Chance: Year 8

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together

www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Chance' from Year 1 to Year 10A** (see page 24)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with Chance:

- Identify and order chance events
- Identify, describe and represent sample spaces
- Randomness and variation
- Observed frequencies and expected probabilities
- Language.

What the Australian Curriculum says about 'Chance'

Content descriptions

Strand | Statistics and probability.

Sub-strand | Chance.

Year 8 ◆ ◆ | ACMSP204
Students identify complementary events and use the sum of probabilities to solve problems.

Year 8 ◆ | ACMSP205
Students describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'.

Year 8 ◆ ◆ | ACMSP292
Students represent events in two-way tables and Venn diagrams and solve related problems.

Year level descriptions

Year 8 ◆ ◆ | Students use two-way tables and Venn diagrams to calculate probabilities.

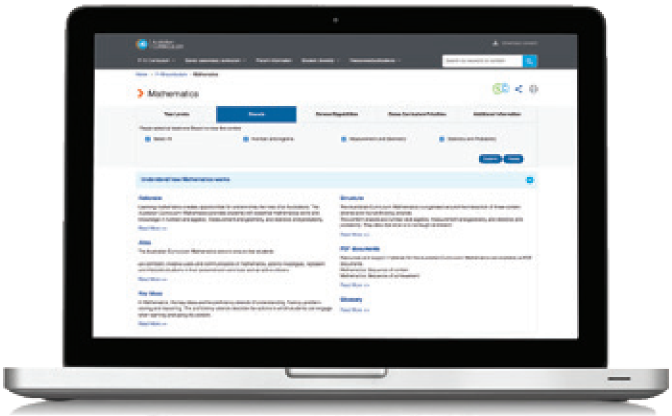
Achievement standards

Year 8 ◆ ◆ | Students model authentic situations with two-way tables and Venn diagrams. Students choose appropriate language to describe events and experiments.

Numeracy continuum

Interpreting statistical information

End of Year 8 ◆ | Students describe and explain why the actual results of chance events are not always the same as expected results (Interpreting statistical information: Interpret chance events).



Source: ACARA, Australian Curriculum: Mathematics

Working with Chance

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 8 'Chance'

In Year 7, in line with students' developing ability to change between fractions, decimals and percentages, they would now be expected to change between these representations for any amount, rather than just for common quantities. Students in Year 7 are introduced to the term 'sample space'.

In Year 8 students use two-way tables to represent possibilities of experiments with two steps. Students are also introduced to the use of the terms 'at least', 'and' and 'or' in relation to chance events. They use two-way tables and Venn diagrams to calculate probabilities satisfying the 'at least', 'and' and 'or' criteria. Students are also introduced to the term 'complementary events' and use the sum of probabilities to solve problems.

In Year 9 students continue to work with two-step chance experiments, but now they consider the probabilities with and without replacement. To support this thinking, students learn to create tree diagrams and use such diagrams to assist in solving problems. Students estimate probabilities by calculating relative frequencies for collected or given data.

In Year 10/10A students describe the results of two- and three-step experiments, both with and without replacement. They investigate the concept of independence and conditional statements.

- **Statistics is an authentic and engaging context to learn about Chance**. Statistics is about variation and uncertainty, and in this way chance and data are intertwined.
- **Chance offers an opportunity to further develop number skills.** Probability is a measure of chance and when students quantify the likelihood of events, this links with their understanding of concepts within numbers, ie fractions, decimals, percentages, ratio and proportion. The development of thinking about probability depends very heavily on proportional reasoning. Students need to be able to compare two things using multiplicative thinking and then apply this to a new situation.
- **Students have very real, informal intuitions about probability.** These intuitions about the likelihood of events are beliefs based on past experiences. There is an ongoing need for teachers to ask students to give their intuitive answers to problems involving chance and variation prior to calculation and get them to talk about what their answers are based upon. Students need to experience that these intuitions become less reliable when events are not equally likely and as contexts and sample spaces become more complicated. Once they experience that their intuition is often misleading for complex events, they become aware that they need more sophisticated ways to identify sample spaces and understand the relationships between compound events.
- **Manipulatives support conceptual understanding of theoretical probabilities.** To develop a more analytical and theoretical approach, teachers need to continue to provide opportunities for students to conduct the experiments themselves. When they work with manipulatives and experience the variation in outcomes, abstract concepts become more concrete. By collecting data, students can determine relative frequencies of events which supports the theoretical probabilities that they work out by considering the sample space. If problems are relating to a pack of playing cards, make sure students have a pack. Even if they have solved the problem, ask them to show you using the pack, how and why that works.
- **Communication is essential for learning.** Have students talk with you and with each other about their learning. Foster the use of the probability specific terms and encourage the modality that you expect them to use in their reports. Discourage students from stating a probability without first describing the experiment with sufficient information, so that a listener could be able to conduct it. Require them to name a few of the possible outcomes besides the ones they are interested in. This encourages them to familiarise themselves with the experiment and think more deeply about the sample space before they try to quantify the probability.
- **Language is a very important part of understanding and describing chance events.** Once students understand the concepts, teachers can make links to the mathematical terminology and formal definitions, connecting the mathematical names to the colloquial and everyday use of the language. Because language is so important, it is a good strategy for the teacher to set up a highly visible class vocabulary list (eg word wall) for students. The word wall links the formal definitions, colloquial language, the students' own understandings and examples from authentic experienced contexts.

Engaging learners

Classroom techniques for teaching Chance

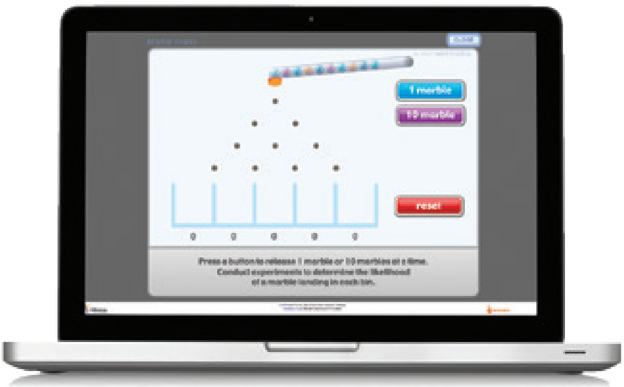
Chance provides opportunities to evoke curiosity and wonder in our students. Many genuine contexts allow students to draw on prior knowledge and intuition to engage them in their learning. Students personalise their learning when they make predictions about outcomes, design and conduct their own experiments and make meaningful inferences from what they have discovered.

Using relative frequency

Many websites such as Cambridge HOTmaths, Scootle, NRICH and the National Library of Virtual Manipulatives have a range of digital activities which students can use as random generators to experiment and collect relative frequencies to explore probabilities.

In most games, skill affects your chance of winning. However even in games where skill plays no part, each outcome is not always equally likely. In ***HOTmaths: Using relative frequency*** marbles fall from a tube and bounce through obstacles to land in 1 of 5 bins. By using the digital activity, students can calculate the relative frequencies for the bins the marbles land in. Students then gather data by conducting several experiments with different numbers of marbles and comment on the relative frequencies, as the number of trials in the experiment increases.

The digital activity can be found at:
http://tlf.dlr.det.nsw.edu.au/learningobjects/Content/L10574/object/



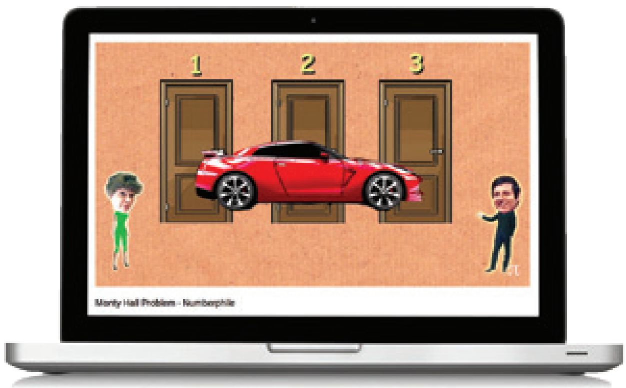
Source: *HOTmaths: Using relative frequency*, HOTmaths – The Le@rning Federation, 2009

Monty Hall problem

Famous problems such as the ***Monty Hall problem*** are counterintuitive and learners are quite often motivated by disbelief to explore the problem. Loosely based on the American television game show *Let's Make a Deal*, it is named after its original host, Monty Hall and focusses on the probability of opening the correct door to win the coveted prize.

Numberphile's examination of this problem features Lisa Goldberg, an adjunct professor in the Department of Statistics at the University of California, Berkeley USA.

The video can be found at:
https://www.youtube.com/watch?v=4Lb-6rxZxx0



Source: *Monty Hall Problem*, Numberphile, 2014

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–5)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the names 'mutually exclusive', 'complementary' and 'independent events'** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged **with questions that will result in them identifying the patterns that simplify finding the probability for different types of events**, so I don't need to design and instruct the details of the investigation for them.

At this stage of development, students can develop an understanding of chance, randomness and variation through conducting their own experiments. When teachers provide opportunities for students to predict, identify, describe and represent the outcomes of the experiments and compare them to theoretical expectations, they require their students to generalise. Telling students the laws of probability, removes this natural opportunity for students to make conjectures and verify connections that they notice.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to fully design a probability investigation, or set tasks that we want students to experience, rather than ask a question (or a series of questions) and support them to planning the stages of the investigation for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students use a specific set of skills, knowledge and procedures during the current unit of work, then it probably is quicker to tell them what to do. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the formulae is a false economy of time.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Complementary events Students identify complementary events and use the sum of probabilities to solve problems.	ACMSP204 ◆ ◆
Example 2: Mutually exclusive events Students identify complementary events and use the sum of probabilities to solve problems.	ACMSP204 ◆ ◆
Example 3: How well do you know your classmates? Students represent events in two-way tables and Venn diagrams and solve related problems.	ACMSP292 ◆ ◆
Example 4: Year 8 students don't eat breakfast Students represent events in two-way tables and Venn diagrams and solve related problems.	ACMSP292 ◆ ◆
Example 5: Area model for two-step experiments Students represent events in two-way tables and Venn diagrams and solve related problems.	ACMSP292 ◆ ◆

Example 1: Complementary events

ACMSP204 ◆ ◆

Students identify complementary events and use the sum of probabilities to solve problems.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

What can you infer?

Instead of ***telling*** students about complementary events, we can challenge students to recognise the relationships between the events for themselves, by ***asking*** questions.

Using a set of 'events' cards, begin matching complementary events without any explanation of the rules. This requires students to notice the connections between the events.

Students learn by noticing: Invite their curiosity

Rather than explaining what complementary events are, demonstrate your own understanding by sorting the cards and asking students what they can notice from it.

Make sure you are explicit about the experiment that is being conducted, eg throwing a die and noting the result.

A set of 'event' cards for this experiment might include:

- Tossing a die and throwing a 5
- Tossing a die and not throwing a 5
- Tossing a die and throwing a 1, 2, 3, 4 or 6
- Tossing a die and throwing an even number
- Tossing a die and throwing an odd number
- Tossing a die and throwing a number bigger than 4
- Tossing a die and throwing a number that is a least 5
- Tossing a die and throwing a number than is no more than 4
- Tossing a die and throwing a number 1, 2, 3 or 4
- Tossing a die and throwing a number less than or equal to 4
- Tossing a die and throwing a number 1, 4 or 6
- Tossing a die and throwing a prime number
- Tossing a die and throwing a multiple of 3
- Tossing a die and throwing a number 1, 2, 4 or 5
- Tossing a die and throwing a multiple of 2
- Tossing a die and throwing a number between 2 and 5 inclusive
- Tossing a die and throwing a 1 or 6
- Tossing a die and throwing a number between 1 and 6.

You can differentiate by using simple events that the students are familiar with, such as tossing a die and compound events when drawing a card for a pack of playing cards (if using this context, make sure students are familiar with standard playing cards). If there is a cultural equivalent that they are more familiar with, consider if the same questions could be used for that context, or even ask students to pose equivalent questions themselves). Always have a set of cards for students to support their thinking. Have more than one option for some matches.

Discuss the following with your students:

- *What do you notice? Can you see a connection in the way I am matching the events?*
- *Are there other matches you could make? Why do they belong together?*
- *Can you describe the relationship between the matched events?*

Share some of the descriptions students come up with. Get other students to respectfully question students about their descriptions and how they apply to certain pairs.

Once students have demonstrated and shared their definitions of complementary events, direct students to look it up in a maths dictionary and compare their descriptions for what is the same and what is different. Give students the opportunity to make their own event card sets.

Ask students to work out the probability of all the events on the cards. Then ask the students:

- *What do you notice? Can you see a connection between the probabilities of the complementary events?*
- *Can you explain this relationship? Can you add this property to your description of complementary events? Why do you think they would be called 'complementary'?*

Where possible, always relate mathematical terms to their colloquial (every day) usage.

Tackling misconceptions: Knowing what it isn't

Asking questions such as *'Why can't I …'*, *'Why is it not …'*, and *'What's wrong with …'* is one good way of hearing a student's reasoning. It can work well if the teacher compiles a number of examples that reflect their students' misconceptions. The examples don't have to be presented as their misconceptions, just as examples of thinking that is commonly seen when people are learning to work with statistics. Once the students have identified the issues, they can reflect on the effect that it has had on their own thinking.

To check for understanding about complementary events, suggest the misconception of matching disjoint or intersecting sets. Present the following problem to students:

A student was uncertain about whether the following matching events for drawing a single card from a playing pack, were complementary.

Event 1: A red card
Event 2: A spade
Event 3: Black picture cards

Then discuss the following with the students:

- *What do you think? Can you explain your thinking?*
- *Can you write an event card that is a complementary event to Event 1: A red card? Is there more than one event that could be a complementary event?*

Example 2: Mutually exclusive events

ACMSP204 ◆ ◆

Students identify complementary events and use the sum of probabilities to solve problems.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about mutually exclusive events, we can challenge students to recognise the relationships between the events for themselves, by ***asking*** questions.

This activity relates to 'Example 1: Complementary events' on page 8 of this narrative.

Using a set of 'events' cards, this activity involves matching mutually exclusive events without any explanation of the rules. This requires students to notice the connections between the events. A set of 'event' cards for this experiment might include:

- Event 1: A red card
- Event 2: A spade
- Event 3: Black picture cards
- Event 4: A black card
- Event 5: A heart.

Using your set of event cards, start to match events which 'have no overlap' and ask the class:

- *Is it possible to match 3, 4 or more events that do not overlap?*

Explain to the students that events that have no overlap are called mutually exclusive events. Then ask:

- *Why would we call them 'mutually exclusive'?*
- *A student said that mutually exclusive events are the same as complementary events. What do you think? Can you explain your thinking?*

This understanding requires the use of logic. All complementary events are mutually exclusive (a red card and a black card) but not all mutually exclusive events are complementary (a red card and a spade). Ask students to use the cards to demonstrate this thinking.

This understanding helps students determine whether they can find the probability of compound events by summing the probabilities (only for mutually exclusive events).

Tackling misconceptions: Knowing what it isn't

Asking questions such as *'Why can't I …'*, *'Why is it not …'*, and *'What's wrong with …'* is one good way of hearing a student's reasoning. It can work well if the teacher compiles a number of examples that reflect their students' misconceptions. The examples don't have to be presented as their misconceptions, just as examples of thinking that is commonly seen when people are learning to work with statistics. Once the students have identified the issues, they can reflect on the effect that it has had on their own thinking.

Present the following scenario to your class, discussing the questions as you go:

A student chose (Event 4: A black card) and (Event 5: A heart) as being mutually exclusive events for drawing a single card from a pack.

- *Do you agree? Explain your thinking.*

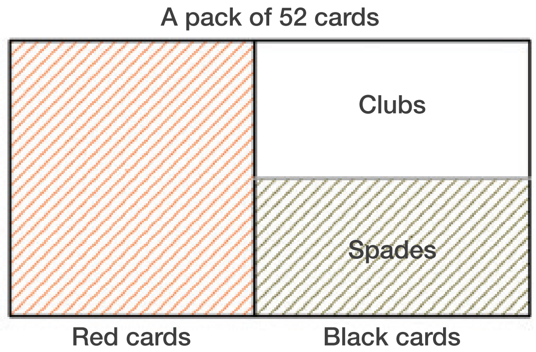
The student then noticed something interesting. They claim that:

P(Black card) + P(Heart) = P(Black card or a Heart)

- *Do you agree with this statement? Does it make sense to you?*
- *Would it always be true? When does it not work?*
- *Is there a way to find the probability for cases when it does not work? Can you show me using the cards?*
- *How would this work for events if we were rolling a die?*

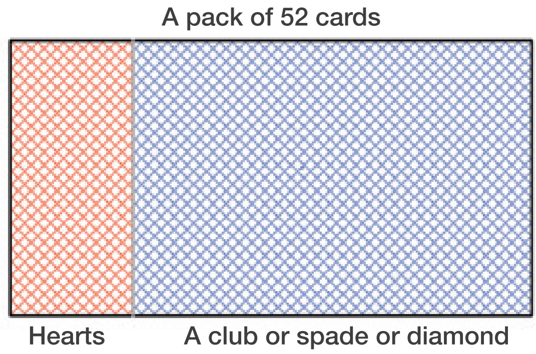
If students are unfamiliar with the notation of probability, it is valuable to re-write the relationship in words.

Using Venn diagrams to represent the sample space and shading in the events, provides a visualisation of what is meant by 'mutually exclusive' and 'complementary'. This can be achieved by sorting a pack of cards into the areas of a Venn diagram that you outline on the desk or the floor (see Figures 1 and 2). Put a range of different diagrams on the Word Wall next to the words and descriptions.



Drawing a 'red card' and drawing a 'spade' are *mutually exclusive* but *not complementary* events.

Figure 1



Drawing a 'heart' and drawing a 'spade' or 'club' or 'diamond' are *complementary* events and therefore also *mutually exclusive*.

Figure 2

Example 3: How well do you know your classmates?

ACMSP292 ◆ ◆
Students represent events in two-way tables and Venn diagrams and solve related problems.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about Venn diagrams, we can challenge students to sort events in a practical way for themselves, by ***asking*** questions.

This activity encourages students to think about complementary and compound events through the use of Venn diagrams. This activity requires the use of three coloured ropes.

Begin by asking your students to make a prediction about the following:

'If I picked a student at random from this class, what do you think my chance is that they will have a dog?'

(If you have a small class you could include data from others in the school or you can access comparable samples and teaching resources from the global CensusAtSchool NZ Random Sampler at http://new.censusatschool.org.nz/tools/random-sampler/)

Students as creators of knowledge: Give students a chance to make a guess

Most of us have an intuition about 'likelihood' that is based on experience; but it can be misleading. As teachers we should provide opportunities for students to discover that by thinking deeper about chance events and by collecting relevant information, they are making more informed judgements.

Explain that the aim of the activity is to collect some information about your class, in particular the answers to the three questions below:

- Do you have a brother?
- Do you own a dog?
- Do you have black hair?

Do a show of hands to tally and record the number of students that answer 'Yes' to each of the questions and ask the students to check the total. Some students will be expecting the total to equal the number of students in the class, which will not be the case if any students answered 'Yes' to more than one question. Discuss the following:

- *What do you notice about the total? What do you wonder about this information?*
- *Is there a better way to represent this information so we know more about our class?*

Using the coloured ropes, make three separate areas so students can be sorted depending on how they answered the questions. Do not overlap them initially.

One-by-one, ask the students to come to the ropes and answer the questions. Hand them an appropriate card (eg a picture of a dog, someone with black hair, or a boy) if they answer *'Yes'* and ask them to stand in the appropriate area. Having a physical record of their responses, helps students recognise that everyone in their area has the same cards and what it is they have in common.

As some students will have two or more cards, gradually move the ropes to overlap the areas (see Figure 3).

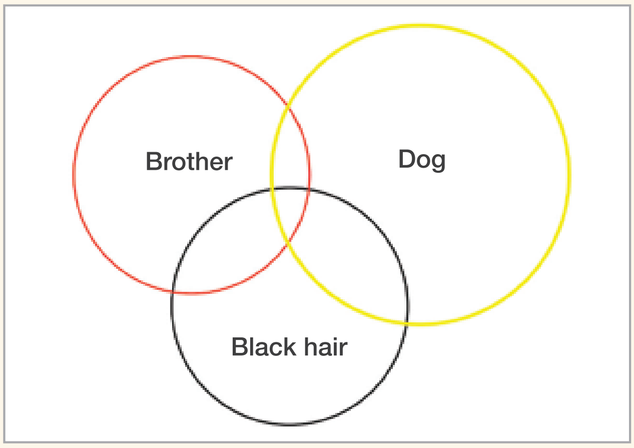


Figure 3

This activity requires each student to have dialogue with you and choose the appropriate area to stand. The other students can be asked if they think the student is in the right area, and assist if the student is not sure where to stand.

Where a student takes all three cards (or takes none), students are challenged to consider that all possible outcomes in the sample space need to be considered.

If some of the areas have no students in them, pose the question about a hypothetical student, or ask a backwards question.

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

Discuss the following with your class:

- *What can you tell me about a student who would stand in this region?*
- *A student holds two cards. How many different regions could they be standing in? Is there another way to describe this?*
- *A student has a dog. How many different regions could they be standing in and what does each of these areas tell us about the student?*

These types of questions encourage students to start thinking about complementary events and the sample space for compound events.

- *How could we represent this information? Is there another way?*

At this stage, students will make the simple connection from the ropes on the ground to intersecting circles, but it is important to allow students to consider alternative ways. Challenge them to find another type of representation, which is not a straightforward task (students will discover how powerful the Venn diagram is in representing this type of information).

Now students have constructed their own understanding, we can point out that they have discovered for themselves a representation that mathematicians call a Venn diagram.

Key language for students to understand and use, includes terms such as 'at least' and 'inclusive or'. This activity creates the opportunity to discuss the use of clear, unambiguous language:

- *Do you have a brother? I have 2. So, is the answer yes or no?* (The answer is yes, but most people want to qualify their answer.)
- *What would be a better question?* ('Do you have at least one brother?' would make sure everyone with one brother or more, would be happy to say 'Yes'.)

In mathematics, 'or' is interpreted as 'inclusive or', for example:

- *Do you have dog or a brother? I have both. So, is the answer yes or no?* (The answer is yes but most people want to qualify their answer.)
- *What would be a better question?* (If we want this information, it would be better to ask 'Do you have a dog, or a brother, or both?' But we need to understand that 'or' means 'either, or both'.)

There is an agreed understanding that the word 'or' is an 'inclusive or', meaning you say yes if you have a dog only, or a brother only, or both a dog and a brother. Ask:

- *What then is the case(s) where you would say 'no'?*
- *What is the probability of a student in our class having a dog?*
- *What is the probability of a student in our class having a brother?*
- *What is the probability of a student in our class having a dog, or a brother?*
- *What do you notice about these probabilities? Can you suggest what the probability is of a student having a dog and black hair? Or black hair and a brother?*
- *What does the pattern seem to be? How could we check?*

Answering our question

'If I picked a student at random from this class, what do you think my chance is that they will have a dog?'

Write the names of each student in the class on a popsicle stick and place it in a jar (these sticks can be used later for random selection when asking questions of individuals in class).

Ask, 'Based on the information we have …':

- *What is the chance that a student selected at random would have a dog?*
- *If we picked a given number of students, say 10, how many would we expect to have a dog?*
- *Identify an event that would be highly likely to occur/ highly unlikely. Explain why.*

These conjectures can be tested by randomly selecting a student, but it is important to discuss expectations before this is done. The chances of a student in class …:

- *owning a dog is 10%, so does that mean the student I randomly select won't have a dog?*
- *having at least one brother is 3 in 5, so if I randomly select 5 students will there be 3 with brothers?*

These discussions encourage students to use language such as 'possible, but not probable', 'more or less likely'. When a student has been selected, the discussion can be about whether the result was expected, or an unexpected result.

This activity is also an opportunity to consider conditional probability:

- *What is the probability that a student chosen at random has black hair? If I have taken all the students who have brothers out of the jar before my selection, is the probability the same?*
- *If I tell you this student has a dog, how many students could it possibly be? What is the chance that one of these students has black hair?*
- *What extra condition could there be so that the probability of having a brother is less than 50%?*

As teachers, when we are talking, we should ensure that we use as much variation in language as possible (ie 1 in 5, 20%, $\frac{1}{5}$, probability of 0.2, odds of 1 to 4, etc) so students become fluent in different ways of describing the probability as a measure of chance.

'Enrichment Maths for Yr 4–9 Level 1 Program 2' is a publication of the Mathematical Association of South Australia which has further examples relating to Venn diagrams. The publication can be ordered at http://www.masanet.com.au/resources/Publications/2017%20Publications%20MASA%20Order%20Form..pdf

Example 4: Year 8 students don’t eat breakfast

ACMSP292 ◆ ◆
Students represent events in two-way tables and Venn diagrams and solve related problems.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about two-way tables, we can challenge students to sort events in a practical way for themselves, by ***asking*** questions.

Pose the following question to your class:

‘Do more boys or girls in our class eat breakfast?’

Then ask students:

- *What do you think? Why?*

Data such as the set used in this example can be generated using the global CensusAtSchool NZ Random Sampler. Generate your own samples and access teaching resources at http://new.censusatschool.org.nz/tools/random-sampler/

The table and graph in Figure 4 below show some of the data collected from a random sample of 100 Year 8 students. Although students will be familiar with multiple column graphs they may not have used two-way tables. By presenting both incomplete representations of the same data, students are able to discover for themselves the way information is recorded in a two-way table. Discuss the following:

- *Tell me something you can read from these representations? How do you know that?*

Use this data in as many ways as possible to answer these questions:

- *Based on this sample of Year 8 students, do you think more boys than girls eat breakfast?*

In this discussion, students consider how well this data fits their class. This is an opportunity to consider why your class might, or might not be, typical of all Australian Year 8’s:

- *Based on this sample of Year 8 students, what is the probability if you picked a boy at random from our class that he would have had breakfast?*
- *Based on this sample of Year 8 students, what is the probability if you picked a girl at random from our class that she would have had breakfast?*

Using the popsicle sticks from the activity in ‘Example 3: How well do you know your class mates’, randomly select a boy and girl from your class. Discuss the likelihood of the event before, and after the selection, to practise the language of probability (highly unlikely, more likely, possible, not probable) and the variation in sampling (surprising result, expected, etc).

	Ate breakfast	Didn’t eat breakfast	Total
Female		11	
Male			40
Total			100

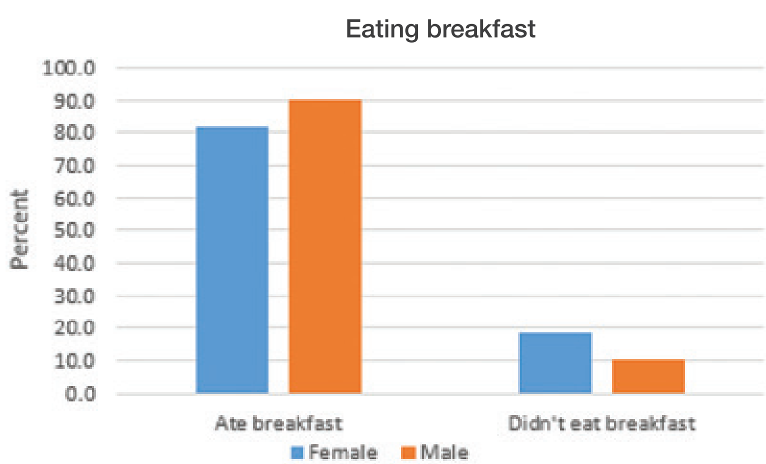


Figure 4

Example 5: Area model for two-step experiments

ACMSP292 ◆ ◆
Students represent events in two-way tables and Venn diagrams and solve related problems.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about independent events, we can challenge students to identify them for themselves, by ***asking*** questions.

This activity draws on students' understanding of an area model that they may have already used to represent the multiplication of larger whole numbers, fractions and expanding algebraic fractions.

The intention of this task is for students to notice that the probability of two independent events both occurring, is equal to the product of the probabilities of the two events, ie P(A and B) = P(A) x P(B), if A and B are independent.

Activities like this **replace** the need for us to make statements such as '**And** means times, and **or** means add'. While we all remember these rules, we do not always understand when to apply them or why they work. Referring back to the notion of a tree diagram or an area model like this, supports students to understand which operation is needed to calculate the required probability.

Conduct an experiment that involves spinning a spinner with equal sections of blue, green and yellow, and drawing a card from a pack of playing cards. (A simpler experiment would be to use the spinner and a die). Ask:

- *What is the chance of getting a blue on the spinner? What is the chance of getting a heart?*

Now consider getting both on one turn. Ask:

- *What is the probability of getting a blue on the spinner and a card which is a heart?*

Record student responses and ask:

- *How could we check this?*

Explain to your class that probabilities can be fractions, which can be represented by shading shapes. This idea can be used to check our answer:

- *Using a rectangle, can you shade an area that represents getting a blue on the spinner? Why does that represent the result? Is there another way?* (The whole rectangle represents all the equal likely events of a colour on the spinner and card from the pack, such as Blue2Hearts, Green5Clubs of which there are 3 x 52 = 156 possibilities. One third of these will have a blue on the spinner and any of the 52 cards.)
- *Now if I want to record which suit the card is (spades, clubs, hearts or diamonds) as well, how could you illustrate that on the same rectangle? How does that represent the possible results? Is there another way?* (Allow students to use physical objects to sort the outcomes. The playing cards in the blue section can be sorted into Blue/Hearts and Blue/Other suits.)

A possible illustration is shown below in Figure 5.

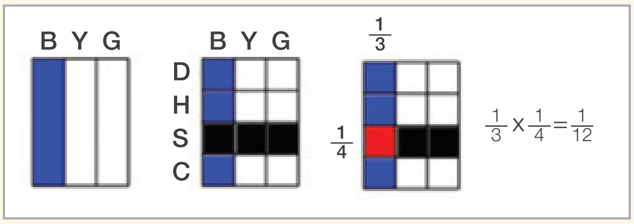


Figure 5

Ask students:

- *Which area on your diagram represents getting a blue on the spinner and a card which is a heart? What fraction is that? Is that what you expected? Does it make sense?*
- *Could we check with another method? Could we try out this method for another outcome (for example, a yellow and a black card)?*

Ask students to make some possible combinations and calculate the probability using more than one way, and then convince someone else that it is correct. Ask them to discuss:

- *What do you notice? Can the probability of all the outcomes be worked out the same way?*
- *Can you write an explanation for a Year 7 student to use your method?*
- *How did you use area to work with fractions?*

Students can also use tree diagrams.

Backwards questions

A ‘backwards’ question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

As a challenge, you can ask students backwards questions. Shade areas (like those shown in Figure 6) and ask students to determine which events have a probability represented by that area.

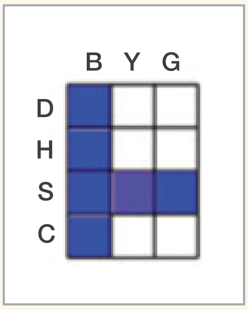


Figure 6

Notice that the area shaded represents the probability that the result is a *blue on the spinner*, ***or*** *the card is a spade*, ***or*** *both*, which illustrates the ‘**inclusive or**’ explained in detail in ‘Example 3: How well do you know your class mates?’

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 6–8)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem-solving examples	
Example 6: Dice battles Students identify complementary events and use the sum of probabilities to solve problems. Students represent events in two-way tables and Venn diagrams and solve related problems.	 ACMSP204 ◆ ◆ ACMSP292 ◆ ◆
Example 7: What sort of detective are you? Can you pick a fake? Students identify complementary events and use the sum of probabilities to solve problems. Students represent events in two-way tables and Venn diagrams and solve related problems.	 ACMSP204 ◆ ◆ ACMSP292 ◆ ◆
Example 8: A chance to win? Students represent events in two-way tables and Venn diagrams and solve related problems.	 ACMSP292 ◆ ◆

Example 6: Dice battles

ACMSP204 ◆ ◆

Students identify complementary events and use the sum of probabilities to solve problems.

ACMSP292 ◆ ◆

Students represent events in two-way tables and Venn diagrams and solve related problems.

Questions from the BitL tool

Problem-solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

Students at this stage of development are familiar with expected outcomes when throwing conventional dice, but it becomes a problem-solving task when outcomes are no longer equally likely.

Consider three different dice, with the numbers on the 6 faces as follows:

Die 1: 3 3 3 3 3 6
Die 2: 2 2 2 5 5 5
Die 3: 1 4 4 4 4 4

In a dice battle between two players, each player picks a die. The two dice are then rolled together and whoever gets the highest value wins. The ultimate winner is the player with the most wins in 10 rolls. Discuss the following with your class:

- Is this a fair game?
- Which die should you choose and why?

These dice are like *Rock, Paper, Scissors* as Die 1 beats Die 2 beats Die 3 beats Die 1 (non-transitive). Students will notice this very quickly but not necessarily be able to understand why it happens. Encourage students to represent the sample space. At Year 8 level, this can be done using a table (Lattice diagram as shown in Figure 7 below), or tree diagram of all possible outcomes so you can see which of the 36 options are won by which die.

Die 2						
5	w2	w2	w2	w2	w2	w1
5	w2	w2	w2	w2	w2	w1
5	w2	w2	w2	w2	w2	w1
2	w1	w1	w1	w1	w1	w1
2	w1	w1	w1	w1	w1	w1
2	w1	w1	w1	w1	w1	w1
	3	3	3	3	3	6
			Die 1			

Figure 7

This may be an opportunity for some students to consider a 'weighted' tree diagram when they find that a tree diagram showing all 36 simple events is large and difficult to draw. As a teacher, you can present an incomplete, or not completely correct sample of work (see Figure 8) from a fictitious student to stimulate discussion:

'A student decided that a tree diagram was too big and decided to draw it like this to work out the probability of Die 1 or 2 winning.'

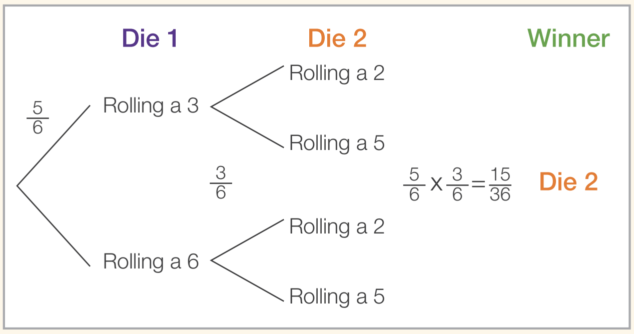


Figure 8

Ask students:

- *Does it look right to you? Would it always work? Would it work for Die 1 and Die 3? Could you use this method to see who would win out of Die 2 and 3?*
- *How is this the same, or different, to the Lattice diagram?*
- *Why does it work?* ($\frac{5}{6}$th of the time you expect to get a 3 and then $\frac{3}{6}$ of that ($\frac{3}{6}$ of $\frac{5}{6} = \frac{3}{6} \times \frac{5}{6}$) you expect to get a 5 so the probability of rolling a 3 on Die 1 and a 5 on Die 2 would be $\frac{15}{36}$.)

When students attempt to work out the probability of Die 1 winning, it leads them to the sum of mutually exclusive events which conceptually can be linked back to different areas in the Lattice diagram.

(A similar activity can be found on page 18 of the <u>Chance: Year 9</u> narrative.)

Interpret

What have you been asked to do? What information is helpful/not useful? (Establish that the student understands how the game is played and even encourage them to guess which one they think might be better.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help if you played a few games? Are there different ways that you could do that? How many games will be enough? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Teachers can become more supportive if students cannot make progress by asking:

- *How could you determine which of Die 1 and Die 2 is most likely to win? Would one game be enough? Is 50 enough? What will you do if you find that Die 1/Die 2 is best? Do you have a new idea now?*

Solve and check

What did you learn from the games? Does this make sense? How could you investigate that? What is unusual about what you discovered? What's that similar to? Could you explain why this happens? How might you represent the ways that Die 1 could win against Die 2? How likely would that be? Is there another way that you could have solved this problem?

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for other strange dice? What if they were 4-sided or 8-sided dice – could you still use your method? What if you rolled two of each?

Example 7: What sort of detective are you? Can you pick a fake?

ACMSP204 ◆ ◆
Students identify complementary events and use the sum of probabilities to solve problems.

ACMSP292 ◆ ◆
Students represent events in two-way tables and Venn diagrams and solve related problems.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This activity uses empirical evidence to challenge students' beliefs about randomness and expected outcomes for independent events.

In this activity, you can challenge students to investigate a suspected counterfeit ring using their own observations, knowledge of random events and problem-solving skills.

This task challenges students' expectations for a process that is very familiar to them, tossing a coin. In 100 tosses, students expect there to be approximately 50 of each outcome: heads and tails. While they do not expect these two outcomes to alternate, they don't expect there will be a long run of one outcome or the other. Surprisingly, in 100 tosses it is almost certain that there will be a run of at least 6 heads or 6 tails.

Distribute a numbered recording sheet to each student in the class for anonymous identification purposes. As a homework task, ask students to complete a record of the outcomes of 100 tosses of a coin. Before they begin recording, instruct them to toss the coin once. If the result is a head, the students must genuinely toss the coin 100 times. If, however, the result is a tail, they are to record the outcomes they might expect if they were tossing the coin. (Impress upon them that it is important that you have accurate data if it is to be a valid investigation.)

If you examine the recording sheets, you can identify the authentic data sets as they will have a significant number of 'clusters', or runs of heads or tails. Students who are 'faking' the data will not have long runs of heads or tails. Use this knowledge to sort the sheets into two piles. Ask the students to identify if you have placed their sheet in the correct pile.

It is important to stress that you are not magical, or performing any trick. You have only used your mathematical skills to make decisions. As detectives, they are challenged to determine a method that can be used by others to identify fake data.

Note: there is a digital activity that students can use as part of their investigation called 'Coin tossing' from the National Library of Virtual Manipulatives: http://nlvm.usu.edu. The activity is also on Scootle: http://www.scootle.edu.au/ec/viewing/L3515/index.html.

(A version of this activity also appears in the *Data representation and interpretation: Year 8* narrative.)

Interpret

What have you been asked to find? What information is helpful/not useful? (Students know if their own data is authentic or not. Observing the sheets in each pile and trying to identify what is the same about the sheets in the authentic pile and how that is different to the sheets in the 'faked' pile, would give significant clues as to the method you used when sorting. The task is most challenging if they receive the unsorted pile of the data sets, as you did.)

Model and plan

Do you have an idea? How might you start? Would it help if you organised the data? Are there different ways that you could do that? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying. Students do not often value the information they get from visually checking the data for patterns, and that is a key skill in approaching this problem.)

Can you see any differences, or similarities, between the authentic and faked data results?

Solve and check

If the data looks different, how could you investigate that? Is there something about the data that you could count and record? How could you represent this so that others could see the pattern? (Students could record the number of runs of different lengths, ie: a run of 1 (head or tail); or a run of 2 (head/head or tail/tail), etc for authentic and ‘faked’ data to compare. Notice that it does not matter if it is a run of heads or a run of tails, it is only the length that is of interest. These runs can be recorded and graphed. Choosing an appropriate table and graph type are evidence of fluency. Once students have developed a conjecture (theory) for how to tell the difference, they can develop a written statement about their process to help others identify a ‘fake’. This is an opportunity for students to demonstrate their reasoning skills.)

How could you check your findings? (To extend this activity, you could provide a new collection of recording sheets; some that have been randomly generated and others that a group of teachers have faked, and challenge the students to catch the ‘criminals’.)

Supporting writing in mathematics

For students who need more support in their writing, develop a wordlist they could choose from, or provide some sentence starters. You could also consider recording the student explaining it to someone else, and then using that to sequence their written explanation.

Reflect

Could you improve your method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Would you always pick the fake correctly? Will your method work for any data sets? (Now that the students have more knowledge, if you conducted the experiment again, the results would not be the same.)

Tackling misconceptions: Knowing what it isn’t

It can work well if the teacher compiles a number of examples that reflect their students’ misconceptions. The examples don’t have to be presented as their misconceptions, just as examples of thinking that are commonly seen when people are learning to work with data. Once the students have identified the issues, they can reflect on the effect that it has had on their own thinking. For example:

One group said they threw 10 heads in row. Another group said that was impossible with a fair coin, so they must have cheated. What do you think? (It is highly unlikely. Possible, but not probable.)

While the students are demonstrating their understanding of the statistical investigation process in this activity, there are important concepts relating to chance that can be drawn from this experience:

- The coin has no memory. The outcome on one toss has no effect on the next. They are independent of each other and this is the official mathematical term. (Students can add this to their Word Wall).
- Check the relative frequency of throwing a head and throwing a tail from the 100 tosses. They should both be approximately ½, as even though there are some long runs of heads, there will also be long runs of tails. The relative frequency is usually approximately the same as the theoretical probability, or expected outcome when there are a large number of trials. Would this have been true if they had only thrown the coin 10 or 20 times?

Example 8: A chance to win?

ACMSP292 ◆ ◆
Students represent events in two-way tables and Venn diagrams and solve related problems.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

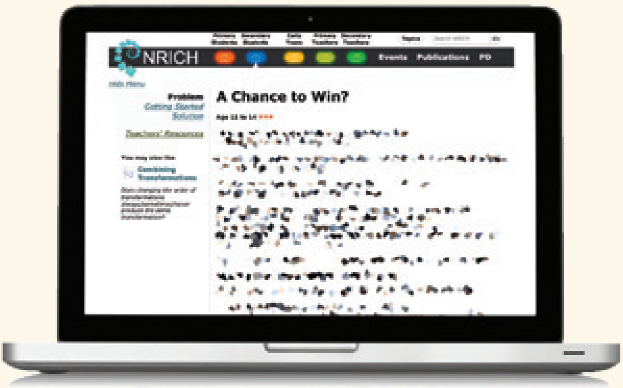
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This activity from the NRICH website is an exploration in chance, which most students enjoy because it involves money and is interactive. The amounts are in British Pounds (GBP) but can be played using Australian Dollars (AUD).

This is an opportunity to not only discuss conversion and exchange rates with money, but also issues relating to gambling and its social implications. Tree diagrams are a useful tool to consider the sample space for multiple trials. The number of cards can be altered to differentiate, or extend the task.

The link to the problem on the NRICH website is: http://nrich.maths.org/5611



Connections between ‘Chance’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use Chance as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 8	
Whilst working with Chance, connections can be made to:	**How the connection might be made:**
Students carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies. ACMNA183	Refer to: **All examples.**
Students solve a range of problems involving rates and ratios, with and without digital technologies. ACMNA188	Refer to: **All examples.**
Students investigate techniques for collecting data, including census, sampling and observation. ACMSP284	Refer to: **Example 3: How well do you know your classmates?** **Example 4: Year 8 students don’t eat breakfast** **Example 6: Dice battles** **Example 7: What sort of detective are you? Can you pick a fake?**
Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes. ACMSP206	Refer to: **Example 3: How well do you know your classmates?** **Example 4: Year 8 students don’t eat breakfast** **Example 6: Dice battles** **Example 7: What sort of detective are you? Can you pick a fake?**
Students explore the variation of means and proportions of random samples drawn from the same population. ACMSP293	Refer to: **Example 3: How well do you know your classmates?** **Example 4: Year 8 students don’t eat breakfast** **Example 6: Dice battles** **Example 7: What sort of detective are you? Can you pick a fake?**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Chance’ from Year 1 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Chance:

Identify and order chance events ◆
In Year 1 to Year 2 students identify and order chance events.

Identify, describe and represent sample spaces ◆
In Year 1 to Year 5 students describe and represent sample space. In Year 6 to Year 8 students mostly describe more complex sample spaces and assign probabilities. In Years 9 to Year 10A students mostly represent complex sample spaces and events.

Randomness and variation ◆
In Year 3 students conduct chance experiments and recognise variation in results. In Year 9 students calculate from given or collected data. In Year 10A students investigate reports of studies in digital media and elsewhere for information on their planning and implementation.

Observed frequencies and expected probabilities ◆
In Year 3 to Year 5 students consider expected probabilities of simple events. In Year 6 students quantify probabilities. In Year 9 and Year 10 students determine probabilities of compound events.

Language ◆
Throughout Year 1 to Year 10A students use the language of chance in increasingly sophisticated ways. In Year 8 students explore the particular language relating to the exclusive or inclusive references to events. In Year 10 students make and appraise inferential statements relating to chance.

Year level	‘Chance’ content descriptions from the AC: Mathematics
Year 1 ◆ ◆	Students identify outcomes of familiar events involving chance and describe them using everyday language such as ‘will happen’, ‘won’t happen’ or ‘might happen’. ACMSP024
Year 2 ◆ ◆ ◆	Students identify practical activities and everyday events that involve chance. Describe outcomes as ‘likely’ or ‘unlikely’ and identify some events as ‘certain’ or ‘impossible’. ACMSP047
Year 3 ◆ ◆ ◆	Students conduct chance experiments, identify and describe possible outcomes and recognise variation in results. ACMSP067
Year 4 ◆ ◆	Students describe possible everyday events and order their chances of occurring. ACMSP092
Year 5 ◆ ◆ ◆	Students list outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions. ACMSP116
Year 6 ◆	Students describe probabilities using fractions, decimals and percentages. ACMSP144
Year 7 ◆ ◆	Students construct sample spaces for single-step experiments with equally likely outcomes. ACMSP167
Year 8 ◆ ◆	Students identify complementary events and use the sum of probabilities to solve problems. ACMSP204
Year 8 ◆	Students describe events using language of ‘at least’, ‘exclusive or’ (A or B but not both), ‘inclusive or’ (A or B or both) and ‘and’. ACMSP205
Year 8 ◆ ◆	Students represent events in two-way tables and Venn diagrams and solve related problems. ACMSP292
Year 9 ◆ ◆	Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events. ACMSP225
Year 9 ◆ ◆ ◆	Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’. ACMSP226

Year 9 ◆ ◆	Students investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians. ACMSP227
Year 10 ◆ ◆ ◆	Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence. ACMSP246
Year 10 ◆	Students use the language of 'if ....then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language. ACMSP247
Year 10A ◆	Students investigate reports of studies in digital media and elsewhere for information on their planning and implementation. ACMSP277

Numeracy continuum: Interpret chance events	
End Foundation	Recognise that some events might or might not happen.
End Year 2	Identify and describe familiar events that involve chance.
End Year 4	Describe possible outcomes from chance experiments using informal chance language and recognising variations in results.
End Year 6	Describe chance events and compare observed outcomes with predictions using numerical representations such as a 75% chance of rain or 50/50 chance of snow.
End Year 8	Describe and explain why the actual results of chance events are not always the same as expected results.
End Year 10	Explain the likelihood of multiple events occurring together by giving examples of situations when they might happen.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

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NRICH is part of the family of activities in the Millennium Mathematics Project.

Dan Meyer's blog: 101 questions

http://www.101qs.com

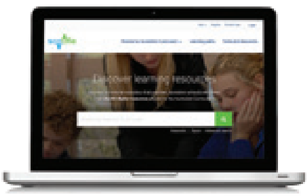
Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Scootle

https://www.scootle.edu.au/ec/p/home

This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.



reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

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Numeracy in the News

http://www.mercurynie.com.au/mathguys/mercury.htm

Numeracy in the News is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.

TIMES modules

http://schools.amsi.org.au/times-modules/

TIMES modules are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.

Top drawer teachers – resources for teachers of mathematics (statistics)

http://topdrawer.aamt.edu.au/Statistics

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.

Double Helix Extra

https://blog.doublehelix.csiro.au/

This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.

CensusAtSchool NZ

http://new.censusatschool.org.nz/tools/random-sampler/

CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics. It aims to:

- *'foster a positive attitude to statistics through using data that is both relevant and real*
- *improve understanding of a data gathering process, its purposes and benefits to society*
- *provide access to large and meaningful multivariate data sets*
- *encourage effective IT teaching and learning*
- *enhance the process of statistical enquiry across the curriculum.'*

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

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Excluded from NEALS

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