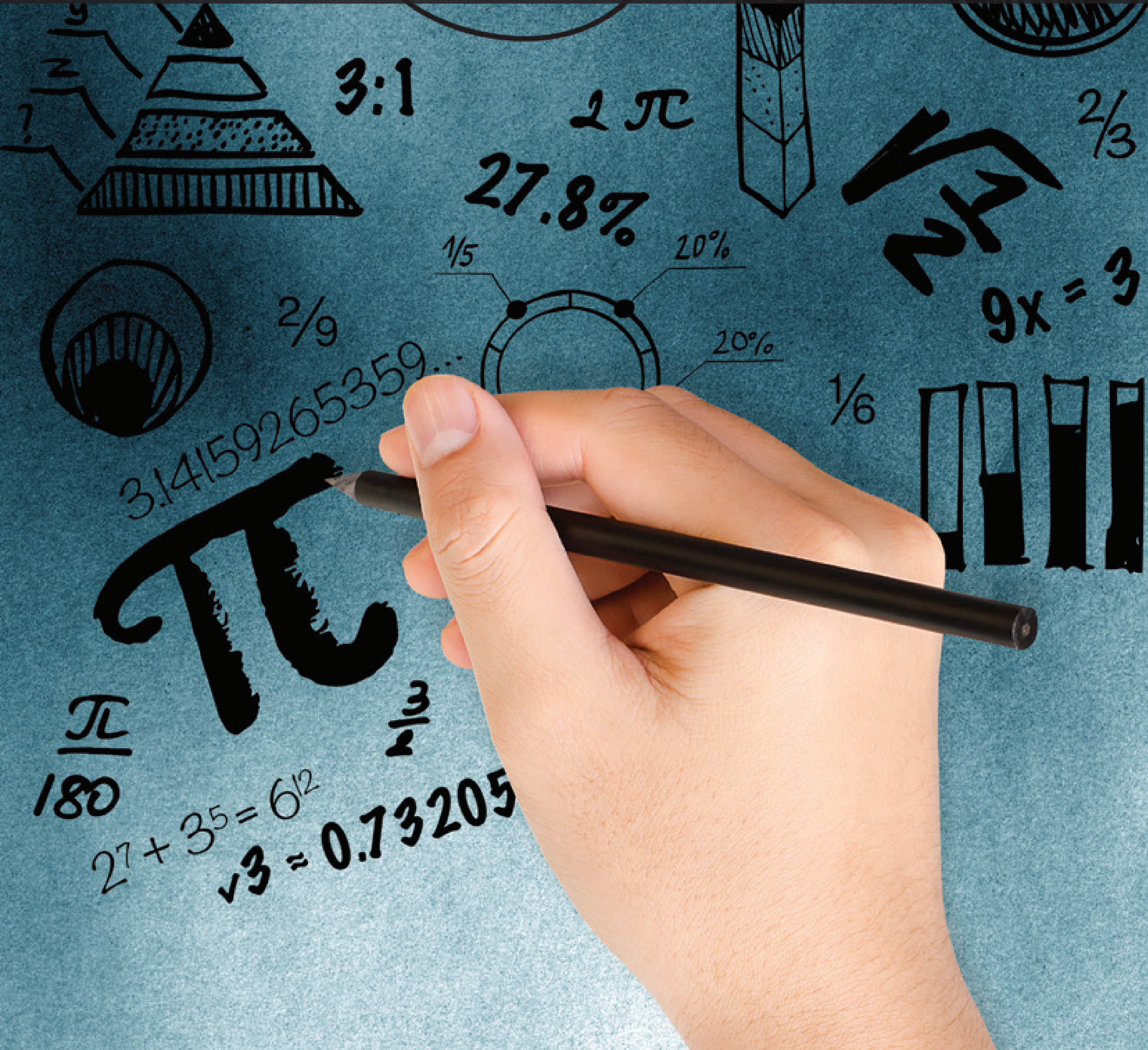
Real numbers: Year 10/10A

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Real numbers' from Year 7 to Year 10A** (see page 18)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with real numbers:

- Recognise, describe and represent real numbers
- Compare and order real numbers
- Convert and calculate using real numbers
- Apply and solve problems using real numbers.

What the Australian Curriculum says about ‘Real numbers’

Content descriptions

Strand | Number and algebra.

Sub-strand | Real numbers.

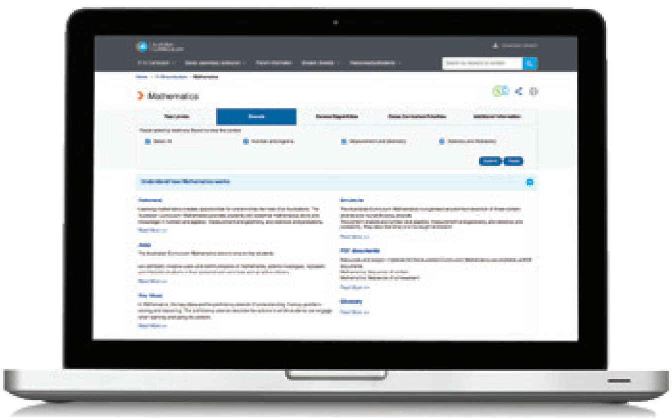
Year 10A ◆ ◆ | ACMNA264
Students define rational and irrational numbers and perform operations with surds and fractional indices.

Year 10A ◆ ◆ ◆ | ACMNA265
Students use the definition of a logarithm to establish and apply the laws of logarithms.

Numeracy continuum

Using fractions, decimals, percentages, ratios and rates

End of Year 10 ◆ | Students illustrate and order relationships for fractions, decimals, percentages, ratios and rates (Interpret proportional reasoning). Students solve problems involving fractions, decimals, percentages, ratios and rates (Applying proportional reasoning).



Source: ACARA, Australian Curriculum: Mathematics

Working with Real numbers

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 10/10A 'Real numbers'

In Year 7 students convert between any fraction, decimal and percentage. Students develop strategies to multiply and divide fractions and decimals, rounding to a given number of decimal places.

In Year 8 students investigate terminating and recurring decimals, introducing the concept of irrational numbers and linking this to the development of an understanding of π.

In Year 9 students build on their fluency in multiplying and dividing decimals by powers of ten and begin to express numbers in scientific notation. An application of this is seen in 'Using units of measurement' when Year 9 students investigate large and small time scales.

In Year 10A students would be expected to define rational and irrational numbers:

- Notice that irrational numbers are in Year 8 and Year 10A but not in Year 9 or 10. Therefore it is important to provide opportunities for students to work with these numbers in other sub-strands like Pythagoras' Theorem (see 'Example 20: Where is $\sqrt{2}$ on the number line?' in the *Pythagoras and trigonometry: Year 9* narrative).
- While indices are in the 'real number' sub-strand in Year 9 and 10A, they appear in the 'number and place value' sub-strand in Year 7 and 8 and the 'patterns and algebra' sub-strand in both Year 9 and Year 10. This is to be expected as indices is a concept that is integral to number and algebra. As teachers we need to be mindful of this in our planning so that learners are engaging with indices in a connected and consistent way.
- Often students are very successful in those applications of the index laws which require trivial transfer of the law and/or applying only one law. When students do not have a conceptual understanding of indices as a mathematical notation; multi-step applications or the application of several different laws, reveal serious misconceptions. As teachers, our learning design must build conceptual understanding, allowing students to recognise patterns and connections for themselves and also allow them to identify the flaws in thinking that lead to common misconceptions.

Engaging learners

Classroom techniques for teaching Real numbers

Power of scientific notation

Showing students videos such as *Powers of Ten™ (1977)* or *Cosmic Voyage* gives students some understanding of the purpose and power of scientific notation, filling them with wonder about the size of our universe and the atoms which it consists of.

The ***Powers of Ten™ (1977)*** video can be found at: https://www.youtube.com/watch?v=0fKBhvDjuy0

An extract from the ***Cosmic Voyage*** video can be found at: https://www.youtube.com/watch?v=qxXf7AJZ73A

Source: *Powers of Ten™ (1977)*, Eames Office, published 2010

Understanding rates and ratios

Scootle digital learning activities can be used in conjunction with concrete materials to hook students into their learning. Being able to work flexibly in a practical situation supports the conceptual understanding of rates and ratios.

Rates and scales activities can be found at: http://www.scootle.edu.au/ec/viewing/L8099/index.html

Ratio and graphs activities can be found at: http://www.scootle.edu.au/ec/viewing/L8103/index.html

Source: *In proportion: rates and scales*, Scootle, Education Services Australia

Estimation 180

Estimating with rates encourages students to connect their intuitive knowledge about rates to practical situations, building fluency and conceptual understanding.

Estimation 180 is a website that has multiple scenarios that can prompt students to estimate, inviting every learner to have an opinion. This resource could be used as a warm-up to lessons, choosing contexts that build their number sense.

The **Estimation 180** activities can be found at: http://www.estimation180.com/blog/going-the-distance-180-challenges

Source: *Estimation 180: Building number sense one day at a time,* by Andrew Stadel

Legend of the chessboard

The famous legend about the origin of chess states that the Emperor of India was so impressed by the new game, that he told the inventor to name his reward. The inventor requested one grain of rice for the first square of the chessboard, two grains for the second, four for the third and so on, doubling the amount for each of the 64 squares. The Emperor agreed, thinking that the man had asked for a small reward.

Ask students to estimate how much that might be, both before and after, doing 'Example 2: Pocket money options – powers of 2'.

After a week, the Emperor's treasurer reported that the reward would add up to an astronomical sum, far greater than all the rice that could conceivably be produced in many centuries! In fact, enough to feed 100 tonnes of rice to every single human on Earth, or 1 kg of rice per day per human for 275 long years and more than a millennium worth of global rice production (approximately 100 million tonnes each year).

The Legend of the Chessboard (Teaser) video can be found at: https://www.youtube.com/watch?v=byk3pA1GPgU

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–5b)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the names and notation for the irrational numbers and logarithms** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged with questions that will result in them identifying the **particular properties of irrational numbers and logarithms that are similar or different to the real numbers they already know**, so I don't need to instruct that information.

At this stage of development, students can **develop an understanding of irrational numbers by noticing similarities and differences to the things they already know about other real numbers**. When teachers provide opportunities for students to explore multiple representations of irrational numbers, they require their students to generalise from the patterns and relationships they observe. Telling students properties and laws removes this natural opportunity for students to make conjectures and verify connections that they notice.

Teachers can support students to identify and define irrational numbers by asking questions as described in understanding proficiency, *What patterns/connections/ relationships can you see?* The intent of this question is to promote learning design that intentionally plans for students to develop a disposition towards looking for patterns, connections and relationships. Using questions such as the ones described here, supports teachers to replace 'telling' the students information, with getting students to notice for themselves.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular rule or theorem during the current unit of work, then it probably is quicker to tell them the rule and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the rule is a false economy of time.

When we challenge our students to **establish a theorem**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Where is √2 on the number line? Students define rational and irrational numbers and perform operations with surds and fractional indices.	ACMNA264 ◆ ◆
Example 2: Paper sizes (√2) Students define rational and irrational numbers and perform operations with surds and fractional indices.	ACMNA264 ◆ ◆
Example 3: Irrational numbers and a musical semitone Students define rational and irrational numbers and perform operations with surds and fractional indices.	ACMNA264 ◆ ◆
Example 4: Logarithms Students use the definition of a logarithm to establish and apply the laws of logarithms.	ACMNA265 ◆ ◆ ◆
Example 5a: Laws of logarithms Students use the definition of a logarithm to establish and apply the laws of logarithms.	ACMNA265 ◆ ◆ ◆
Example 5b: Logarithms are indices Students use the definition of a logarithm to establish and apply the laws of logarithms.	ACMNA265 ◆ ◆ ◆

Example 1: Where is √2 on the number line?

ACMNA264 ◆ ◆
Students define rational and irrational numbers and perform operations with surds and fractional indices.

Questions from the BitL tool
Understanding proficiency:
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about irrational numbers, we can challenge students to recognise the relationships between different representations and the numbers they describe, by ***asking*** questions.

In this activity, instead of telling students about irrational numbers, we can challenge them to identify this for themselves by asking the following questions:

- *What two integers does √2 lie between on a number line? How do you know?*
  When asked to find √2 on a number line a student drew this.

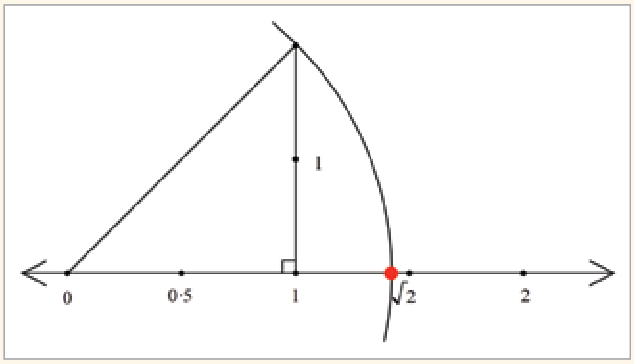


Figure 1

- *How might you check whether the location is correct using this method?*
- *Can you explain the student's thinking?*
- *Now that you have seen this, what other irrational numbers might you locate on the number line using this method?*

Further tasks that support students to create their own understanding of the content of 10A mathematics can be found in the Mathematical Association of South Australia publication, 'Rich Tasks for 10A Mathematics' which can be accessed at: https://masanet.wildapricot.org/resources/Publications/Current%202017%20Publications_Booklet..pdf

(This activity also appears in the *Pythagoras and Trigonometry: Year 9* narrative.)

Example 2: Paper sizes (√2)

ACMNA264 ◆ ◆
Students define rational and irrational numbers and perform operations with surds and fractional indices.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

In what ways can your thinking be generalised?

Instead of ***telling*** students about irrational numbers, we can challenge students to recognise their significance through investigating similar shapes for themselves, by ***asking*** questions.

Begin a discussion about paper sizes that the students may be familiar with. Ask students:

- *What different paper sizes are you familiar with? What are they used for? Why?*

Issue the students with A3, A4 and A5 sheets of paper, scissors, rulers and sticky tape. Ask students:

- *What is the same/different about these sheets of paper?* (Students often identify an **additive** relation between the different sizes, for example, 'The difference goes up each time'. Encourage students to find a **multiplicative** relationship.)
- *If you know the measurements of these three sheets, what might be the size of A2 and A6 sheets? Why?*
- *What might be an estimate of the size (dimensions) and area of an A0 size sheet? How could you check?*
- *What is the scale factor for the enlargement for A5 to A4, A4 to A3? Can you generalise the relationship between 'consecutive' paper sizes, for example An and A(n+1)?* (The area doubles but the scale factor of the enlargement is √2. This can be verified by measuring the lengths of sides of consecutive paper sizes and finding the ratio. At this level, students should be encouraged to prove this algebraically. This concept is encountered in both Example 1 and Example 4 in the *Geometric reasoning: Year 9* narrative and is a good connection with irrational numbers.)

For students who are seeking an intellectual challenge, ask:

- *Are paper sizes the same all over the world?*
- *What is the difference/same between paper sizes A, B and C? Are they similar rectangles? Can you generalise the ratios between the paper sizes B and C?*

Links to authentic contexts

Using examples/analogies from real life situations engages students and can contextualise the purpose of a mathematical process. Students are most likely to be familiar with the A5, A4 and A3 sizing of paper, but may not be aware that the ratio of the sides is an irrational number.

(This activity also appears in the *Geometric reasoning: Year 9* and *Real numbers: Year 8* narratives.)

Example 3: Irrational numbers and a musical semitone

ACMNA264 ◆ ◆
Students define rational and irrational numbers and perform operations with surds and fractional indices.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

In what ways can your thinking be generalised?

Instead of ***telling*** students about irrational numbers, we can challenge students to recognise their significance, by ***asking*** questions.

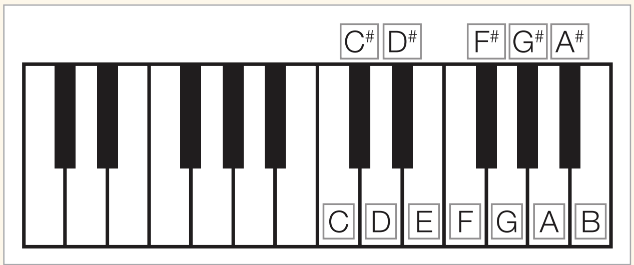


Figure 2: https://pixabay.com/en/piano-musical-instrument-keyboard-312543/
Source: Clker-Free-Vector-Images, Pixabay

This activity explores irrational numbers in musical semitones.

When you pluck a string, it vibrates at a certain frequency according to its length and tension. The modern piano has been designed to have an equally-tempered scale.

Two adjacent notes are referred to as a half-step, or semitone (eg C to C$^{\#}$) and **12 half-steps** make up an octave.

Instruct your class to find the frequency of the semitones for at least an octave of notes, by asking:

- *What patterns do you notice?*
- *Do they go up by the same amount? Are they multiplied by the same number? How would you find out?* (They don't increase by the same amount as their differences are not constant, but their ratio is (eg $\frac{C^{\#}}{C} = \frac{D}{C^{\#}}$ etc.))

Compare the frequency of the C notes in successive octaves. Ask:

- *What do you notice?* (It is roughly double.)
- *If the ratio is the same for each semitone, how does that ratio $\frac{C^{\#}}{C}$ relate to 2?* (One student said it was $\frac{1}{12}$ of 2.)
- *Does that make sense? How could we check?*
- *What might it be?*
  $2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a \times 2^a$
  $= 2$ so $a = \frac{1}{12}$
  The ratio is $2^{\frac{1}{12}}$

Further tasks that support students to create their own understanding of the content of 10A mathematics can be found in the Mathematical Association of South Australia publication, 'Rich Tasks for 10A Mathematics' https://masanet.wildapricot.org/resources/Publications/Current%202017%20Publications_Booklet..pdf

Example 4: Logarithms

ACMNA265 ◆ ◆ ◆
Students use the definition of a logarithm to establish and apply the laws of logarithms.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

In what ways can your thinking be generalised?

Instead of ***telling*** students about logarithms, we can challenge students to recognise their significance, by ***asking*** questions.

Before defining what a logarithm is, allow students to explore the log button on their calculators.

Using examples which includes numbers besides powers of 10, stimulates further discussion and exploration. For example:

Have you ever noticed the log button on your calculator? If you have an idea what it does, write down what you think log100 might be and why, then support others as we explore this idea further.

Write these expressions on separate post-it notes and evaluate them using your calculator. Sort them in some way and explain your thinking. Make a conjecture.

log0 log1 log2 log3 log4 log5 log6

log10 log0.1 log100 log20 log1000 log0.01

Ask students:

- *What patterns do you notice?* (When students notice the pattern for powers of 10 ($\log 10^n = n$), ask them if they could predict the log of another number, then verify it with the calculator. If students interpreted log as something like 'The power of 10 that gives …', ask them if this makes sense for the other values.)
- *If log means 'The power of 10 that gives …', why does it make sense that log20 would be between 1 and 2? Or that log0.1 would be a negative number? Does it make sense that log1 would be 0 and that the calculator couldn't give a value for log0?* ((10^1 =10) and (10^2 =100). As 20 lies between 10 and 100, its logarithm will lie between 1 and 2.)
- *What other questions might you have?* (Could you find the log of a negative number, for example, log($^-10$)? What 'Power of 10 gives ...' a negative result? No power of a positive real number gives a negative result and again the calculator does not give a value. Students may notice that (log4 = 2 x log2), or that (log20 = 1+ log2), etc.)
- *What patterns do you notice? How might you verify this? Why does this work?* (Any unresolved conjectures can be displayed so that as they learn more about logarithms, they can come back to these ideas.)
- *How do Mathematicians define a 'log'? How might you find out?*
- *How does this compare to your definition?*
- *Where are logarithms used?*

(Before calculators were used in schools, logarithms were one way of simplifying numerical calculations. Having tables to change from logarithms to numbers and back, it was much easier to add and subtract the logs then it was to multiply and divide the numbers.

Logarithmic scales include:

- Richter Scale to measure the intensity of an earthquake
- decibels to measure the intensity of sound
- Google PageRank to indicate the authority or significance of a page
- scientific contexts such as pH, which is a logarithmic scale of acidity
- any quantity growing or decaying exponentially.)

Further tasks that support students to create their own understanding of the content of 10A Mathematics can be found in the Mathematical Association of South Australia publication, 'Rich Tasks for 10A Mathematics' https://masanet.wildapricot.org/resources/Publications/Current%202017%20Publications_Booklet..pdf

Example 5a: Laws of logarithms

ACMNA265 ◆ ◆ ◆
Students use the definition of a logarithm to establish and apply the laws of logarithms.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

In what ways can your thinking be generalised?

Instead of ***telling*** students about the laws of logarithms, we can challenge students to identify them for themselves, by ***asking*** questions.

This activity relates to Example 4: Logarithms.

Share with the class that from explorations in Example 4, some students identified that:

log4 = (2 x log2)   log20 = (1 + log2)   log6 = (log2 + log3)
(log100 - log20) = log5

For one of these relationships (or one of your own) consider the following questions:

- *What patterns do you notice?*
- *What conjecture can you make? How might you verify this?*
- *How might you generalise this discovery? Why does this work?*
- *Pair up with someone who thought differently to you or someone who investigated a different pattern.*

  (Have groups present their thinking to the class, describing their conjecture in both words and symbols.

  If you add the log of one number to the log of another, then the result will be the log of the two numbers times together.
  log a + log b = log(a x b)

  The log of (a power of a number eg, an) is the exponent times by the log of the base number.
  log an = n x log a

  When you take away the logs of two numbers, you get the log of the two numbers divided.
  log a - log b = log(a÷b)).

- *Which of these laws might explain that the log20 = log2 + 1?*

  log20 = log(2 x 10)
  = log2 + log10 (using log a + log b = log(axb))
  = log2 + 1

Further tasks that support students to create their own understanding of the content of 10A mathematics can be found in the Mathematical Association of South Australia publication, ‘Rich Tasks for 10A Mathematics’ https://masanet.wildapricot.org/resources/Publications/Current%202017%20Publications_Booklet..pdf

Example 5b: Logarithms are indices

ACMNA265 ◆ ◆ ◆
Students use the definition of a logarithm to establish and apply the laws of logarithms.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

In what ways can your thinking be generalised?

Instead of ***telling*** students the laws of logarithms are similar to the index laws, we can challenge students to make a connection for themselves, by ***asking*** questions.

This activity relates to Example 4: Logarithms and Example 5a: Laws of logarithms.

Explain to the class that in previous examples we have seen how logarithms relate to powers of 10:
ie if $100 = 10^2$ then $\log 100 = 2$

At this level, students are familiar with the index laws. It often makes sense to students to see that the logarithms are the indices of the powers, and then realise the connection between the index laws they are familiar with, and the laws of logarithms.

The following card sort supports students to connect: an exponential equation, a logarithmic equation, index laws and logarithmic laws.

This task can be differentiated by using cards that only relate to one logarithmic law. It can be made more challenging by using bases other than 10:

$\log_a a^n = n$	$10^5 = 100000$	$\log_{10} 10000 = 5$	$\log(10^5) = 5$
$\log_a a^n = n$	$2^4 = 16$	$\log_2 16 = 4$	$\log_2 2^4 = 4$
$a^n \times a^m = a^{(n+m)}$	$10^3 \times 10^4 = 10^7$	$\log 10^3 + \log 10^4 = \log 10^7$	$\log 1000 + \log 10000 = \log 10^7$
$a^n \times a^m = a^{(n+m)}$	$a^3 \times a^4 = a^{3+4}$	$\log m + \log n = \log (m \times n)$	
$\log \frac{m}{n} = \log m - \log n$	$\frac{10^7}{10^5} = 10^2$	$\log(10^7) - \log(10^5) = \log \frac{10^7}{10^5}$	$\log \frac{10^7}{10^5} = \log 10^2 = 2$

These cards can be used to stimulate discussion and make connections through logarithmic law and how they relate to index properties. You can also challenge students to make their own cards.

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 6–7)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem-solving examples	
Example 6: Irrational constructions Students define rational and irrational numbers and perform operations with surds and fractional indices.	ACMNA264 ◆ ◆
Example 7: Irrational roots Students define rational and irrational numbers and perform operations with surds and fractional indices.	ACMNA264 ◆ ◆

Example 6: Irrational constructions

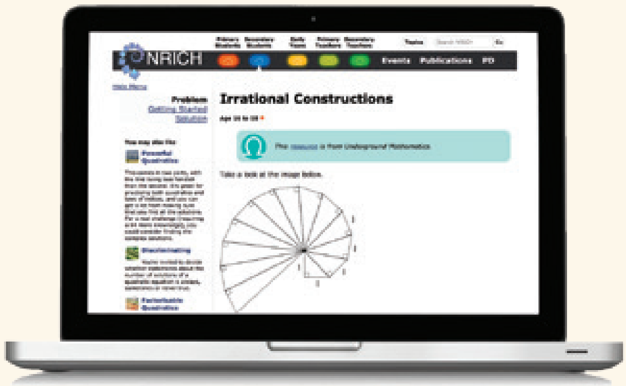
ACMNA264 ◆ ◆
Students define rational and irrational numbers and perform operations with surds and fractional indices.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge students to identify the information they'll need for themselves, by ***asking*** questions.

This activity from the NRICH website requires students to consider and solve an irrational construction.

The link to the problem on the NRICH website is: http://nrich.maths.org/11605



Interpret

What question might you ask yourself? What do you need to show to answer that question? What information is helpful? What information is not useful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Questions to be used only after students have grappled with the problem for a few minutes: How might you simplify this problem? Which part can you draw? (Ask students to speak to someone who they think is being a good problem-solver today and ask them to show what they are trying.)

Solve and check

How will you record, summarise and display your findings? What do you think is the most important feature of your strategy? Is there another way that you could have solved this problem? Could you test your strategy in a different way?

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Example 7: Irrational roots

ACMNA264 ◆ ◆
Students define rational and irrational numbers and perform operations with surds and fractional indices.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge students to identify the information they'll need for themselves, by ***asking*** questions.

This activity from the NRICH website challenges students to identify quadratic equations by working backwards from irrational roots.

The link to the problem on the NRICH website is: http://nrich.maths.org/11596

Connections between 'Real numbers' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use real numbers as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 10/10A	
Whilst working with Real numbers, connections can be made to:	**How the connection might be made:**
Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes. ACMMG244	Refer to: **Example 2: Paper sizes (√2)**
Students solve right-angled triangle problems including those involving direction and angles of elevation and depression. ACMMG245	Refer to: **Example 1: Where is √2 on a number line?** **Example 6: Irrational constructions**
Students solve simple quadratic equations using a range of strategies. ACMNA241	Refer to: **Example 7: Irrational roots**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Real numbers’ from Year 7 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Real numbers:

Recognise, describe and represent real numbers ◆
In Foundation to Year 3 students recognise, describe and represent fractions and decimals. In Year 10A students are mostly recognising and describing more abstract real numbers.

Compare and order real numbers ◆
In Year 4 to Year 5 students are also expected to compare and order fractions and decimals.

Convert and calculate using real numbers ◆
In Year 6 students mostly convert and calculate using fractions and decimals. In Year 7 students mostly convert and calculate using fractions, decimals and percentages.

Apply and solve problems using real numbers ◆
In Years 8 to Year 9 students are mostly solving problems using percentages, rates and ratios.

Year level	‘Fractions and decimals’ content descriptions from the AC: Mathematics: Year 1 to Year 6
Year 1 ◆	Students recognise and describe one-half as one of two equal parts of a whole. ACMNA016
Year 2 ◆	Students recognise and interpret common uses of halves, quarters and eighths of shapes and collections. ACMNA033
Year 3 ◆	Students model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole. ACMNA058
Year 4 ◆	Students investigate equivalent fractions used in contexts. ACMNA077
Year 4 ◆ ◆ ◆	Students count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line. ACMNA078
Year 4 ◆ ◆	Students recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation. ACMNA079
Year 5 ◆ ◆	Students compare and order common unit fractions and locate and represent them on a number line. ACMNA102
Year 5 ◆ ◆	Students investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator. ACMNA103
Year 5 ◆	Students recognise that the place value system can be extended beyond hundredths. ACMNA104
Year 5 ◆ ◆	Students compare, order and represent decimals. ACMNA105
Year 6 ◆ ◆	Students compare fractions with related denominators and locate and represent them on a number line. ACMNA125
Year 6 ◆ ◆	Students solve problems involving addition and subtraction of fractions with the same or related denominators. ACMNA126
Year 6 ◆	Students find a simple fraction of a quantity where the result is a whole number, with and without digital technologies. ACMNA127
Year 6 ◆	Students add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers. ACMNA128
Year 6 ◆	Students multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies. ACMNA129
Year 6 ◆	Students multiply and divide decimals by powers of 10. ACMNA130
Year 6 ◆	Students make connections between equivalent fractions, decimals and percentages. ACMNA131

Year level	‘Real numbers’ content descriptions from the AC: Mathematics: Year 7 to Year 10A
Year 7 ◆ ◆	Students compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line. ACMNA152
Year 7 ◆ ◆	Students solve problems involving addition and subtraction of fractions, including those with unrelated denominators. ACMNA153
Year 7 ◆	Students multiply and divide fractions and decimals using efficient written strategies and digital technologies. ACMNA154
Year 7 ◆	Students express one quantity as a fraction of another, with and without the use of digital technologies. ACMNA155
Year 7 ◆	Students round decimals to a specified number of decimal places. ACMNA156
Year 7 ◆	Students connect fractions, decimals and percentages and carry out simple conversions. ACMNA157
Year 7 ◆	Students find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. ACMNA158
Year 7 ◆ ◆	Students recognise and solve problems involving simple ratios. ACMNA173
Year 8 ◆	Students investigate terminating and recurring decimals. ACMNA184
Year 8 ◆	Students investigate the concept of irrational numbers, including π. ACMNA186
Year 8 ◆	Students solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies. ACMNA187
Year 8 ◆	Students solve a range of problems involving rates and ratios, with and without digital technologies. ACMNA188
Year 9 ◆	Students solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. ACMNA208
Year 9 ◆	Students apply index laws to numerical expressions with integer indices. ACMNA209
Year 9 ◆	Students express numbers in scientific notation. ACMNA210
Year 10A ◆ ◆	Students define rational and irrational numbers and perform operations with surds and fractional indices. ACMNA264
Year 10A ◆ ◆ ◆	Students use the definition of a logarithm to establish and apply the laws of logarithms. ACMNA265

Numeracy continuum: Using fractions, decimal, percentages, ratios and rates	
End Foundation	Recognise that a whole object can be divided into equal parts. Identify quantities such as more, less and the same in everyday comparisons.
End Year 2	Visualise and describe halves and quarters. Solve problems using halves and quarters.
End Year 4	Visualise, describe and order tenths, hundredths, 1-place and 2-place decimals. Solve problems using equivalent fractions for tenths, hundredths, 1-place and 2-place decimals.
End Year 6	Visualise, describe and order equivalent fractions, decimals and simple percentages. Solve problems using equivalent fractions, decimals and simple percentage.
End Year 8	Visualise and describe the proportions of percentages, ratios and rates. Solve problems using simple percentages, ratios and rates.
End Year 10	Illustrate and order relationships for fractions, decimals, percentages, ratios and rates. Solve problems involving fractions, decimals, percentages, ratios and rates.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

http://www.101qs.com

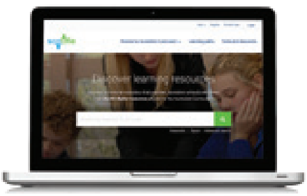
Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Scootle

https://www.scootle.edu.au/ec/p/home

This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.



Estimation 180

http://www.estimation180.com

Estimation 180 is a website with a bank of daily estimation challenges to help students to improve both their number sense and problem-solving skills.

reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by Learning Improvement