# Chance: Year 10/10A

## MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together


www.acleadersresource.sa.edu.au


Government of South Australia
Department for Education

# Contents



## Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

**More information about 'Transforming Tasks':**
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

**Bringing it to Life (BitL) key questions are in bold orange text.**

*Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.*

**More information about the 'Bringing it to Life' tool:**
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Chance' from Year 1 to Year 10A** (see page 24)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with Chance:

- Identify and order chance events
- Identify, describe and represent sample spaces
- Randomness and variation
- Observed frequencies and expected probabilities
- Language.

# What the Australian Curriculum says about ‘Chance’

## Content descriptions

**Strand** | Statistics and probability.

**Sub-strand** | Chance.

**Year 10** ◆ ◆ ◆ | ACMSP246
Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**Year 10** ◆ | ACMSP247
Students use the language of ‘if … then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language.

**Year 10A** ◆ | ACMSP277
Students investigate reports of studies in digital media and elsewhere for information on their planning and implementation.

## Year level descriptions

**Year 10** ◆ | Students determine probabilities of two- and three-step experiments.

**Year 10** ◆ | Students investigate independence of events.

**Year 10** ◆ ◆ | Students interpret and evaluate media statements.

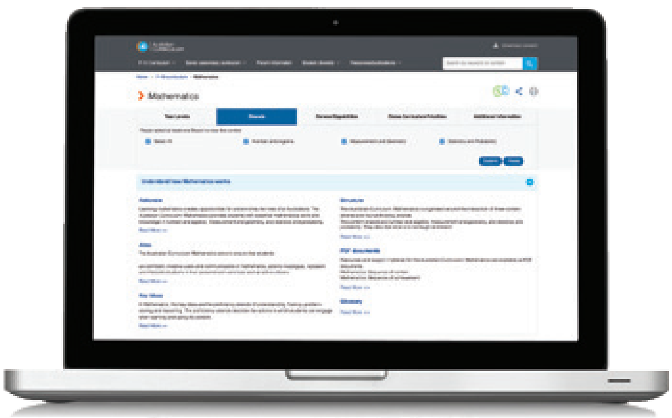
## Achievement standards

**Year 10** ◆ ◆ | Students list outcomes for multi-step chance experiments and assign probabilities for these experiments.

## Numeracy continuum

**Interpreting statistical information**

**End of Year 10** ◆ ◆ | Students explain the likelihood of multiple events occurring together by giving examples of situations when they might happen (Interpreting statistical information: Interpret chance events).



Source: ACARA, Australian Curriculum: Mathematics

# Working with Chance

**Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum**

## What we are building on and leading towards in Year 10/10A 'Chance'

**In Year 7**, in line with students' developing ability to change between fractions, decimals and percentages, they would now be expected to change between these representations for any amount, rather than just for common quantities. Students in Year 7 are introduced to the term 'sample space'.

**In Year 8** students use two-way tables to represent possibilities of experiments with two steps. Students are also introduced to the use of the terms 'at least', 'and' and 'or' in relation to chance events. They use two-way tables and Venn diagrams to calculate probabilities satisfying the 'at least', 'and' and 'or' criteria. Students are also introduced to the term 'complementary events' and use the sum of probabilities to solve problems.

**In Year 9** students continue to work with two-step chance experiments, but now they consider the probabilities with and without replacement. To support this thinking, students learn to create tree diagrams and use such diagrams to assist in solving problems. Students estimate probabilities by calculating relative frequencies for collected or given data.

**In Year 10/10A** students describe the results of two- and three-step experiments, both with and without replacement. They investigate the concept of independence and conditional statements.

Even at this stage, **it is still important for students to conduct the experiments themselves** if possible, so they can compare relative frequencies with their theoretical calculations. While the curriculum does not require students to conduct their own experiments, the importance of using manipulatives supports students dealing with abstract concepts in a concrete way, in a context they are familiar with.

There is a clear emphasis at this stage, that **students can describe and articulate their learning using the language of probability**. Once students understand the concepts, teachers can make links to the mathematical terminology and formal definitions and connect the mathematical names to the colloquial and everyday use of the language, such as independent/dependent, mutually exclusive, disjoint and complementary. Students can demonstrate their understanding of conditional probability by using, 'Now I know that …' and 'If … then …' statements. We need to signal that they are using conditional probability when they use these statements, and also when they deal with dependent events and change probabilities once they have more information.

**Students need to have a range of methods for identifying and describing compound events** that are not necessarily equally likely (eg systematic lists, tables, tree or lattice diagrams). As teachers, we need to ensure students are making deliberate choices that fit the context of the problem by asking them to justify their choice of representation and encouraging them to modify standard representations, or even create their own, to support their thinking.

**We have always encouraged students to identify and share their strong intuitions about the likelihood of events occurring** which were often accurate for simple problems. We should still continue to do this. Now they will be dealing with large, complicated sample spaces where intuition can be misleading, and it is not practical to list or represent every outcome. This helps students appreciate the need for a more theoretical consideration of probability. It is common for people to have unsupported intuitions and beliefs about situations involving variation and chance. To be critical consumers of information from digital media and other sources, we must be aware that these intuitions can be misleading and can used to persuade and exploit us. As teachers, we need to show students examples of how this can happen and how we can be better informed with a conceptual understanding of the theory of probability.

# Engaging learners

**Classroom techniques for teaching Chance**

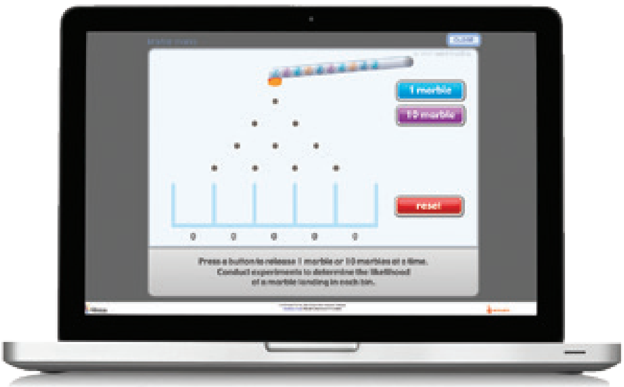
**Chance provides opportunities to evoke curiosity and wonder in our students.** Many genuine contexts allow students to draw on prior knowledge and intuition to engage them in their learning. Students personalise their learning when they make predictions about outcomes, design and conduct their own experiments and make meaningful inferences from what they have discovered.

## Using relative frequency

Many websites such as Cambridge HOTmaths, Scootle, NRICH and the National Library of Virtual Manipulatives have a range of digital activities which students can use as random generators to experiment and collect relative frequencies to explore probabilities.

In most games, skill affects your chance of winning. However even in games where skill plays no part, each outcome is not always equally likely. In ***HOTmaths: Using relative frequency*** marbles fall from a tube and bounce through obstacles to land in 1 of 5 bins. By using the digital activity, students can calculate the relative frequencies for the bins the marbles land in. Students then gather data by conducting several experiments with different numbers of marbles and comment on the relative frequencies, as the number of trials in the experiment increases.

The digital activity can be found at:
http://tlf.dlr.det.nsw.edu.au/learningobjects/Content/L10574/object/



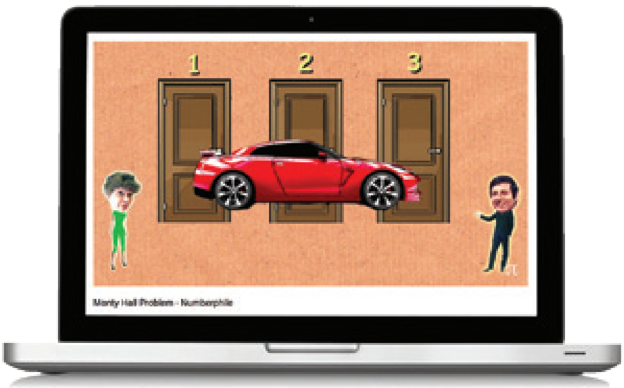

Source: *HOTmaths: Using relative frequency*, HOTmaths – The Le@rning Federation, 2009

## Monty Hall problem

Famous problems such as the ***Monty Hall problem*** are counterintuitive and learners are quite often motivated by disbelief to explore the problem. Loosely based on the American television game show *Let's Make a Deal*, it is named after its original host, Monty Hall and focusses on the probability of opening the correct door to win the coveted prize.

Numberphile's examination of this problem features Lisa Goldberg, an adjunct professor in the Department of Statistics at the University of California, Berkeley USA.

The video can be found at:
https://www.youtube.com/watch?v=4Lb-6rxZxx0




Source: *Monty Hall Problem*, Numberphile, 2014

# From tell to ask

**Transforming tasks by modelling the construction of knowledge (Examples 1–4)**

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

*What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?*

For example, no amount of reasoning will lead my students to **create the names 'mutually exclusive', 'complementary' and 'independent events'** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged **with questions that will result in them identifying the patterns that simplify finding the probability for different types of events**, so I don't need to design and instruct the details of the investigation for them.

At this stage of development, students can develop an understanding of chance, randomness and variation through conducting their own experiments. When teachers provide opportunities for students to predict, identify, describe and represent the outcomes of the experiments and compare them to theoretical expectations, they require their students to generalise. Telling students the laws of probability, removes this natural opportunity for students to make conjectures and verify connections that they notice.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to fully design a probability investigation, or set tasks that we want students to experience, rather than ask a question (or a series of questions) and support them to planning the stages of the investigation for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students use a specific set of skills, knowledge and procedures during the current unit of work, then it probably is quicker to tell them what to do. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the formulae is a false economy of time.

## Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

| From tell to ask examples | |
|---|---|
| **Example 1: What are your chances now?**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | **ACMSP246** ◆ ◆ ◆<br>**ACMSP247** ◆ |
| **Example 2: In a Box**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | **ACMSP246** ◆ ◆ ◆<br>**ACMSP247** ◆ |
| **Example 3: Who picks their card first?**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | **ACMSP246** ◆ ◆ ◆<br>**ACMSP247** ◆ |
| **Example 4: Are the Seven Dwarfs brothers?**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | **ACMSP246** ◆ ◆ ◆<br>**ACMSP247** ◆ |

# Example 1: What are your chances now?

**ACMSP246** ◆ ◆ ◆

Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆

Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Understanding** proficiency:

**What patterns/connections/relationships can you see?**

**Can you represent/calculate in different ways?**

**Reasoning** proficiency:

**In what ways can your thinking be generalised?**

**In what ways can you communicate?**

Instead of ***telling*** students about conditional probability, we can challenge students to recognise for themselves how sample spaces and hence probabilities change with more information, by ***asking*** questions.

Have you ever played the game 'Guess Who'? The aim of the game is for you to determine the identity of a mystery person by asking 'Yes' or 'No' questions to improve your chance of guessing their identity. This is a case where your questioning gives you more knowledge and reduces the number in the sample space, which increases the probability of you guessing correctly. Some people do not guess until the probability of being correct is 1.

Two people can play a similar game with a deck of playing cards. One player chooses a card from the pack. The other completes a table like the one shown in Figure 1. Students enjoy playing games and if they record the choices and results as they play, they are collecting data which will encourage them to think more deeply about features of the game and possible strategies.

Each turn involves calculating the probability of guessing it correctly, making a guess, and then asking a question that will only be answered with 'Yes' or 'No'. This continues until the player successfully guesses the card.

| Probability of guessing | Guess | Correct/Incorrect |
|---|---|---|
| $\frac{1}{52}$ | Ace of hearts | ✘ |
| Question: Is it a red card? Yes | | |
| $\frac{1}{25}$ as I know it is not the AH | 3 of diamonds | ✘ |
| Question: Is it a diamond? No | | |
| $\frac{1}{12}$ as I know it is a heart but not AH | 10 of hearts | ✘ |
| Question: Is it a picture card? Yes | | |
| $\frac{1}{3}$ as it is either JH, QH or KH | J of hearts | ✔ |

*Figure 1*

If students have difficulty determining the probabilities, let them use a pack of cards and discard the cards they eliminate as they receive more information.

Ask students:

- *Why is the probability always $\frac{1}{n}$?*

  However many possibilities it could be, you are trying to guess **the one** that your opponent picked from the pack.
- *What is the minimum number of questions you could ask to be sure you would know the card? What could these questions be? Is there another way?*
- *What questions might you ask so that the probability of guessing correctly would be $\frac{1}{15}$? Is there another way?*

**Backwards questions**

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

# Example 2: In a Box

**ACMSP246** ◆ ◆ ◆

Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆

Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Understanding** proficiency:

**What patterns/connections/ relationships can you see?**

**Can you represent/ calculate in different ways?**

**Reasoning** proficiency:

**In what ways can your thinking be generalised?**

**In what ways can you communicate?**

Instead of ***telling*** students about conditional probability, we can challenge students to recognise for themselves how sample spaces and hence probabilities change with more information, by ***asking*** questions.

In a problem such as this one from the NRICH website, it is difficult to predict whether picking two ribbons of the same colour or two different ones, would be equally likely. It is an example of having to consider the theoretical probabilities in this particular context. One might presume that having the same number of each colour would be fair, but again intuitions about chance can be misleading.


**The link to the problem on the NRICH website is: https://nrich.maths.org/919**

## Understanding the experiment more fully

When students first encounter a chance problem, ask them to describe the experiment with enough information so that a listener would be able to conduct it for themselves:

*'If I draw two ribbons from the box, what is the probability that both are red?'*

Require them to name a few of the possible outcomes besides the one they are interested in *(I could get (a red and blue) on a draw, but that is not what I am interested in)*. This encourages them to familiarise themselves with the experiment and think more deeply about the sample space before they try to quantify the probability. Even if they have solved the problem, ask them to show you using the box of ribbons, how and why that works. Ask them to draw a representation of the sample space. This encourages students to consider the experiment holistically and not seek 'quick' methods based on rules such as, '**and** means times', and, '**or** means add', which are often applied with no understanding.

# Example 3: Who picks their card first?

**ACMSP246** ◆ ◆ ◆

Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆

Students use the language of ‘if … then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Understanding** proficiency:

**What patterns/connections/relationships can you see?**

**Can you represent/calculate in different ways?**

**Reasoning** proficiency:

**In what ways can your thinking be generalised?**

**What can you infer?**

Instead of ***telling*** students about dependent events, we can challenge students to recognise the relationships between the events for themselves, by ***asking*** questions.

This is a game played by two or more students, where each player nominates the suit (hearts, diamonds, clubs or spades) that they would like to draw from the pack. Each player draws a card which everyone sees and is not replaced. The first player to draw their nominated suit is the winner.

Before each draw, the player must determine the probability of drawing their suit on that particular draw (eg if there have been 4 cards drawn in the game and 2 of them were hearts, there would be 48 (=52–4) cards left and 11 (=13–2) of them would be hearts, so my chances of getting a heart on the next draw are $\frac{11}{48} \approx 23\%$).

Discuss the following with your students:

- *Does it matter who goes first? How can we make that fair?*
- *Am I better to choose the same suit as someone else or a different one?*

  For the second person there are various probabilities depending on what the first person drew and whether the players chose the same suit:
  - 1st and 2nd choose different suits (1st didn’t get their suit and 2nd did) $\frac{3}{4} \times \frac{13}{51} = \frac{39}{204} \approx 19\%$
  - 1st and 2nd choose same suits (1st didn’t get their suit and 2nd did) $\frac{3}{4} \times \frac{12}{51} = \frac{36}{204} \approx 18\%$
- *Is it a long or short game?*

  Drawing a suit is generally not a long game, as the first player has a 1 in 4 chance of drawing their suit. You may wish to collect class data on game length and use your knowledge of statistics to inform your decisions in this context.
- *How could we make it a more interesting game? Will it be longer or shorter?*

  Students may wish to choose two options, such as a suit and a face value (eg a heart **or** a jack: 14 cards). Here is an opportunity to demonstrate understanding of mathematical language relating to chance, such as the ‘inclusive or’. They may choose a face value or range of values, a picture card, a colour (eg red **and** picture card: 6 cards). Allow students to make their own decisions about how the game could be changed. This is an opportunity to differentiate their learning, compare the length of the game for the different choices and relate this to theoretical probabilities.
- *Is it fair if players can choose different characteristics for their card? Is it fair if one chose picture cards and the other chose hearts? Are there any situations where this would be fair? In general, what has to be true?*

  Initially, the characteristics must have the same probability. The probability of a picture card is $\frac{12}{52}$, where hearts is $\frac{13}{52}$ and so the player choosing hearts would have a game advantage.

(A similar activity can be found on page 11 of the *Chance: Year 9* narrative.)

# Example 4: Are the Seven Dwarfs brothers?

**ACMSP246** ◆ ◆ ◆

Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆

Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Understanding** proficiency:

**What patterns/connections/ relationships can you see?**

**Can you represent/ calculate in different ways?**

**Reasoning** proficiency:

**In what ways can your thinking be generalised?**

**What can you infer?**

Instead of ***telling*** students about dependent events, we can challenge students to recognise the relationships between the events for themselves, by ***asking*** questions.

Consider a family that comprises of 7 children. If we assume that having a boy or a girl is equally likely, how unusual is it for a family with 7 children to have 7 boys? While having 7 children is unusual these days, what percentage of these families would have 7 boys? Guess.

When mathematicians have a difficult problem, they usually solve an easier one first. So, let's start with a smaller family with 4 children. Ask if anyone in the class knows a family with 4 children and collect some relevant information about that family (eg 3 boys and 1 girl, the girl is the youngest).

Discuss the following with your students:

- *How could we record that? What questions do you have?*

  Record their questions first and some of your own.
- *What percentage of families with 4 children would have 3 boys and 1 girl? In how many of these would the girl be the youngest?*
- *What percentage of families with 4 children would have 3 girls and 1 boy, with the boy being the youngest? Is that the same question?*
- *If you knew there were 3 boys and 1 girl, what is the probability that she was the youngest?*
- *What other combination would be equally likely? How many families have at least one girl?*

### Breaking down a big class inquiry

After identifying questions that arise in a class inquiry, considering them all at once can be overwhelming. If you want the class to all work on the same question, you can pick one and ask them to help you answer it. If not, this is a chance to differentiate learning. Write the questions on pieces of paper and ask groups to select one that interests them. If they finish quickly, ask them to select another or make up a difficult question for themselves.

If you consider 3 boys and 1 girl in any order:

- *How unusual do you think that is? How many different possible families are there? How could we represent that? Is there another way? Are they all equally likely? How could we calculate that?*

  By listing systematically or doing a tree diagram, students can establish that there are 16 different families (2 x 2 x 2 x 2) so the probability of each of these different families is $\frac{1}{16}$ = 6.25%.

  There are, however, 4 different families with 3 boys and 1 girl (4 x $\frac{1}{16}$ = 25%).

  In half of all possible different families, the girl is the youngest. ($\frac{1}{2} \times 16 = 8$).

  However, in only **one** of the families with 3 boys and 1 girl, is the girl the youngest.

  This is a **conditional probability** question: *'Given that there are 3 boys and 1 girl, what is the probability that the girl is the youngest?'*

  Students may have noticed the symmetry of the tree diagram, because boys and girls are equally likely, p(3B and 1G) = p(3G and 1B). Note if we are considering order, while p(BBBG) = p(GGGB), it is true for every one of the 16 equally likely families (ie p(GBGB) = p(GGGG), etc.)

  Students can now consider the Seven Dwarfs question. They might also consider the assumption that we made about the equal probability of a girl and a boy. In June 2016, there were 96.9 males for every 100 females in the greater Adelaide region. Ask students:

  - *How would this affect our calculations?*

The Australian Bureau of Statistics can be accessed at: http://www.abs.gov.au/Ausstats/abs@.nsf/mf/3235.0#PARALINK1

# Proficiency: Problem-solving

**Proficiency emphasis and what questions to ask to activate it in your students (Examples 5–10)**

**Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.**

## Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

## Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

**Growth mindset: Learning that not knowing is the beginning of a learning opportunity**

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

## Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

| Proficiency: Problem-solving examples | |
|---|---|
| **Example 5: Is this a fair game?**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | ACMSP246 ◆ ◆ ◆<br>ACMSP247 ◆ |
| **Example 6: The famous Monty Hall problem**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.<br>Students investigate reports of studies in digital media and elsewhere for information on their planning and implementation (10A). | ACMSP246 ◆ ◆ ◆<br>ACMSP247 ◆<br>ACMSP277 ◆ |
| **Example 7: It's blue, but which bag did it come from?**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | ACMSP246 ◆ ◆ ◆<br>ACMSP247 ◆ |
| **Example 8: Pick a number at random**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | ACMSP246 ◆ ◆ ◆<br>ACMSP247 ◆ |
| **Example 9: Mystery bag**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | ACMSP246 ◆ ◆ ◆<br>ACMSP247 ◆ |
| **Example 10: What sort of detective are you? Can you pick a fake?**<br>Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.<br>Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. | ACMSP246 ◆ ◆ ◆<br>ACMSP247 ◆ |

# Example 5: Is this a fair game?

**ACMSP246** ◆ ◆ ◆
Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆
Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Problem-solving** proficiency:
**Interpret; Model and plan; Solve and check; Reflect.**

**Reasoning** proficiency:
**What can you infer?**

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This is a game played between two people. The object of the game is to choose the sequence that will appear from tossing a coin 3 times, eg heads, tails and tails (HTT). Each player discloses which sequence they think will win, prior to playing the game. In this task, students have the advantage of knowing the sequence their opponent has selected (HTT) and can use this knowledge to identify a tactic that will allow them to choose a winning sequence.

The pair then toss coin(s) recording the outcomes. The player whose sequence is the first to occur is the winner. Discuss with students:

**If I choose my sequence after you, I will win more games than you do.**

- *How many different sequences are there to choose from? How can you be sure you have them all?*

  This is an opportunity to encourage students to list outcomes systematically and to apply the product principle to determine the number of permutations: 2 choices for each toss (2 x 2 x 2 = 8)

  My strategy is as follows:

  If you choose HHH, I choose THH
  If you choose HTT, I choose THT or HHT
  If you choose HHT, I choose THH or HHH
  If you choose THT, I choose TTH or HTH
  If you choose HTH, I choose THT or HHT
  If you choose TTH, I choose TTT or HTT
  If you choose THH, I choose TTH or HTH
  If you choose TTT, I choose HTT

**How am I making my decisions? Will I win more if I use this strategy? If I do, why does this work?**

## Interpret

*What have you been asked to calculate? What information is helpful/no use? Can you spot a pattern in my choices? Why is there only one choice for HHH and TTT?* (Establish that the student is aware of how the game is played by playing at least one game as a trial. If you win, discuss whether you will always win. If you lose, does that mean that your strategy is not working? In both cases, a decision cannot be made on the result of one game and so the there is a decision to be made about how many trials need to be done and a chance to pool class data.

Note that in my strategy, whatever you choose [1, 2, 3], I will choose [H, 1, 2] or [T, 1, 2], except I can't choose the same as you.)

## Model and plan

*Do you have an idea? How might you start? What equipment will be helpful? Do you need to check every combination? Are there different ways that you could do that? What do you think would be easiest/most efficient/always work for you?* (Ask students to speak to someone who you think is being a good problem solver today and ask them to show you what they are trying. Teachers can become more supportive if students cannot make progress.)

## Solve and check

*What have you noticed? How could you investigate that? Does the strategy appear to work? Have you done enough testing? Why do you think that? Why does it work? Is there another way that you could have solved this problem?*

## Reflect

*What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any values? What if you used a sequence of 2 or 4, could you still use this method? Is it a fair game? How confident are you in your findings?* (The reasoning behind my strategy is that if my opponent has chosen [1, 2, 3], then [1, 2, _] must be tossed for them to win. If I have chosen [H, 1, 2] or [T, 1, 2] then I have at least a 50% chance of winning before my opponent's sequence comes up. Of course, there is a 1 in 8 chance that they will beat me in the first three tosses [1, 2, 3] and I did not get a chance to get either [H, 1, 2] or [T, 1, 2].)

Ask students:

- *Can you improve my strategy?*
- *Consider this strategy: If you choose HHT, I choose THH or HHH. Is THH or HHH a better choice? Convince me. How will you prove it?*

(A version of this activity appears in the *Data representation and interpretation: Year 8* narrative – 'Example 9: Three in a row – a winning strategy').

# Example 6: The famous Monty Hall problem

**ACMSP246** ◆ ◆ ◆

Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆

Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**ACMSP277** ◆

Students investigate reports of studies in digital media and elsewhere for information on their planning and implementation (10A).

**Questions from the BitL tool**

**Problem-solving** proficiency: **Interpret; Model and plan; Solve and check; Reflect.**

**Reasoning** proficiency: **What can you infer?**

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

Consider a game show where there are three doors to choose from. There is a car behind one of the doors and a goat behind each of the other two. You win whatever is behind the door you choose.

After you have chosen your door the game show compère opens one of the other two doors, to reveal a goat. You now have the opportunity to stick with the door you originally chose or swap to the other unopened door.

Ask students:

**Is it better to swap or stay with your original choice? Does it make any difference?**

- *What do you think?*
- *How could we investigate this question?*

Plan an investigation and see if you still feel the same.

There are various websites that explain the mathematics behind this problem and some have digital objects that students can experiment with. The theoretical probability suggests that it is better to swap; in fact, your chance of winning doubles.

Numberphile's examination of this problem can be found at: https://www.youtube.com/watch?v=4Lb-6rxZxx0

# Example 7: It's blue, but which bag did it come from?

**ACMSP246** ◆ ◆ ◆
Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆
Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Problem-solving** proficiency:
**Interpret; Model and plan; Solve and check; Reflect.**

**Reasoning** proficiency:
**What can you infer?**

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This activity from the NRICH website can be modelled in the classroom using concrete materials. Students can be challenged by asking 'backwards questions', such as:

- *What do you think the content of the bags might be, so that you know it came from Bag A?*
- *What do you think the content of the bags might be, so that it was equally likely to come from either bag?*
- *Would it be possible to fill the bags so that the marble was twice as likely to come from Bag A than it is from Bag B?*


**The link to the problem on the NRICH website is: http://nrich.maths.org/506**

# Example 8: Pick a number at random

**ACMSP246** ◆ ◆ ◆
Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆
Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Problem-solving** proficiency:
**Interpret; Model and plan; Solve and check; Reflect.**

**Reasoning** proficiency:
**What can you infer?**

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

If you were to pick a number at random, what is the probability that it is a multiple of 5, or a multiple of 3? If you played $1 to generate a number at random and got $1.40 back if it was a multiple of 3 or 5, would it be expected to be profitable in the long run? If not, what should the payout be?

## Interpret

*What have you been asked to calculate? What information is helpful/no use? Do you think it is likely or not?* (Randomly generate a few numbers between 1 and 100 and discuss the result.)

## Model and plan

*Do you have an idea? How might you start? What equipment will be helpful? How could you better understand the process? Are there different ways that you could do that? What do you think would be easiest/most efficient/always work for you?* (Speak to someone who you think is being a good problem solver today and ask them to show you what they are trying. Teachers can become more supportive if students cannot make progress.)

*Generate a few numbers and see what you notice. Do you feel like it is likely or not? How do you know? Do you have a new idea now? Do you have enough information? What if it was multiples of two other numbers? Which ones might be more/less likely? Why?*

## Solve and check

*What have you noticed? How could you investigate that? Have you done enough testing? Why do you think that? Is there another way that you could have solved this problem?* (Most students at this level will generate some random numbers and collect data to check their intuitive decision as to whether it would be profitable or not. If they do not, then begin to consider a more analytical or theoretical approach. Remind them that they might just have been very lucky/unlucky, as such results can occur due to natural variation.)

*Convince me.*

## Reflect

*What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any values? What if you used three numbers (eg 3, 5 and 7), could you still use this method? Is it a profitable game? How confident are you in your findings?* (Most students understand that the probability will depend on how many multiples of 3 and 5 there are in the first 100 numbers, but they may not count them correctly. Some may list the numbers and these students will notice that the two events are not mutually exclusive and that some numbers are both multiples of 5 and 3 and cannot be counted twice. Students with good number sense will realise that there are 20 multiples of 5 (100 ÷ 5) and 33 multiples of 3 ($100 \div 3 = 33\frac{1}{3}$) and that there are 6 multiples of 15 which are counted twice ($100 \div 15 = 6\frac{2}{3}$), so there are 47 (20 + 33 - 6) favourable numbers. This is best shown on a Venn diagram, especially if there are three numbers involved. As the probability is $\frac{47}{100}$ you expect 47 wins every 100 games, making 40 cents each time ($18.80) but spending $53.)

# Example 9: Mystery bag

**ACMSP246** ◆ ◆ ◆
Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆
Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Problem-solving** proficiency:
**Interpret; Model and plan; Solve and check; Reflect.**

**Reasoning** proficiency:
**What can you infer?**

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

The Year 8 Mystery bag activity explores sampling and simple principles of probability, as well as providing practice in ratios. Students make individual and collaborative decisions about an unseen population, based on small samples. The activity can be extended for Year 10 students to investigate the probability of multi-step dependent events, while consolidating understanding about sampling and using statistics to solve problems.

## Materials

Create a 'mystery bag' by filling an empty bag with 60 coloured items as follows:

- 20 Red
- 15 Blue
- 10 Green
- 10 Yellow
- 4 White
- 1 Black

## Activity

Present the mystery bag to the class and tell them there are 60 items in the bag.

Next, pass the bag around the class, instructing each student to take out 5 items. Without letting anyone else see what they have, ask each student to record the colour/s of their items, before replacing all the items back in the bag and passing it to the next student. (To make this task easier, you could consider using a mystery bag at each student table).

Once the students have completed this task, you can ask:

- *Is selecting the 5 items simultaneously the same or different to choosing them one at a time? Are they independent or dependent events?*
- *Does your result affect the result for the next student? Are they independent or dependent?*
- *What is the connection between the sample you have and the contents of the bag (population)?*
- *Who thinks they have an unlikely result? Who is not sure? Why?*
- *Can you predict the contents of the bag?*
- *Which colour has the greatest and the least number of items in the bag?*

Each student is to make a prediction about the contents of the mystery bag based on their selection and record it with their reasoning.

At this stage, the students will have different levels of confidence about their predictions. Based on their sample (eg 3 red, 1 blue and 1 white), ask them to make a statement that:

- they know is true (there are red, blue and white items in the bag)
- they are very confident about (there are more red items than blue or white)
- one that they are somewhat confident about (there are more than 10 red items in the bag)
- one that they are not confident about (there are 36 red items (3 x 12), 12 blue items (1 x 12) and 12 white items (1 x 12).

Students will quite often state the last point as one that they are confident about, because it has been calculated numerically, which for them gives it more credibility. It is, however, unlikely that a sample of 5 exactly predicts a population of 60. As teachers, we should encourage students to use their understanding of ratios to interpret numerically what they have observed from their samples, but also help them to realise the limitations of this sampling when predicting the population.

**‘Think, Pair, Share’: A process to support all students to think deeply**

With a classroom brainstorm, students tend to share a range of rapid first responses. This may not allow all students to think more deeply about the problem. A practice called ‘Think, Pair, Share’ allows all students time to consider the problem individually, as well as a safe way to discuss their thoughts.

‘Think’ time is when each student thinks silently about the problem.

‘Pair’ time is for students to discuss their ideas with one other student.

‘Share’ time is an opportunity for the teacher to facilitate students sharing, comparing and contrasting the ideas that they have had, or they have heard.

Students then pair up and share their information to make new predictions based on the extra information.

Share some of the results from students’ samples with the class:

- *Now that you have more information, who thinks they had an unlikely result? Which one might be an ‘odd one out’?* (A sample that contains a colour no one else has, or a sample with all the items the same colour.)
- *What if we combine the results of the samples? Are the predictions for the contents of the bag still the same as our first predictions?* (Students are now aware that there were colours in other samples that they did not have. More information means they now know something about the population they did not know before and should be given the option to change their prediction.)
- *Were our first predictions wrong?* (Students often feel the process is ‘unfair’ as their sample did not contain one of the colours. It is important for students to realise that when making inferences based on data, that their first predictions were not ‘wrong’. They were completely valid for the information they had at the time.

  This illustrates that even if the samples are random, natural variation means that predictions based on samples may not be accurate.

  Reveal the contents of the mystery bag. At this stage, ask students to calculate the probability of their first result to check our intuition about what constitutes unlikely results. Set groups this challenge before discussing possible methods or difficulties:

  The probability of getting 5 red items is ($\frac{20}{60} \times \frac{19}{59} \times \frac{18}{58} \times \frac{17}{57} \times \frac{16}{56}$). This is less than 0.3% and is only expected to occur 3 times in 1000 samples.

  Students at this level are generally familiar with the concept of dependent events and can determine this probability by considering one branch of a weighted tree diagram.

  Consider however a result such as 3 red, 1 blue and 1 white (RRRBuW). A complete tree diagram starts with 6 branches and is both tedious and complicated to complete, but students may choose this method and be successful. (Note: there are only 4 white balls and 1 black ball to be drawn, so some branches are reduced.))
- *How many different ways have you found? What do you notice about the probabilities of each? The fractions are different, so what about the values?* (eg RRRBuW ($\frac{20}{60} \times \frac{19}{59} \times \frac{18}{58} \times \frac{15}{57} \times \frac{4}{56}$) and BuRRWR ($\frac{15}{60} \times \frac{20}{59} \times \frac{19}{58} \times \frac{4}{57} \times \frac{18}{56}$))
- *Is this always the case? So, is there another way to calculate the probability?* (Students can use multiplicative thinking to realise that they only need to count how many different ways there are of arranging the order. At this stage, it is best done by systematic listing (20 ways). After students have calculated their own probabilities, form a class number line between 0 and 1 to determine who had the most unlikely results.)
- *Would the result be the same, or different, if we had drawn them and recorded them one at a time?* (Knowing the order eliminates the need to determine how many different ways 3 red, 1 blue and 1 white can be ordered, that is why 5 red was easier to calculate.)

(A similar activity can be found on page 13 of the Data representation and interpretation: Year 9 narrative.)

# Example 10: What sort of detective are you? Can you pick a fake?

**ACMSP246** ◆ ◆ ◆
Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Students investigate the concept of independence.

**ACMSP247** ◆
Students use the language of 'if … then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

**Questions from the BitL tool**

**Problem-solving** proficiency:
**Interpret; Model and plan; Solve and check; Reflect.**

**Reasoning** proficiency:
**What can you infer?**

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This activity uses empirical evidence to challenge students' beliefs about randomness and expected outcomes for independent events.

Students are asked to investigate a suspected counterfeit ring using their own observations, knowledge of random events and problem-solving skills.

This task challenges students' expectations for a process that is very familiar to them, tossing a coin. In 100 tosses, students expect there to be approximately 50 of each outcome: heads and tails. While they do not expect these two outcomes to alternate, they don't expect there will be a long run of one outcome or the other either. Surprisingly in 100 tosses, it is almost certain that there will be a run of at least 6 heads or 6 tails.

Distribute a numbered recording sheet to each student in the class for anonymous identification purposes. As a homework task, ask students to complete a record of the outcomes of 100 tosses of a coin. Before they begin recording, instruct them to toss the coin once. If the result is a head, the students must genuinely toss the coin 100 times. If, however, the result is a tail, they are to record the outcomes they might expect if they were tossing the coin. (Impress upon them that it is important to follow these instructions for accurate data if it is to be a valid investigation.)

If you examine the recording sheets, you can identify the authentic data sets as they will have a significant number of 'clusters' or runs of heads or tails. Students who are 'faking' the data will not have long runs of heads or tails. Use this knowledge to sort the sheets into two piles. Ask the students to identify if you have placed their sheet in the correct pile.

It is important to stress that you are not magical or performing any trick. You have only used your mathematical skills to make decisions. As detectives, they are challenged to determine a method that can be used by others to identify fake data.

Note: there is a digital activity that students can use as part of their investigation called 'Coin Tossing' from the National Library of Virtual Manipulatives: http://nlvm.usu.edu. The activity is also on Scootle: http://www.scootle.edu.au/ec/viewing/L3515/index.html

(This activity also appears in the *Chance: Year 8* narrative.)

## Interpret

*What have you been asked to find? What information is helpful/not useful?* (Students will know if their own data is authentic or not. Observing the sheets in each pile and trying to identify what is the same about the sheets in the authentic pile, and how that is different to the sheets in the 'faked' pile, would give significant clues as to the method you used when sorting. The task is most challenging if they receive the unsorted pile of the data sets, as you did.)

## Model and plan

*Do you have an idea? How might you start? Would it help if you organised the data? Are there different ways that you could do that?* (Ask students to speak to someone who you think is being a good problem solver today and ask them to show you what they are trying. Students do not often value the information they get from visually checking the data for patterns and that is a key skill in approaching this problem.)

*Can you see any differences, or similarities, between the authentic and faked data results?*

## Solve and check

*If the data looks different, how could you investigate that? Is there something about the data that you could count and record? How could you represent this so that others could see the pattern?* (Students could record the number of runs of different lengths, ie: a run of 1 (head or tail); or a run of 2 (head/head or tail/tail), etc for authentic and for 'faked' data to compare. Notice that it does not matter if it is a run of heads or a run of tails, it is only the length that is of interest. These runs can be recorded and graphed. Choosing an appropriate table and graph type are evidence of fluency. Once students have developed a conjecture (theory) for how to tell the difference, they can develop a written statement about their process to help others identify a 'fake'. This is an opportunity for students to demonstrate their reasoning skills.)

*How could you check your findings?* (To extend this activity, you could provide a new collection of recording sheets; containing some that have been randomly generated and others that a group of teachers have faked, and challenge the students to catch the 'criminals'.)

## Reflect

*Could you improve your method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Would you always pick the fake correctly? Will your method work for any data sets?* (Now that the students have more knowledge, if you conducted the experiment again, the results would not be the same.)

While the students are demonstrating their understanding of the statistical investigation process in this activity, there are important concepts relating to chance that can be drawn from this experience.

The coin has no memory – the outcome on one toss has no effect on the next. They are independent of each other. In this case, our intuition can be misleading and the challenge now is to use our knowledge of theoretical probabilities to determine the expectations of long runs of heads or tails:

- *Is it possible that the coin is biased? How could we check that?*
  (Check the relative frequency of throwing a head and throwing a tail over the 100 tosses; they should both be approximately $\frac{1}{2}$.)
- *How is this possible with having such a long run of heads, eg 6?*
  (Even though there are some long runs of heads, there will also be long runs of tails.)
- *You get a lot of variation when you conduct experiments, perhaps you just got a weird result with a run of 6 heads. What do you think?*
  (That is possible with one trial but not probable with the whole class doing it, as there was a large number of trials.)
- *Can you estimate what the probability of getting a run of 6 heads or 6 tails would be using the class data?*
  (With a large number of trials, the average number of 'long runs of 6' in 100 tosses is approximately the theoretical probability or expected outcome.)
- *From our data we expect it to be … Can you determine the theoretical probability of getting at least one run of 6 heads or 6 tails in 100 tosses?*

For students who need more encouragement:

- *What is making the problem difficult? Is there an easier but similar problem they could tackle?*
  (Students can work out the probability of getting 6 heads or 6 tails in 6 tosses by $((\frac{1}{2})^6 \times 2 = \frac{1}{32}$.)
  Most students understand that you would expect that if you tossed the 6 coins 32 times (or one coin 192 times) you would expect it to occur once.)
- *So, what does that mean if we toss it 100 times (not 192)? Does that make sense to you? Is that what our trials suggest? So, what are we missing?*
  (Suggest students go back to their trials and divide the 100 tosses into lots of 6.)
- *What do you notice? How can you adjust for this?*
  Partitioning into 6 could split a run of 6 as shown below:

| | | |
|---|---|---|
| HTHHTT | TTTTHH | TH…. |

There are in fact 95 overlapping sets of 6. This is not a trivial problem as these overlapping sets are not independent events. If we were to consider another problem, where we would expect 1 in every 32 to have a run of 6 heads or tails, the event is seeming more likely.

### An intellectual stretch for us all …

By considering the complementary event, the chances of getting a run of 6 which is not 6 heads or 6 tails is $\frac{62}{64} = \frac{31}{32}$

The probability that 95 independent runs of 6 do not have these runs (6H or 6T) is $(\frac{31}{32})^{95} \approx$ 4.9% Therefore, the probability that at least one of them **will**, is the complement: 95.1%, or highly likely.

You can access further discussions about runs in independent events at Quarantine:
http://gregegan.customer.netspace.net.au/QUARANTINE/Runs/Runs.html

Related activities from Scootle can be accessed here:
http://www.scootle.edu.au/ec/viewing/L3659/index.html
http://www.scootle.edu.au/ec/viewing/L3665/index.html


Source: *Random or not: explore alternating jubes (1:1)*, Education Services Australia Ltd, 2013

# Connections between ‘Chance’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use Chance as a starting point.

Here are just some of the possible connections that can be made:

| Mathematics: Year 10/10A | |
|---|---|
| **Whilst working with Chance, connections can be made to:** | **How the connection might be made:** |
| Students evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data. ACMSP253 | Refer to:<br>**All examples.** |

**Making connections to other learning areas**

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

# ‘Chance’ from Year 1 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Chance:

**Identify and order chance events** ◆
In Year 1 to Year 2 students identify and order chance events.

**Identify, describe and represent sample spaces** ◆
In Year 1 to Year 5 students describe and represent sample space. In Year 6 to Year 8 students mostly describe more complex sample spaces and assign probabilities. In Years 9 to Year 10A students mostly represent complex sample spaces and events.

**Randomness and variation** ◆
In Year 3 students conduct chance experiments and recognise variation in results. In Year 9 students calculate from given or collected data. In Year 10A students investigate reports of studies in digital media and elsewhere for information on their planning and implementation.

**Observed frequencies and expected probabilities** ◆
In Year 3 to Year 5 students consider expected probabilities of simple events. In Year 6 students quantify probabilities. In Year 9 and Year 10 students determine probabilities of compound events.

**Language** ◆
Throughout Year 1 to Year 10A students use the language of chance in increasingly sophisticated ways. In Year 8 students explore the particular language relating to the exclusive or inclusive references to events. In Year 10 students make and appraise inferential statements relating to chance.

| Year level | ‘Chance’ content descriptions from the AC: Mathematics |
|---|---|
| Year 1 ◆ ◆ | Students identify outcomes of familiar events involving chance and describe them using everyday language such as ‘will happen’, ‘won’t happen’ or ‘might happen’. ACMSP024 |
| Year 2 ◆ ◆ ◆ | Students identify practical activities and everyday events that involve chance. Describe outcomes as ‘likely’ or ‘unlikely’ and identify some events as ‘certain’ or ‘impossible’. ACMSP047 |
| Year 3 ◆ ◆ ◆ | Students conduct chance experiments, identify and describe possible outcomes and recognise variation in results. ACMSP067 |
| Year 4 ◆ ◆ | Students describe possible everyday events and order their chances of occurring. ACMSP092 |
| Year 5 ◆ ◆ ◆ | Students list outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions. ACMSP116 |
| Year 6 ◆ | Students describe probabilities using fractions, decimals and percentages. ACMSP144 |
| Year 7 ◆ ◆ | Students construct sample spaces for single-step experiments with equally likely outcomes. ACMSP167 |
| Year 8 ◆ ◆ | Students identify complementary events and use the sum of probabilities to solve problems. ACMSP204 |
| Year 8 ◆ | Students describe events using language of ‘at least’, ‘exclusive or’ (A or B but not both), ‘inclusive or’ (A or B or both) and ‘and’. ACMSP205 |
| Year 8 ◆ ◆ | Students represent events in two-way tables and Venn diagrams and solve related problems. ACMSP292 |
| Year 9 ◆ ◆ | Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events. ACMSP225 |
| Year 9 ◆ ◆ ◆ | Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’. ACMSP226 |

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| **Year 9** ◆ ◆ | Students investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians. ACMSP227 |
| **Year 10** ◆ ◆ ◆ | Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence. ACMSP246 |
| **Year 10** ◆ | Students use the language of ‘if … then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language. ACMSP247 |
| **Year 10A** ◆ | Students investigate reports of studies in digital media and elsewhere for information on their planning and implementation. ACMSP277 |

| **Numeracy continuum: Interpret chance events** | |
|---|---|
| **End Foundation** | Recognise that some events might or might not happen. |
| **End Year 2** | Identify and describe familiar events that involve chance. |
| **End Year 4** | Describe possible outcomes from chance experiments using informal chance language and recognising variations in results. |
| **End Year 6** | Describe chance events and compare observed outcomes with predictions using numerical representations such as a 75% chance of rain or 50/50 chance of snow. |
| **End Year 8** | Describe and explain why the actual results of chance events are not always the same as expected results. |
| **End Year 10** | Explain the likelihood of multiple events occurring together by giving examples of situations when they might happen. |

Source: ACARA, Australian Curriculum: Mathematics

# Resources

## NRICH website

**http://nrich.maths.org**

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.



## Dan Meyer's blog: 101 questions

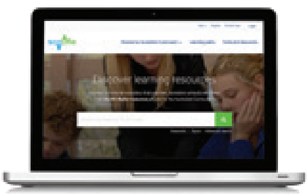
**http://www.101qs.com**

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

## Scootle

**https://www.scootle.edu.au/ec/p/home**



This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.

## reSolve: maths by inquiry

**https://www.resolve.edu.au**

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

## Plus Magazine

**https://plus.maths.org**

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.



## Numeracy in the News

**http://www.mercurynie.com.au/mathguys/mercury.htm**

**Numeracy in the News** is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.

## TIMES modules

**http://schools.amsi.org.au/times-modules/**

**TIMES modules** are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.

## Top drawer teachers – resources for teachers of mathematics (statistics)

**http://topdrawer.aamt.edu.au/Statistics**

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.

## Double Helix Extra

**https://blog.doublehelix.csiro.au/**

This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.

## CensusAtSchool NZ

**http://new.censusatschool.org.nz/tools/random-sampler/**

CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics. It aims to:

- *'foster a positive attitude to statistics through using data that is both relevant and real*
- *improve understanding of a data gathering process, its purposes and benefits to society*
- *provide access to large and meaningful multivariate data sets*
- *encourage effective IT teaching and learning*
- *enhance the process of statistical enquiry across the curriculum.'*

**Do you want to feel more confident about the maths you are teaching?**

**Do you want activities that support you to embed the proficiencies?**

**Do you want your students thinking mathematically rather than just doing maths?**

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.


Produced by the Learning Improvement division