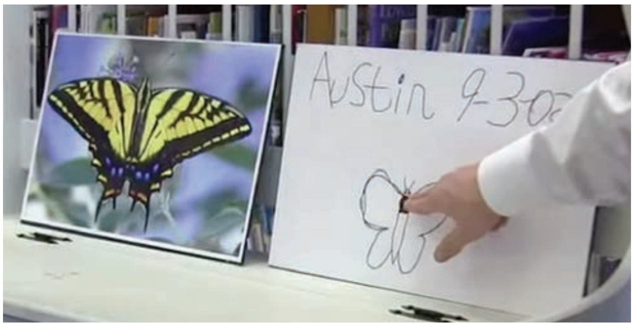
Engaging learners

Classroom techniques for teaching geometric reasoning

Developing geometric reasoning is very challenging for students, requiring risk taking, persistence and the expectation that you will make mistakes and learn from them.

Austin's butterfly

This video tells the story of a first-grade student who received constructive feedback from his peers in his efforts to draw a butterfly. It is a great story to share with your class when they are about to tackle a significant learning challenge.

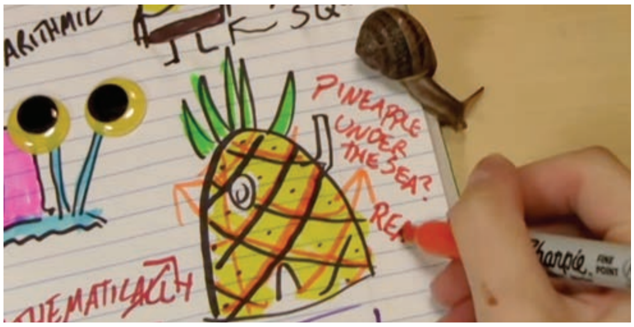


Source: *Critique and Descriptive Feedback: The Story of Austin's Butterfly*, Ron Berger at Presumpscot Elementary, Portland, ME https://www.google.com.au/webhp?sourceid=chrome-instant&ion=1&espv=2&ie=UTF-8#q=austin%27s%20butterfly%20youtube

Vi Hart's art

Introduce your students to the stimulating work of Vi Hart, an artist, musician and mathematician.

Vi's doodling in mathematics fascinates students with her rapid speech, skilful sketching and unusual explorations into questions such as 'Is SpongeBob SquarePants' home really a pineapple?'



Source: Vi Hart (2012) *Open Letter to Nickelodeon, Re: SpongeBob's Pineapple under the Sea*, accessed at https://www.youtube.com/watch?v=gBxeju8dMho

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–5)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

Students already have an understanding of the everyday usage of statements such as 'convince me' or 'prove it' but no amount of reasoning will lead my students to create the mathematical protocols of **geometric proof** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged with questions that will result in them identifying the features of mathematical proof, so I don't need to instruct that information.

At this stage of development, students can develop an understanding of **the difference between demonstration and formal proof**. When teachers provide opportunities for students to **make and verify conjectures from specific cases and then require their students to generalise and justify the patterns to others, they are experiencing the difference between verification and proof**. When teachers demonstrate **formal geometric proofs** for students before students identify connections, they remove the natural opportunity for students to construct a convincing argument for the conjectures that they notice for themselves.

Teachers can support students to develop an understanding of **formal geometric proof** by asking questions as described in the **Understanding** proficiency. **In what ways can your thinking be generalised? In what ways can you prove?**

When we are feeling 'time poor' it's tempting to believe that it will be quicker to demonstrate a range of geometric proofs (hoping they will learn through examples), rather than designing opportunities ask questions and support them to establish convincing arguments for themselves (before we demonstrate the formal setting out). Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular series of steps during the current unit of work, then it probably is quicker to tell them the rules and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the rule is a false economy of time. When we challenge our students to *provide* a convincing argument, we model that mathematical reasoning can be powerful and useful. We provide our students with an authentic context for working mathematically.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Always a right angle Students formulate proofs involving congruent triangles and angle properties (Year 10) Students prove and apply angle and chord properties of circles (Year 10A)	ACMMG243 ◆ ACMMG272 ◆
Example 2: Arrowhead in a circle Students formulate proofs involving congruent triangles and angle properties (Year 10) Students prove and apply angle and chord properties of circles (Year 10A)	ACMMG243 ◆ ACMMG272 ◆
Example 3: Angles in a segment Students formulate proofs involving congruent triangles and angle properties (Year 10) Students prove and apply angle and chord properties of circles (Year 10A)	ACMMG243 ◆ ACMMG272 ◆
Example 4: Hex Students formulate proofs involving congruent triangles and angle properties (Year 10) Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (Year 10)	ACMMG243 ◆ ACMMG244 ◆
Example 5: Incircle Students formulate proofs involving congruent triangles and angle properties (Year 10) Students prove and apply angle and chord properties of circles (Year 10A)	ACMMG243 ◆ ACMMG272 ◆

Example 1: Always a right angle

ACMMG243 ◆

Year 10 | Students formulate proofs involving congruent triangles and angle properties

ACMMG272 ◆

Year 10A | Students prove and apply angle and chord properties of circles

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

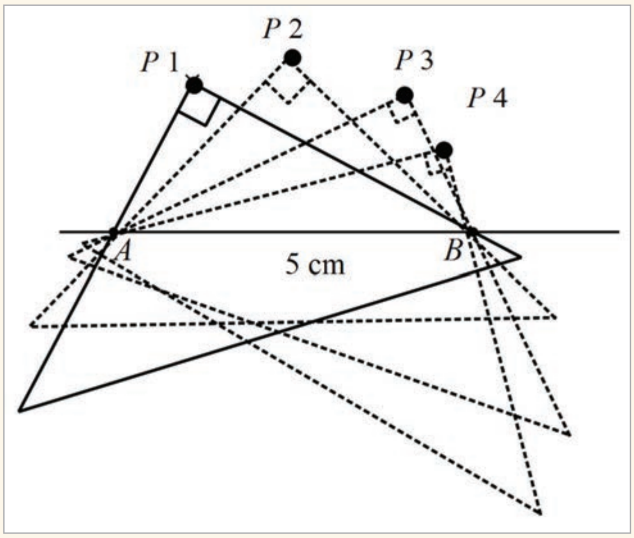
Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the properties of angles in a circle, we can challenge students to recognise the relationships, by ***asking*** questions.

In this activity, students draw a line segment AB about 5 cm long and use a set square to find different places where a point, P, could be placed so that the ∠*APB* is a right angle.



Ask students:

- *What do you notice about all the possible positions for X? How could you check?* (They look like they are in a semi-circle. Using a compass and the midpoint of AB as the centre of the circle, draw a semi-circle to see if the points all lie on its circumference.)

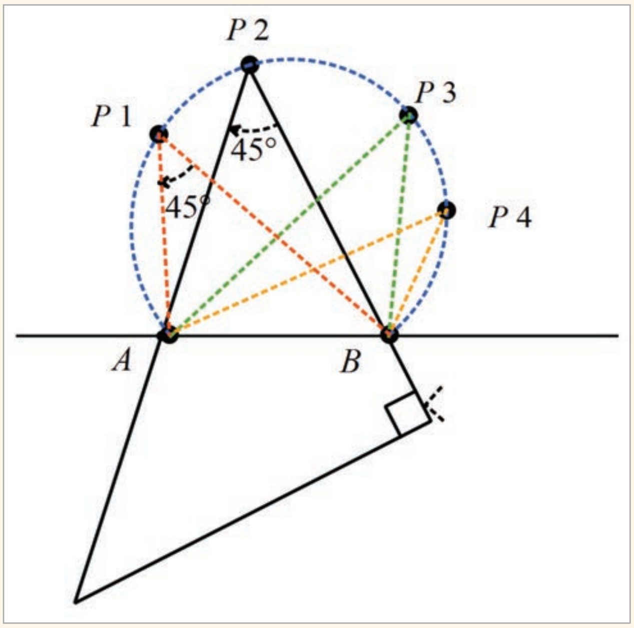
Ask students to check this in reverse, ie draw a semi-circle and check that all the angles drawn from A to the circumference and then to B would be 90°. Then ask:

- *What do you notice? Would this always be the case?* (Students may want to change the length of the line, or use another angle. This can be repeated using the acute angles on the set square.)

Develop the concept before the detail

Persisting with experimentation and verification, rather than proving or using the property, can help to keep students focused on the relationship rather than its application to solve problems, which will probably involve abstract representation.

This activity can be repeated using the acute angles on the set square. The points will form a major arc of a circle.



Students can check these findings in reverse by drawing a circle and marking a chord AB and measuring multiple possible angles ∠*APB*.

Ask students to make a conjecture:

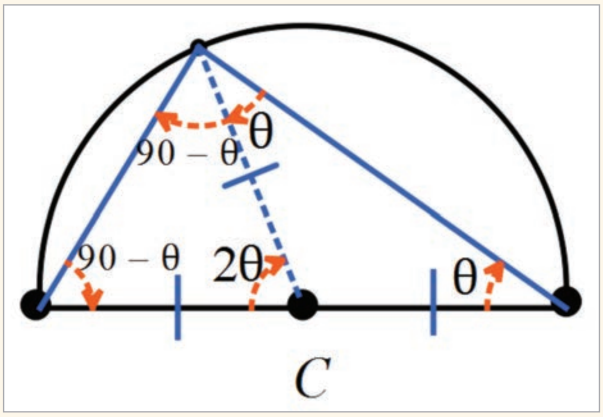
- *What can you infer?*

We have verified that if the angle at the circumference is a right angle, it is subtended by (ie end points come from) a diameter but can we prove it?

Remind students of the following three questions:

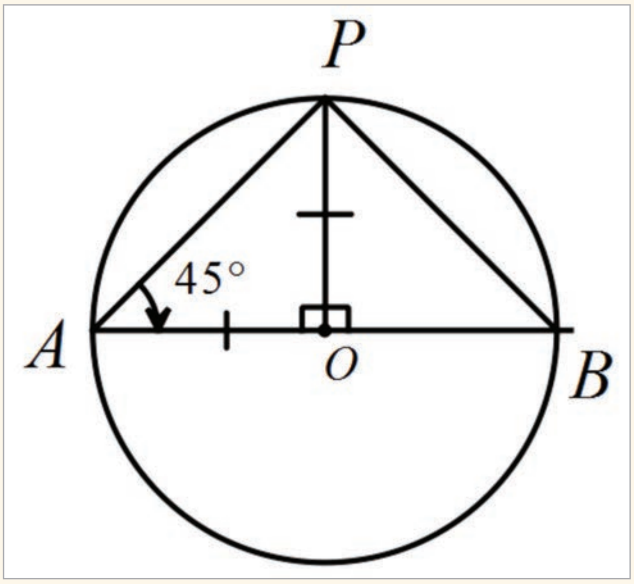
- *What do you know is true? [ie What you are given or what you can deduce? How could you show that on the diagram?]* (The radii are equal, the diameter is a straight line with the centre in the middle.)

  If students need more support, ask:
  Can you draw in another radius that might be helpful in solving our problem? Is there anything more you know now you have drawn that in? Can you record your thinking as you go along? How can you show angles are the same even when you don't know what they are? If I told you to use the exterior angle rule for a triangle, how might that help you?



- *What do you think might be true but you don't know for sure?* (If $\angle AXB$ is going to be 90°, I think the two top angles should add to 90.)
- *What do you want to prove?* (That $\angle AXB$ = 90°.)

To draw attention to possible misconceptions, ask students to comment on another student's work.



Show the diagram above, then ask:

- *What advice would you give to a student who drew this diagram to help them prove their conjecture?*

Students often use particular values for the angle. This may be a good way to enter a problem. They can then try to generalise their thinking. Ask:

- *What feedback would you give to this student about their thinking?*

This special case only proves it is true if the angle is 45°. Try to use the same reasoning for any angle by using a pronumeral such as θ.

Example 2: Arrowhead in a circle

ACMMG243 ◆

Year 10 | Students formulate proofs involving congruent triangles and angle properties

ACMMG272 ◆

Year 10A | Students prove and apply angle and chord properties of circles

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

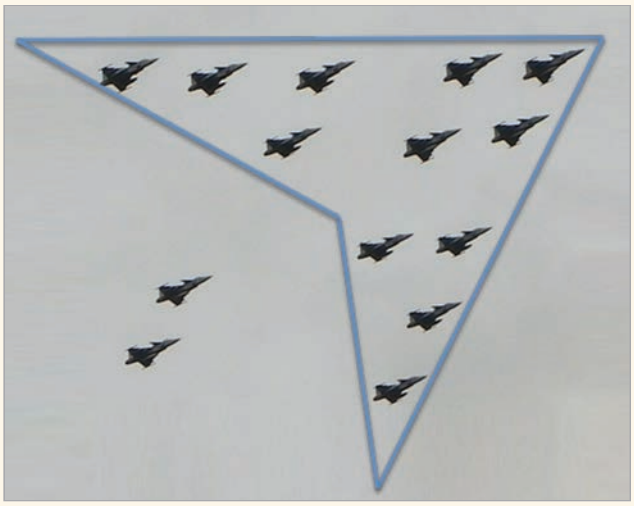
Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the properties of angles in a circle, we can challenge students to recognise the relationships, by ***asking*** questions.



Ask students to draw an arrowhead so that the vertices are three points on the circumference of a circle and the fourth is the centre.

Measure the three acute interior angles of your arrowhead and the exterior obtuse angle. Use and discuss this mathematical language with your students:

- *What do you notice? Did everyone get the same results? Why?* (If the arrowhead is symmetrical two of the acute angles will be equal but all students should notice that the obtuse exterior angle is double the acute interior angle subtended by the same chord.)

Ask students to make a conjecture:

- *What can you infer?*

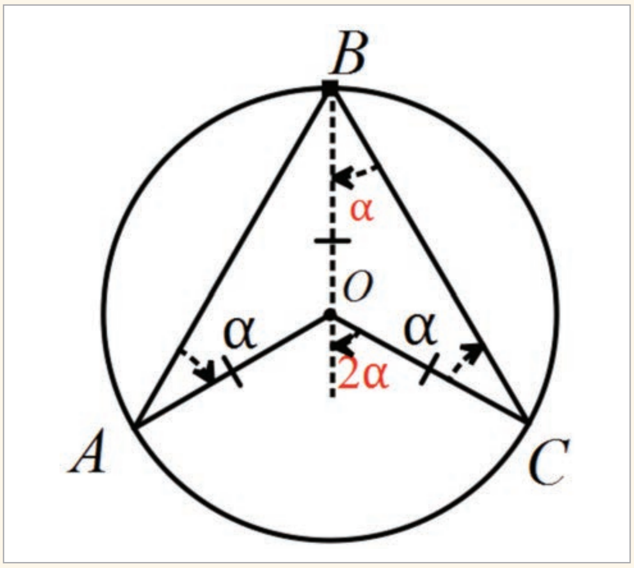
As this proof is similar to 'Example 1: Always a right angle', students should be encouraged to attempt a proof remembering the following three questions:

- *What do you know is true? [ie What you are given or what you can deduce? How could you show that on the diagram?]*
- *What do you think might be true but you don't know for sure?*
- *What do you want to prove?*

Tackling misconceptions: Knowing what it isn't

We can challenge our students reasoning by asking *'Why can't I…? / Why is it not…? / What's wrong with…?'* questions. This type of question can be used to address common misconceptions. Students can be given problems which are answered incorrectly, without working out. They can be asked to give this (fictional student) feedback summarising the mistakes that they have made and how to avoid that mistake in the future.

To draw attention to possible misconceptions, ask students to comment on another student's work.



Show the diagram above, then ask:

- *What advice would you give to a student who drew this diagram to help them prove their conjecture?*
- *What feedback would you give to this student about their thinking?* (While the student has correctly marked the equal radii and used the exterior angle rule, they have assumed that the both angles would be α, which would only be the case for a symmetrical arrowhead.)

Example 3: Angles in a segment

ACMMG243 ◆

Year 10 | Students formulate proofs involving congruent triangles and angle properties

ACMMG272 ◆

Year 10A | Students prove and apply angle and chord properties of circles

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you answer backwards questions?

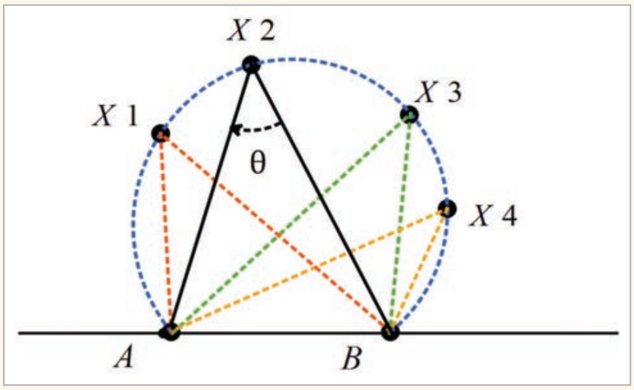
Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the properties of angles in a circle, we can challenge students to recognise the relationships, by ***asking*** questions.

In this activity, students draw a line segment AB about 5 cm long and use a set square to find the different places where a point, X, could be placed so that the $\angle AXB$ is always the same.



Ask students:

- *What do you notice about all the possible positions for X? How could you check?* (They look like they are in a circle. Students might try to find the centre by folding or by trial and error somewhere on the perpendicular bisector of the chord AB.)

Ask students to check this in reverse, ie draw a circle and check that all the angles drawn from A to the circumference and then to B would be the same size. Then ask:

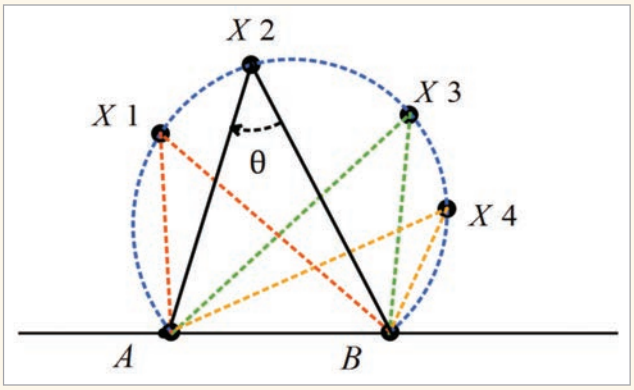
- *Would this always be the case? What do you notice? How is it the same/different for another angle?* (Students may want to change the length of the line, or use another angle. This can be repeated using the angles they draw themselves and cut out of cardboard. They might try obtuse angles.)

Students can check these findings in reverse by drawing a circle and marking a chord AB and measuring multiple possible angles $\angle AXB$.

Ask students to make a conjecture:

- *What can you infer?*

Students can complete the circle and measure the angle in the alternate (minor) segment to find it is not the same but is the supplementary angle (180- θ).



Ask students:

- *What do you notice about the angle in the alternate segment? What can you infer? Convince me.*

We have verified that the angles at the circumference, subtended by the same chord of a circle are equal but can we prove it?

Building a body of knowledge: Creating new knowledge by adapting and applying something you have already proven to be true

As a community of learners, mathematicians build on new knowledge they have or someone else has proven to be true. Having made a conjecture and proving it to be true, students should have the opportunity to consider, 'Now that I know that, what else might be true?'

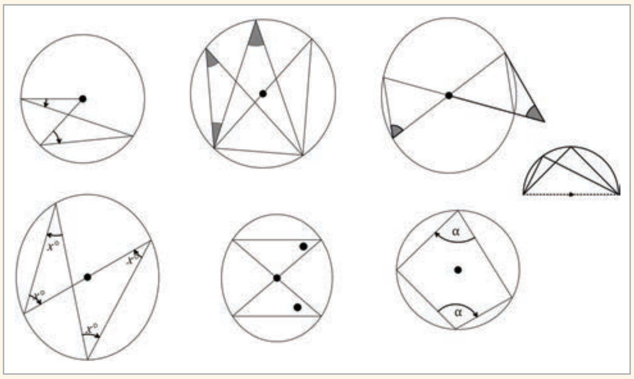
Remind students of the following three questions:

- *What do you know is true? Only what you are given or what you can deduce. How could you show that on the diagram?* (Can you use a similar approach to another circle proof you have done? Could you use *any result that you have proven about angles in circles?* Students may start to use congruent/similar triangles or exterior angles which are logical places to start. Allow them to explore these approaches until they are successful or feel they need a different approach.

- *What do you think might be true but you don't know for sure?* (All the angles at the circumference would be the same.)
- *What do you want to prove?* (That ∠*AXB* = ∠*AYB*.)

If students need more support, ask:
If I told you to use the angle at the centre rule that you proved for arrowhead in a circle, how might that help you?

To draw attention to possible non-standard arrangements, ask students to comment on another student's work.



Show the diagram above, then discuss:

Students who have just heard about your conjecture have thought that it would mean the following angles would be equal, therefore:

- *What feedback would you give to students who marked these angles as equal?*

Example 4: Hex

ACMMG243 ◆

Year 10 | Students formulate proofs involving congruent triangles and angle properties

ACMMG244 ◆

Year 10 | Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

Questions from the BitL tool

Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students tudents, the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

In this activity from the NRICH website, students are asked to reform a regular hexagon into 3 congruent hexagons using a range of congruent triangles.

This activity could be extended by asking:

- *How can you prove your thinking, numerically and algebraically?*

The link to this problem on the NRICH site is: http://nrich.maths.org/795

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Example 5: Incircle

ACMMG243 ◆

Year 10 | Students formulate proofs involving congruent triangles and angle properties

ACMMG272 ◆

Year 10A | Students prove and apply angle and chord properties of circles

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the properties of angles in a circle, we can challenge students to recognise the relationships, by ***asking*** questions.

This activity begins with the following question:

Did you know it is possible to draw a circle inside any triangle such that the circle touches all three sides?

Ask students to draw a triangle and see if they can do it by trial and error.

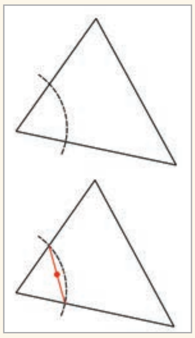
Trial and error supports engagement with the problem

When asked to make predictions or test a claim, students are challenged to think more deeply about the process they are about to observe. Asking what the circle 'might look like' rather than what it 'will be' invites thinking rather than 'seeking the correct answer'. Students are often motivated to experiment and take notice of the results to see how well the results match their attempts but we can support a growth mindset through valuing their thinking rather than the accuracy of their trials.

Let's check that out.

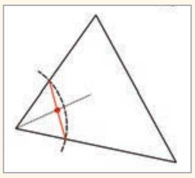
In an open space in the yard, create a triangle using a **chalk line** (not oxide).

For each side hold the chalk line anchored between two vertices of the triangle and get a student to 'ping' (stretch and release) the line from a central point of the line. Repeat for the other two sides.



Using a piece of string which is not flexible, at least a metre in length, swing an arc (marking it with chalk) from one of the vertices that cuts the adjacent sides.

Find the midpoint between the two points where the arc cuts the sides.

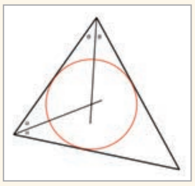


Use the chalk line to mark the line that passes through the midpoint and the vertex. Ask students:

- *What do you notice about the line and the angle of the triangle?*

Repeat the entire process for another vertex, asking:

- *What do you predict might happen if you repeat the process for the third vertex?* (The intersection of the lines is the centre of your incircle.)



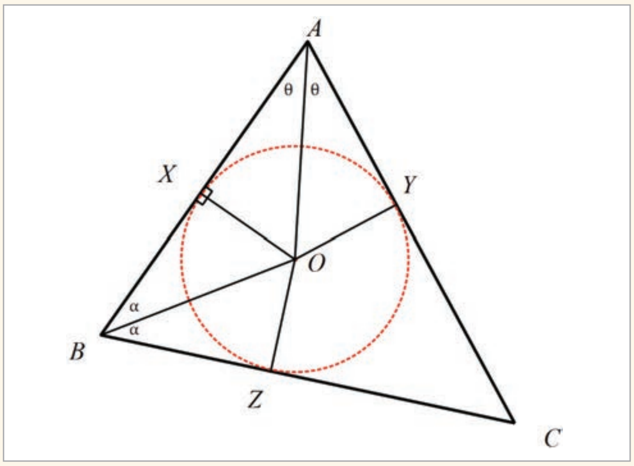
Using the non-flexible string, anchor at the centre and starting at one of the vertices, draw the incircle of the triangle. Ask students:

- *How accurate was the incircle you drew? Why?*

Repeat this process by drawing a triangle on a piece of paper and using a compass to swing the arcs, bisect the angle and draw the incircle. Ask students:

- *How accurate was the incircle you drew? Why?*
- *Would this always work?*
- *How could you prove this? Could you use congruent triangles?*

A student started to draw this diagram:



- *How does this relate to the incircle you drew? Can you explain why the student marked the information on the diagram and whether there are other things that could be included?*
- *Can you see any triangles you think might be congruent?*
- *How could you prove that you could always draw the red circle inside the triangle?*
- *What properties about two tangents from an external point could you infer from this proof?*

A similar activity can be found in the Mathematical Association of South Australia (MASA) publication, ***Rich Tasks for 10A Mathematics*** (available at http://www.masanet.com.au/publications-booklet/).

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 6–10)

Problems are described in the AC: Mathematics as ‘meaningful’ or ‘unfamiliar’. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as ‘fun’ nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, ‘I don’t know’, or ‘I’m not sure’. Students respond differently to this feeling, some shut down, others begin to ask, ‘But how could I work that out?’

In developing powerful learners we are aiming for all of our students to learn that ‘not knowing’ is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, ‘What could I do to work this out?’

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they’ll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they’ll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 6: Great squares Students formulate proofs involving congruent triangles and angle properties Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	ACMMG243 ◆ ACMMG244 ◆
Example 7: Shadows Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	ACMMG244 ◆
Example 8: Federation Square Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	ACMMG244 ◆
Example 9: Square partitions Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	ACMMG244 ◆
Example 10: Trapezium four Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	ACMMG244 ◆

Example 6: Great squares

ACMMG243 ◆

Students formulate proofs involving congruent triangles and angle properties

ACMMG244 ◆

Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

In this activity from the NRICH website, students are introduced to a growing shape pattern and are invited to notice and explore aspects that interest them.

This activity could be extended by asking:

- *How would this work with triangles; pentagons; etc?*

The link to this problem on the NRICH site is: http://nrich.maths.org/72

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Planning for intellectual stretch

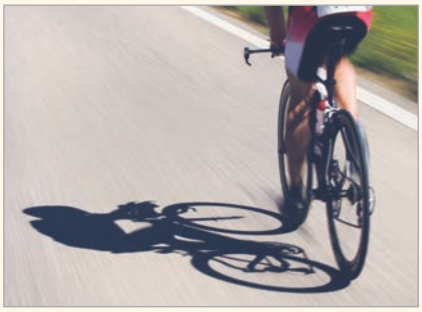
Always consider possible ways in which an initial inquiry can be extended. Through doing this, we can provide students with learning experiences where they think more deeply about a question they already know a lot about. They are encouraged to transfer, adapt and extend a problem they are familiar with, rather than returning to an entry level and a familiarisation phase for a new problem.

Example 7: Shadows

ACMMG244 ◆
Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.



Provide students with a range of images with shadows (such as those above), or ask students to take some of their own.

Ask students:

- *What's the first question that comes to mind?* (Knowing the height of one of two objects and the length of the shadows might be used to determine the height of an unknown object, such as, a tree in the yard or a school building.)

Interpret

What question have you selected? What might you need to work out to answer that question? What information is helpful? What information is not useful? What extra information do you need? What information can you reasonably infer? (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? Would it help if you thought about a similar problem for a simpler figure first? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

What height for the person is too high? What height is too low?

Questions to be used only after students have grappled with the problem for a few minutes:

Does that seem right to you? Do other people think that too? Could you give a range of possible values? Can you estimate where the light source is, where the wall is? What could you do to check your thinking?

Reflect

Ask students to pair up with someone who did it differently to discuss:

How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same or a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Example 8: Federation Square

ACMMG244 ◆

Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

This activity could be considered a ***Three-Act Maths Task*** like those on Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

Identifying the question to solve

The group can share questions and sort them into mathematical and non-mathematical questions. Then, of the mathematical questions, students can sort their questions into those that cannot be answered with the given information and those that could be answered using the given information or additional information that could be inferred.

Dan Meyer has a technique, that we have seen many teachers adopt when generating and collecting questions from students. First he asks students to individually write down questions that come to mind. Then, as he invites students to share their questions, he writes students' names next to the questions. He also asks if anyone else likes that question. *'Did you write it down, or if you didn't perhaps you still think that it's a good question.'* Through doing this, both Dan and his class get a sense of the questions that are of interest to the students.

Begin by asking if any students have visited Federation Square in Melbourne and discuss the purpose and history of the space with them. Ask whether anyone had noticed the unique sandstone building façades. Show a picture of the façade that has not been marked.

Source: http://fedsquare.com/about/history-design

Ask students:

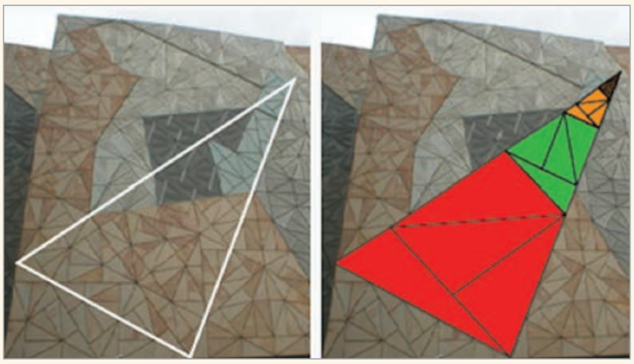
- *What's the first question that comes to mind?* (Is the pattern random? What shapes is the wall made up of? Are the triangles similar or congruent? How many triangles are on the wall? …on a section of the wall?)

Keeping control of the question

If students' questions sit outside of the area that we want them to work in, we have some choices to make. We can always add our own question to the list and ask for that question to be solved. But we probably want to minimise that, as students may lose interest in generating possible questions if they know that we'll always replace their questions with our own.

Students may have generated variations on the question that we had intended and if we can see that they will still use the concepts that we had intended, we could either let them answer their own question or ask them to answer our question and then reflect on whether or not their question was also answered in the process.

Show students the following images:



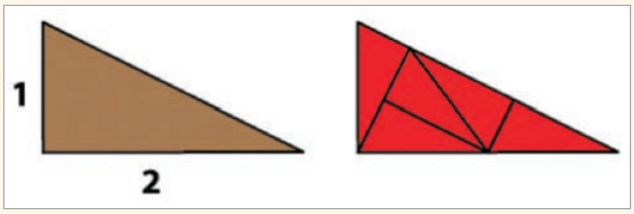
Used with permission and adapted from Polster B & Ross M (2012) 'Federation Square's hidden gem', *The Age*, 16 July edition, retrieved from http://www.qedcat.com/tempMMindex.html

Then ask the following questions:

- *How is this pattern an example of congruent triangles inside a similar triangle?*

- *What might you need to work out to answer that question? What information is helpful? What information is not useful? What extra information do you need? What information can you reasonably infer?* (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

To challenge students' thinking, you could ask them to identify congruent triangles and then find a larger triangle that contains 5 congruent triangles.



This triangular piece can be divided into 5 congruent triangles, similar to the original triangle.

- Construct a right angled triangle with the given proportions and divide it into 5 congruent triangles. (This will need to be large to allow for two more such divisions.)
- Divide each of these five triangles into five and then each of those into five to recreate the tiling.

This process can be applied to other figures.

An extended task relating to the tiles in Federation Square can be found in the Mathematical Association of South Australia (MASA) publication, ***Stage 1 Folio Tasks – SACE*** (available at http://www.masanet.com.au/publications-booklet/).

Example 9: Square partitions

ACMMG244 ◆
Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

Questions from the BitL tool

Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

This problem is from **Dan Meyer's blog: 101 questions**. It contains a short animation that can be presented to students along with the question: *What's the first question that comes to mind?*

In this example the question is suggested to the students, but they could certainly extend the question by asking their own 'What if…?' question in relation to changing the original equally divided square.

This problem is about area but also the use of similarity in relation to solving problems involving 2D shapes. Refer to the notes on the website to assist the writing of prompt questions. Also note that other users of the blog make comments about using the problems from this site with their students.

The link to this problem on Dan Meyer's blog is: http://www.101qs.com/72-square-partitions

Example 10: Trapezium four

ACMMG244 ◆

Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

In this activity from the NRICH website, students are asked to consider whether the challenges posed about the 4 triangular areas created by the diagonals of a trapezium are possible or not.

The link to this problem on the NRICH site is: http://nrich.maths.org/4960

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Connections between 'geometric reasoning' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use geometric reasoning as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 10/10A	
Whilst working with geometric reasoning, connections can be made to:	**How the connection might be made:**
Solve right-angled triangle problems including those involving direction and angles of elevation and depression. ACMMG245	Refer to: **Example 5: Greater squares.**
Define rational and irrational numbers and perform operations with surds and fractional indices. ACMNA264	Refer to: **Example 5: Greater squares.**

Making connections

We know that when students meet a concept frequently, and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individuals or small groups of students.

Year 10 AC Mathematics focusses on students solving problems and proving conjectures by applying logical reasoning. In solving these problems they draw on the content knowledge and skills they have developed through the sub-strands of 'Shape', 'Using units of measurement' and 'Geometric reasoning' at earlier levels. As well as consolidating their understanding in 'Measurement and geometry', they are developing confidence in the communication, representation and recording of a formal proof consistent with mathematical protocols.

‘Geometric reasoning’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with geometric reasoning.

Estimate/measure/compare/identify/classify using geometrical reasoning ◆
In Foundation to Year 2 through the ‘shape’ content descriptions, students sort/recognise/name/describe/draw/classify. In Year 3 to Year 5 students estimate/measure/compare/identify/classify using geometrical reasoning.

Investigate and generalise using geometrical reasoning ◆ ◆
In Year 6 to Year 8 students mostly investigate and solve.

Solve and prove using geometrical reasoning ◆
In Year 9 and Year 10 students mostly solve and prove by applying logical reasoning.

Year level	‘Shape’ content descriptions from the AC: Mathematics
Foundation ◆	Students sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment. ACMMG009
Year 1 ◆	Students recognise and classify familiar two-dimensional shapes and three-dimensional objects using obvious features. ACMMG022
Year 2 ◆	Students describe and draw two-dimensional shapes, with and without digital technologies. ACMMG042
Year 2 ◆	Describe the features of three-dimensional objects. ACMMG043
Year level	**‘Geometric reasoning’ content descriptions from the AC: Mathematics**
Year 3 ◆	Students identify angles as measures of turn and compare angle sizes in everyday situations. ACMMG064
Year 4 ◆	Students compare angles and classify them as equal to, greater than or less than a right angle. ACMMG089
Year 5 ◆	Students estimate, measure and compare angles using degrees. Construct angles using a protractor. ACMMG112
Year 6 ◆ ◆	Students investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Students use results to find unknown angles. ACMMG141
Year 7 ◆	Students identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal. ACMMG163
Year 7 ◆ ◆	Students investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning. ACMMG164
Year 7 ◆ ◆	Students demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral. ACMMG166
Year 7 ◆	Students classify triangles according to their side and angle properties and describe quadrilaterals. ACMMG165
Year 8 ◆	Students define congruence of plane shapes using transformations. ACMMG200
Year 8 ◆	Students develop the conditions for congruence of triangles. ACMMG201
Year 8 ◆ ◆	Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning. ACMMG202
Year 9 ◆ ◆	Students use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar. ACMMG220

Year level	‘Geometric reasoning’ content descriptions from the AC: Mathematics *continued*
Year 9 ◆	Students solve problems using ratio and scale factors in similar figures. ACMMG221
Year 10 ◆	Students formulate proofs involving congruent triangles and angle properties. ACMMG243
Year 10 ◆	Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes. ACMMG244
Year 10A ◆	Students prove and apply angle and chord properties of circles. ACMMG272

Numeracy continuum: Using spatial reasoning	
End Foundation	Visualise 2D shapes and 3D objects: sort and name simple 2D shapes and 3D objects.
End Year 2	Visualise 2D shapes and 3D objects: identify, sort and describe common 2D shapes and 3D objects.
End Year 4	Visualise 2D shapes and 3D objects: visualise, sort, identify and describe symmetry, shapes and angles in the environment.
End Year 6	Visualise 2D shapes and 3D objects: visualise, sort, describe and compare the features of objects such as prisms and pyramids in the environment.
End Year 8	Visualise 2D shapes and 3D objects: visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects.
End Year 10	Visualise 2D shapes and 3D objects: visualise, describe and analyse the way shapes and objects are combined and positioned in the environment for different purposes.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Each time we refer to one of the NRICH resources we have provided you with a link to that activity.

Dan Meyer's blog: 101 questions

http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by Office for Education and Early Childhood

Excluded from NEALS