Linear and non-linear relationships: Year 10/10A

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together

www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Linear and non-linear relationships' from Year 7 to Year 10A** (see page 15)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with linear and non-linear relationships:

- Plotting and sketching skills and strategies
- Connection between numerical, algebraic and graphical representations and methods
- Strategies for solving equations
- Using digital technologies.

What the Australian Curriculum says about 'Linear and non-linear relationships'

Content descriptions

Strand | Number and algebra.

Sub-strand | Linear and non-linear relationships.

Year 10 ◆ | ACMNA235
Students solve problems involving linear equations, including those derived from formulas.

Year 10 ◆ ◆ | ACMNA236
Students solve linear inequalities and graph their solutions on a number line.

Year 10 ◆ ◆ ◆ | ACMNA237
Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology.

Year 10 ◆ | ACMNA238
Students solve problems involving parallel and perpendicular lines.

Year 10 ◆ ◆ | ACMNA239
Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate.

Year 10 ◆ | ACMNA240
Students solve linear equations involving simple algebraic fractions.

Year 10 ◆ ◆ | ACMNA241
Students solve simple quadratic equations using a range of strategies.

Year 10A ◆ | ACMNA270
Students solve simple exponential equations.

Year 10A ◆ | ACMNA267
Students describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations.

Year 10A ◆ ◆ | ACMNA268
Students apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation.

Year 10A ◆ | ACMNA269
Students factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts.

Year level descriptions

Year 10 ◆ | Students find unknowns in formulas after substitution.

Year 10 ◆ | Students make the connection between equations of relations and their graphs.

Year 10 ◆ | Students use a range of strategies to solve equations.

Year 10 ◆ ◆ | Students use algebraic and graphical techniques to find solutions to simultaneous equations and inequalities.

Achievement standards

Year 10 ◆ | Students solve problems involving linear equations and inequalities.

Year 10 ◆ | Students solve simple quadratic equations and pairs of simultaneous equations.

Year 10 ◆ | Students make the connections between algebraic and graphical representations of relations.

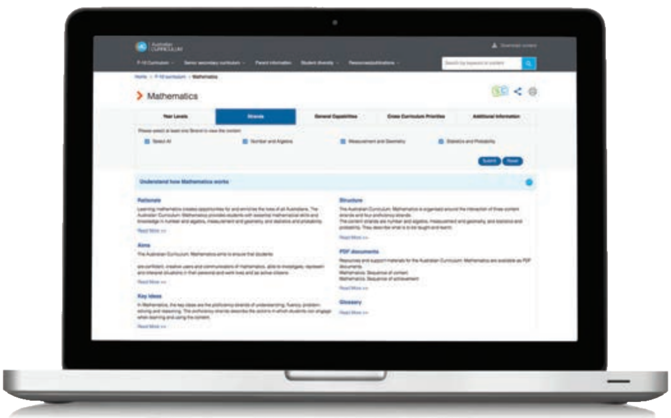
Year 10 ◆ | Students recognise the relationships between parallel and perpendicular lines.

Year 10 ◆ | Students find unknown values after substitution into formulas.

Numeracy continuum

Recognise and use patterns and relationships

End of Year 10 ◆ | Students explain how the practical application of patterns can be used to identify trends. (Recognise and use patterns and relationships: Linear and non-linear relationships).



Source: ACARA, Australian Curriculum: Mathematics

Working with Linear and non-linear relationships

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 10/10A 'Linear and non-linear relationships'

In Year 7 students plot given coordinates and make observations about where points lie on a straight line or otherwise. Students make the step from solving problems that have been expressed as number sentences, to solving simple linear (algebraic) equations.

In Year 8 students continue to plot coordinates on a Cartesian plane, but now they are expected to generate coordinates from linear equations. This will be done with and without digital technologies. Developing this skill enables students to explore the use of graphical techniques as well as algebraic methods for solving simple linear equations.

In Year 9 students continue to solve linear equations graphically, but they develop efficiency through understanding that a linear graph can be sketched from plotting just two points. Students investigate gradients of linear graphs and develop techniques for calculating the distance between two points and the midpoint of lines on a Cartesian plane.

In Year 10 students use their understanding of gradients to solve problems involving parallel and perpendicular lines. They continue to solve linear equations that now may involve simple algebraic fractions. Students transfer their skills from solving linear equations to solving linear inequalities. They apply their ability to plot linear equations (with and without digital technologies) to solve simultaneous equations graphically and explore the use of algebraic technique. Quadratic equations, equations of circles, and exponents are introduced and students explore the connection between the algebraic and graphical representation. At this stage, students are only expected to sketch and solve equations of quadratics.

In Year 10A students sketch parabolas, circles and exponential functions, and solve simple exponential equations. They investigate the features of graphs and develop an understanding of the connection between the graphical and algebraic representation of polynomials so that they are able to sketch a range of curves and describe the features of the curves from looking at their equation. Factorisation of quadratics extends to non-monic quadratic equations.

- Developing linear relationships from practical and real-life situations, supports students in gaining a conceptual understanding that the straight line is representing a relation between two varying quantities. Once this understanding has been established, concepts of coordinates, equations, slope and Y-intercept have some significance to the learner; particularly if they have encountered linear relationships in a range of different contexts.
- As well as providing situations where linear relationships occur, it is valuable to identify non-linear relationships as well, so that learners realise that not all relationships will be linear. In the James Nizam sculpture of the stacking goblets (see 'Example 8: Stacking sculpture' in the *Linear and non-linear relationships: Year 8* narrative), the number of goblets in each level is a linear relationship (Level 1 has 1 goblet, Level 2 has 2 goblets, etc) but the number of goblets in the entire sculpture as it grows is not (after one level the structure has 1 goblet, after two levels it has 3, after three levels it has 6 …).

Engaging learners

Classroom techniques for teaching Linear and non-linear relationships

Grade, pitch and slope

Grade, pitch, and slope are important components in landscape and garden design, building and landscape architecture and for engineering and aesthetic design factors. In environmental design, drainage, slope stability, navigation by people and vehicles, complying with building codes and design integration are all aspects of slope considerations.

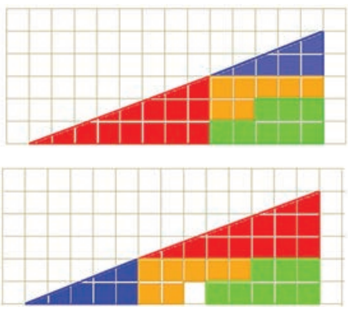
'Jobs that use maths slopes' can be found at: https://sciencing.com/jobs-use-math-slopes-7231483.html

Source: Alicia Rudnicki (2017) 'Jobs that use maths slopes', *Sciencing*, Leaf Group Education

Triangle puzzler

Students can construct this intriguing puzzle. The answer as to where the extra unit came from can be explained by considering the slope of the **two** lines that make up the 'hypotenuse' of the larger triangle.

Placing a ruler along this 'side' reveals that it is not a straight line and that the blue and red hypotenuses do not have the same slope ($\frac{3}{8} \neq \frac{2}{5}$).



See 'Example 3: Triangle puzzler – gradient' in the *Linear and non-linear relationships: Year 9* narrative for further information.

Baldwin Street

According to Guinness World Records, Baldwin Street in Dunedin, New Zealand is the steepest street in the world.

The street has the following properties:

- 161.2m in length with an elevation gain of 47.22m
- An average gradient of 1 in 3.41
- A maximum gradient of 1 in 2.86.

It is the site of the annual Cadbury Jaffa Race, where up to 25,000 Jaffas are released at the top of the incline and bounce their way in a sea of orange to the bottom. The 2013 Cadbury Jaffa Race down Baldwin Street video can be found at: https://www.youtube.com/watch?v=zYZCcABDuWE

Cyclists from all over the world travel there in order to conquer the climb. Watch the strategies of the riders as they begin to struggle. Why does this work?

Cycling up the steepest street in the World in New Zealand video can be found at: https://www.youtube.com/watch?v=a9uO3KCJImA

Baldwin St World Record video can be found at: https://www.youtube.com/watch?v=hhXRZjcgKgY

Source: Rudy Pospisil (2017) *Baldwin St World Record*, https://www.youtube.com/watch?v=hhXRZjcgKgY

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–3)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **write the equation of a line in algebraic form** themselves. They need to receive this information in some way. However, it *is* possible my students can be challenged with questions that will result in them identifying the **relationship between the coordinates**, so I don't need to instruct that information.

At this stage of development, students can **develop an understanding of the patterns and relationships that can exist on a coordinate system**. When teachers provide opportunities for students to **recognise, create and describe relationships between coordinates of points**, they require their students to generalise. Telling students rules and relationships removes this natural opportunity for students to make conjectures and verify and apply connections that they notice. Using questions such as the ones described here, supports teachers to replace 'telling' the students information, with getting students to notice for themselves.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular rule or theorem during the current unit of work, then it probably is quicker to tell them the rule and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the rule is a false economy of time.

When we challenge our students to **establish a theorem**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Parallel and perpendicular lines Students solve problems involving parallel and perpendicular lines.	ACMNA238 ◆
Example 2: Perpendicular bisectors Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology. Students solve problems involving parallel and perpendicular lines.	ACMNA237 ◆◆◆ ACMNA238 ◆
Example 3: Handshakes Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate. Students solve simple quadratic equations using a range of strategies	ACMNA239 ◆ ◆ ACMNA241 ◆ ◆

Example 1: Parallel and perpendicular lines

ACMNA238 ◆
Students solve problems involving parallel and perpendicular lines.

Questions from the BitL tool

Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?

Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about the equations of a linear relationship, we can challenge students to recognise the relationships between the coordinates of the points for themselves, by ***asking*** questions.

This series of NRICH explorations allow students to explore digital activities to create their own knowledge or consolidate their conceptual understanding of parallel and perpendicular lines and their equations:

Parallel lines

This digital activity allows students to move points to create a pair of lines, displaying their equations so students can notice the special features when the pair are parallel.

The link to this problem on the NRICH website is: http://nrich.maths.org/5609

Perpendicular lines

This digital activity allows students to move points to create a pair of lines, displaying their equations so students can notice the special features when the pair are perpendicular.

The link to this problem on the NRICH website is: http://nrich.maths.org/5610

Enclosing squares

This activity requires students to apply their knowledge of perpendicular lines and their equations using technology.

A free graphics calculator can be found at: https://www.desmos.com/calculator

The link to this problem on the NRICH website is: http://nrich.maths.org/763

Surprising transformations

Given four different transformations and the equations of two lines, the task is to identify the order that transforms one line into the other.

The link to this problem on the NRICH website is: http://nrich.maths.org/6544

Using a series of activities such as these and having the students work in groups, allows for differentiation of the task and the opportunity for them to share their thinking as well as learn from others.

Not every group needs to do every challenge. Rating the challenges from ‘one chilli’ to ‘four chillis’ allows groups to choose their own level of challenge. Groups can give feedback to the whole class about their processes and provide a generalisation of their learning (not necessarily the solution).

Example 2: Perpendicular bisectors

ACMNA237 ◆ ◆ ◆

Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology.

ACMNA238 ◆

Students solve problems involving parallel and perpendicular lines.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the equations of a linear relationship, we can challenge students to recognise the relationships between the coordinates of the points for themselves, by ***asking*** questions.

Instruct the students to draw any triangle on their page, and ask:

Is it possible to draw a perfect circle around that triangle so that all the vertices lie on the circle? Try it. How did you go?

This activity explores a way for the students to discover how to draw that circle for any triangle.

Begin this activity by plotting points A (-5, 1), B (3, 9) and C (3, -3) using a scale which results in the triangle being about ½ page in size. Ask students:

- *How might you find the midpoint, M, of the side AB? How many different ways can you do this?* (Students might count squares across and up to be the same for each half, measure, fold the side in half or even use a formula.)

Find the line that passes through M and is at right-angles to AB. This is called the **perpendicular bisector**. Ask students:

- *Why might it be given that name?* (Perpendicular means upright or 90°, and bisect means cut in half.)
- *How might you draw this line? How many different ways can you do this?* (Students might fold the side in half and crease the fold to find the line. They may start at the midpoint (M) they found and make sure they have a perpendicular slope.)

Find the perpendicular bisector of AC. You might do this using the same or a different method. Find the point of intersection, X, of the perpendicular bisectors you have now found. Ask students:

- *What do you notice about the distance between X and A, X and B, and X and C? Convince me.* (X was the same distance from all 3 points.)

Draw all the points and lines you have found on another set of axes. Using X as the centre, draw a circle passing through point A. Ask students:

- *What do you notice?*
- *Why might you have suspected this when you found the distances earlier?* (X was the same distance from all 3 points, so X can be the centre and the common distance is the radius of the circle.)

This circle is called the **circumcircle** of the triangle ABC (see Figure 1). Ask students:

- *Why might it be given that name?* ('Circum' – as in *circum*ference and *circum*navigate means to 'go around'. This is the circle that 'goes around' the triangle.)

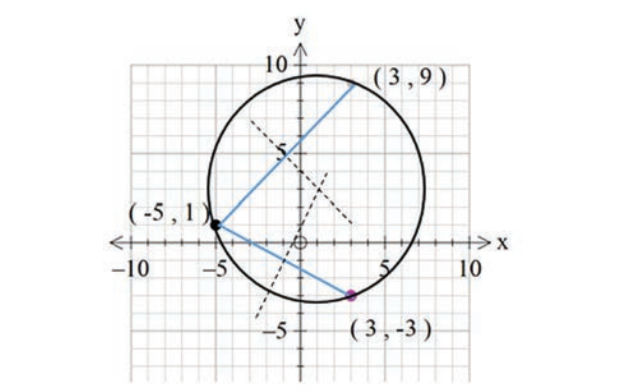


Figure 1

- *Use what you have learned to draw the circumcircle for the triangle you drew at the start.*

(A variation of this activity also appears in the *Linear and non-linear relationships: Year 9* narrative.)

Example 3: Handshakes

ACMNA239 ◆ ◆
Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate.

ACMNA241 ◆ ◆
Students solve simple quadratic equations using a range of strategies.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students how to use a table of values to plot a line, we can challenge students to go from the unknown to the known for themselves, by ***asking*** questions.

This activity is from the NRICH website.

Students collect data from a number of mathematicians shaking hands with each other. They can use a range of enactments, representations and models to explore this. The relationship between the number of people and the number of handshakes is not linear. The challenge is to identify and describe this non-linear relationship.

This is an opportunity for students to fit models to data using a graphics calculator or Excel spreadsheet and relate the relationship to the context of shaking hands.

As all *n* people shake everyone else's hand, there will be *n x* (*n*-1) shakes, except that each hand shake will be counted twice, giving ½ *n x* (*n*-1) shakes.

Draw a connected scatterplot and fit a trendline, a quadratic (or polynomial degree 2), showing the equation (see Figure 2).

No. of people	No. of shakes
2	1
3	3
4	6
5	10
6	15

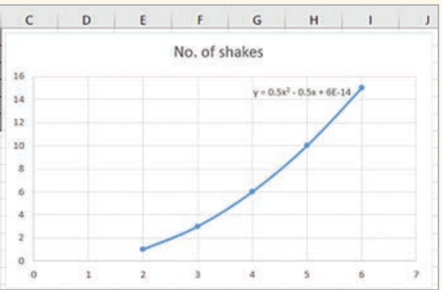
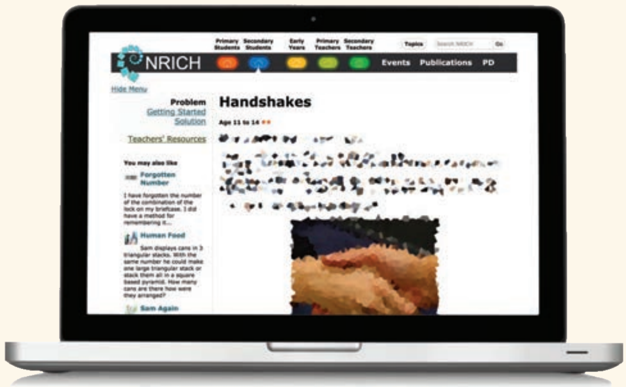


Figure 2

The relationship using technology is:
$y = 0.5x^2 - 0.5x + 6E\text{-}14$.

This is an opportunity to discuss with students that 6E-14 is 6×10^{-14} is extremely small in this negligible.

The remaining expression $0.5x^2 - 0.5x$ is equivalent to ½ *x*(*x*-1).



The link to the problem on the NRICH website is: http://nrich.maths.org/6708

(A variation of this activity also appears in the *Linear and non-linear relationships: Year 9* narrative.)

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 4–6)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem-solving examples	
Example 4: Parabolic patterns Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate.	ACMNA239 ◆ ◆
Example 5: Intersections – simultaneous equations Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology.	ACMNA237 ◆
Example 6: Parabol-arc Students apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation.	ACMNA268 ◆ ◆

Example 4: Parabolic patterns

ACMNA239 ◆ ◆
Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take for themselves, by ***asking*** questions.

In this activity from the NRICH website, students are presented with graphed parabolas and challenged to identify the equations of a range of quadratics.

The link to the problem on the NRICH website is: https://nrich.maths.org/773

Example 5: Intersections – simultaneous equations

ACMNA237 ◆
Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take for themselves, by ***asking*** questions.

In this activity from the NRICH website, students are provided with the provocation that two systems of linear equations that appear very similar, have very different solutions.

The link to the problem on the NRICH website is: http://nrich.maths.org/5438

Example 6: Parabol-arc

ACMNA268 ◆ ◆
Students apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take for themselves, by ***asking*** questions.

This investigation explores a graphical method for determining the roots of quadratic equations of the form $x^2 - kx + c = 0$, as an alternative to the use of the quadratic formula or technology.

Consider the function: $y = x^2 - 4x + 3$

Draw the graph of the function for $-1 \leq x \leq 4$, indicating the axes intercepts, vertex and axis of symmetry. On the same set of axes, draw a circle with its diameter PQ through the points P(0,1) and Q(4,3).

This circle will be referred to as the parabol-arc of the quadratic function. Ask students:

- *What do you notice? What does this make you think?* (Students notice that the circle intersects with the parabola on the x-axis. These values for x are the roots of the equation $x^2 - 4x + 3 = 0$.)

Suggest that this might be another way to solve quadratic equations:

- *How could we test this idea?*

Interpret

What is your conjecture? What might you need to work out to form, verify or prove your conjecture? What information is helpful? What information is not useful? What extra information do you need? What information can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task. A possible conjecture might be:

For a quadratic function of the form $y = x^2 - kx + c$, the circle constructed on the diameter of PQ where P(0,1) and Q(k,c) will pass through the zeros of the equation: $x^2 - kx + c = 0$.

It may not be worded in formal mathematical language and students may have formed different conjectures. As long as the conjecture is consistent with their observations, it is important to encourage and support them in their efforts to prove or disprove the conjecture. If it is not consistent, use questioning to clarify their thinking.)

Model and plan

Do you have an idea? How might you start? Would it help if you thought about a similar/another quadratic where this might work? (Ask students to speak to someone who they think is being a good problem-solver today and ask them to show what they are trying. As a prompt, give another example of a quadratic with integer roots, such as $x^2 - 5x + 6$. Ask the student why they think you chose that quadratic.))

Solve and check

Would this work for all quadratics? What are the conditions for it to work? Can you generalise? Can you prove your conjecture? How did you disprove it? What if I told you that for $x^2 - 5x + 6$ you need to draw the diameter P(0,1) and Q(5,6)? What would that make you think? What might you do to check your thinking? (A formal algebraic proof for this conjecture can be achieved using quadratic theory.)

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Provide intellectual stretch by challenging students to consider:

Quadratic functions of the form:

i) $y = x^2 + bx + c$, where b and $c > 0$

ii) $y = -x^2 + bx - c$, where b and $c > 0$

iii) $y = ax^2 + bx + c$

Is it possible to find more than one parabola that has a parabol-arc with a diameter of PQ where P(0,1) and Q(4,3)? Justify your answer.

A version of this challenge with the scaffolding for a formal proof, can be found at MASA in 'Stage 1 Folio Tasks' – Topic 10, 10.3 The Parabol-arc: https://masanet.wildapricot.org/resources/Publications/Current%202017%20Publications_Booklet..pdf

(This activity also appears in the *Patterns and algebra: Year 10/10A* narrative.)

Connections between 'Linear and non-linear relationships' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use linear and non-linear relationships as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 10/10A	
Whilst working with Linear and non-linear relationships, connections can be made to:	**How the connection might be made:**
Students prove and apply angle and chord properties of circles. ACMMG272	Refer to: **Example 2: Perpendicular bisectors**
Students expand binomial products and factorise monic quadratic expressions using a variety of strategies. ACMNA233	Refer to: **Example 6: Parabol-arc**
Students graph simple non-linear relations with and without the use of digital technologies and solve simple related equations. ACMNA296	Refer to: **Example 3: Handshakes**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Linear and non-linear relationships’ from Year 7 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Linear and non-linear relationships:

Plotting and sketching skills and strategies ◆
In Year 7 to Year 9 students focus mostly on plotting and sketching skills and strategies.

Connection between numerical, algebraic and graphical representations and methods ◆
In Year 8 to Year 10A students focus mostly on using and comparing multiple representations and methods.

Strategies for solving equations ◆
In Years 10/10A students focus mostly on strategies for solving equations.

Using digital technologies ◆
In Year 8 to Year 10A students use digital technologies to identify and represent functions and relations and also solve related problems.

Year level	‘Linear and non-linear relationships’ content descriptions from the AC: Mathematics
Year 7 ◆	Students solve simple linear equations. ACMNA179
Year 7 ◆	Students investigate, interpret and analyse graphs from authentic data. ACMNA180
Year 8 ◆ ◆	Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. ACMNA193
Year 8 ◆ ◆	Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution. ACMNA194
Year 9 ◆ ◆	Students find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software. ACMNA214
Year 9 ◆ ◆	Students find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software. ACMNA294
Year 9 ◆ ◆	Students sketch linear graphs using the coordinates of two points and solve linear equations. ACMNA215
Year 9 ◆ ◆ ◆	Students graph simple non-linear relations with and without the use of digital technologies and solve simple related equations. ACMNA296
Year 10 ◆	Students solve problems involving linear equations, including those derived from formulas. ACMNA235
Year 10 ◆ ◆	Students solve linear inequalities and graph their solutions on a number line. ACMNA236
Year 10 ◆ ◆ ◆	Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology. ACMNA237
Year 10 ◆	Students solve problems involving parallel and perpendicular lines. ACMNA238
Year 10 ◆ ◆	Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate. ACMNA239
Year 10 ◆	Students solve linear equations involving simple algebraic fractions. ACMNA240
Year 10 ◆ ◆	Students solve simple quadratic equations using a range of strategies. ACMNA241
Year 10A ◆	Students solve simple exponential equations. ACMNA270
Year 10A ◆	Students describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations. ACMNA267

Year 10A ◆ ◆	Students apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation. ACMNA268
Year 10A ◆	Students factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts. ACMNA269

Numeracy continuum: Recognising and using patterns and relationships	
End Year 6	Identify and describe pattern rules and relationships that help to identify trends.
End Year 8	Students identify trends using number rules and relationships.
End Year 10	Students explain how the practical application of patterns can be used to identify trends.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Visual patterns

http://www.visualpatterns.org/

This website has multiple visual patterns that students can consider and describe in a way that makes sense to them. Students will have multiple interpretations of each image, promoting multiple ways of visualising, solving problems and stimulating dialogue. They can be used as lesson starters.

Scootle

https://www.scootle.edu.au/ec/p/home

This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.

reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

Numeracy in the News

http://www.mercurynie.com.au/mathguys/mercury.htm

Numeracy in the News is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.

TIMES modules

http://schools.amsi.org.au/times-modules/

TIMES modules are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.

Top drawer teachers – resources for teachers of mathematics (statistics)

http://topdrawer.aamt.edu.au/Statistics

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.

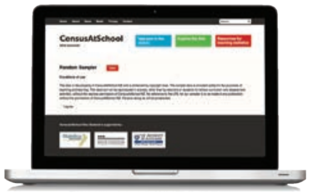
Double Helix Extra

https://blog.doublehelix.csiro.au/

This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.

CensusAtSchool NZ

http://new.censusatschool.org.nz/tools/random-sampler/



CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics.
It aims to:

- *'foster a positive attitude to statistics through using data that is both relevant and real*
- *improve understanding of a data gathering process, its purposes and benefits to society*
- *provide access to large and meaningful multivariate data sets*
- *encourage effective IT teaching and learning*
- *enhance the process of statistical enquiry across the curriculum.'*

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by the Learning Improvement division