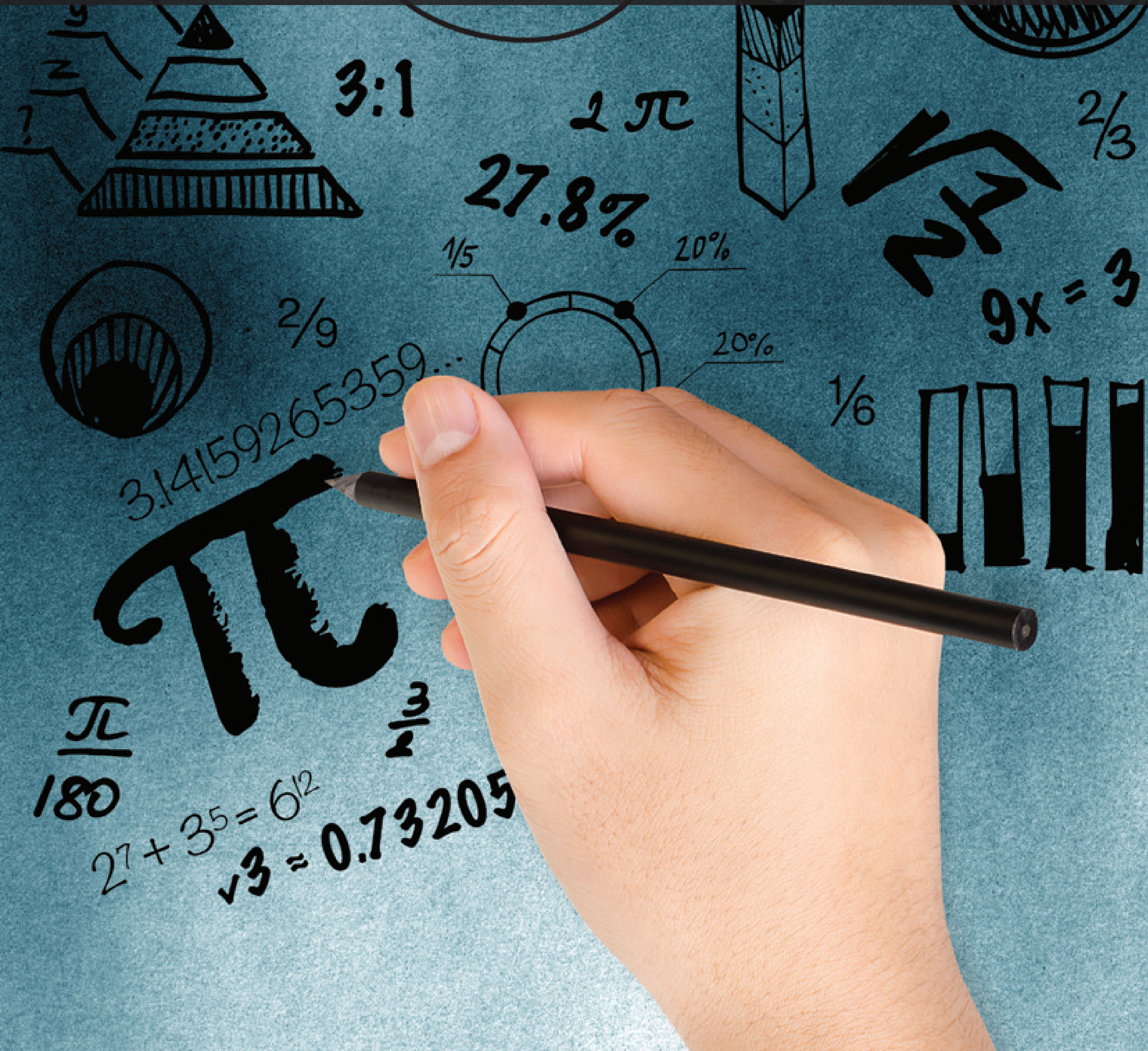
Real numbers: Year 9

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Real numbers' from Year 7 to Year 10A** (see page 20)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with real numbers:

- Recognise, describe and represent real numbers
- Compare and order real numbers
- Convert and calculate using real numbers
- Apply and solve problems using real numbers.

What the Australian Curriculum says about ‘Real numbers’

Content descriptions

Strand | Number and algebra.

Sub-strand | Real numbers.

Year 9 ◆ | ACMNA208
Students solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems.

Year 9 ◆ | ACMNA209
Students apply index laws to numerical expressions with integer indices.

Year 9 ◆ | ACMNA210
Students express numbers in scientific notation.

Year level descriptions

Year 9 ◆ | Students apply the index laws to expressions with integer indices and expressing numbers in scientific notation.

Year 9 ◆ | Students describe the relationship between graphs and equations.

Achievement standards

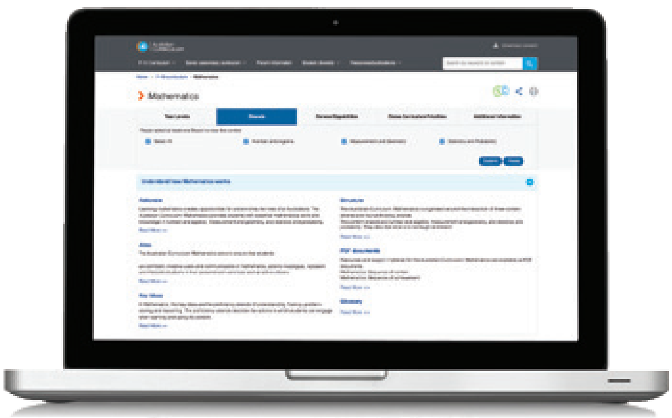
Year 9 ◆ | Students interpret ratio and scale factors in similar figures.

Year 9 ◆ | Students apply the index laws to numbers and express numbers in scientific notation.

Numeracy continuum

Using fractions, decimals, percentages, ratios and rates

End of Year 10 ◆ | Students illustrate and order relationships for fractions, decimals, percentages, ratios and rates (Interpret proportional reasoning). Students solve problems involving fractions, decimals, percentages, ratios and rates (Applying proportional reasoning).



Source: ACARA, Australian Curriculum: Mathematics

Working with Real numbers

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 9 ‘Real numbers’

In Year 7 students convert between any fraction, decimal and percentage. Students develop strategies to multiply and divide fractions and decimals, rounding to a given number of decimal places.

In Year 8 students investigate terminating and recurring decimals, introducing the concept of irrational numbers and linking this to the development of an understanding of π.

In Year 9 students build on their fluency in multiplying and dividing decimals by powers of ten and begin to express numbers in scientific notation. An application of this is seen in ‘Using units of measurement’ when Year 9 students investigate large and small time scales.

In Year 10A students would be expected to define rational and irrational numbers.

- Notice that irrational numbers are in Year 8 and Year 10A but not in Year 9 or 10. Therefore it is important to provide opportunities for students to work with these numbers in other sub-strands like Pythagoras’ Theorem (see ‘Example 20: Where is $\sqrt{2}$ on the number line?’ in the *Pythagoras and trigonometry: Year 9* narrative).
- While indices are in the ‘real number’ sub-strand in Year 9 and 10A, they appear in the ‘number and place value’ sub-strand in Year 7 and 8 and the ‘patterns and algebra’ sub-strand in both Year 9 and Year 10. This is to be expected as indices is a concept that is integral to number and algebra. As teachers we need to be mindful of this in our planning so that learners are engaging with indices in a connected and consistent way.
- Often students are very successful in those applications of the index laws which require trivial transfer of the law and/or applying only one law. When students do not have a conceptual understanding of indices as a mathematical notation, multi-step applications, or the application of several different laws reveal serious misconceptions. As teachers, our learning design must build conceptual understanding, allowing students to recognise patterns and connections for themselves and also allow them to identify the flaws in thinking that lead to common misconceptions.

Engaging learners

Classroom techniques for teaching Real numbers

Power of scientific notation

Showing students videos such as *Powers of Ten™ (1977)* or *Cosmic Voyage* gives students some understanding of the purpose and power of scientific notation, filling them with wonder about the size of our universe and the atoms which it consists of.

The ***Powers of Ten™ (1977)*** video can be found at: https://www.youtube.com/watch?v=0fKBhvDjuy0

An extract from the ***Cosmic Voyage*** video can be found at: https://www.youtube.com/watch?v=qxXf7AJZ73A

Source: *Powers of Ten™ (1977)*, Eames Office, published 2010

Understanding rates and ratios

Scootle digital learning activities can be used in conjunction with concrete materials to hook students into their learning. Being able to work flexibly in a practical situation supports the conceptual understanding of rates and ratios.

Rates and scales activities can be found at: http://www.scootle.edu.au/ec/viewing/L8099/index.html

Ratio and graphs activities can be found at: http://www.scootle.edu.au/ec/viewing/L8103/index.html

Source: *In proportion: rates and scales*, Scootle, Education Services Australia

Estimation 180

Estimating with rates encourages students to connect their intuitive knowledge about rates to practical situations, building fluency and conceptual understanding.

Estimation 180 is a website that has multiple scenarios that can prompt students to estimate, inviting every learner to have an opinion. This resource could be used as a warm-up to lessons, choosing contexts that build their number sense.

The **Estimation 180** activities can be found at: http://www.estimation180.com/blog/going-the-distance-180-challenges

Source: *Estimation 180: Building number sense one day at a time,* by Andrew Stadel

Legend of the chessboard

The famous legend about the origin of chess states that the Emperor of India was so impressed by the new game, that he told the inventor to name his reward. The inventor requested one grain of rice for the first square of the chessboard, two grains for the second, 4 for the third and so on, doubling the amount for each of the 64 squares. The Emperor agreed, thinking that the man had asked for a small reward.

Ask students to estimate how much that might be, both before and after, doing 'Example 2: Pocket money options – powers of 2'.

After a week, the Emperor's treasurer reported that the reward would add up to an astronomical sum, far greater than all the rice that could conceivably be produced in many centuries! In fact, enough to feed 100 tonnes of rice to every single human on Earth, or 1 kg of rice per day per human for 275 long years and more than a millennium worth of global rice production (approximately 100 million tonnes each year).

The Legend of the Chessboard (Teaser) video can be found at: https://www.youtube.com/watch?v=byk3pA1GPgU

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–5)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the rules for standard notation** by themselves. They need to receive this information in some way. However, it *is* possible my students can be challenged with questions that will result in them identifying the **particular characteristics from examples** for themselves, so I don't need to instruct that information.

At this stage of development, students can **develop fluency from conceptual understanding of different types and representations of real numbers**. When teachers provide opportunities for students to **identify the patterns in the operations with numbers in index notation**, they require their students to generalise from the connections they have made. Telling students index laws removes this natural opportunity for students to make conjectures and verify connections that they notice.

Teachers can support students to choose, and use, different numerical notations in authentic contexts by asking questions as described in understanding proficiency, *What patterns/connections/relationships can you see?* The intent of this question is to promote learning design that intentionally plans for students to develop a disposition towards looking for patterns, connections and relationships. Using questions such as the ones described here, supports teachers to replace 'telling' the students information, with getting students to notice for themselves.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular rule or theorem during the current unit of work, then it probably is quicker to tell them the rule and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the rule is a false economy of time.

When we challenge our students to **establish a theorem**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Question of scale – scientific notation Students express numbers in scientific notation.	ACMNA210 ◆
Example 2: Pocket money options – powers of 2 Students apply index laws to numerical expressions with integer indices. Students express numbers in scientific notation.	ACMNA209 ◆ ACMNA210 ◆
Example 3: Ferrari ride – determining rates to make a prediction Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.	ACMNA208 ◆
Example 4: What were they thinking? – common misconceptions involving indices Students apply index laws to numerical expressions with integer indices.	ACMNA209 ◆
Example 5: Prime factorisation and index laws Students apply index laws to numerical expressions with integer indices.	ACMNA209 ◆

Example 1: Question of scale – scientific notation

ACMNA210 ◆
Students express numbers in scientific notation.

Questions from the BitL tool

Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?

Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about scientific notation, we can challenge them to recognise the relationships between different representations and the magnitudes they describe, by ***asking*** questions.

This activity is from the NRICH website.

This activity challenges students to use their knowledge of scientific notation and scale in a scientific context. It challenges them to judge the length of the physical objects, with sizes ranging from 1 Angstrom to 1 million km.

The link to the problem on the NRICH website is: http://nrich.maths.org/6349

Example 2: Pocket money options – powers of 2

ACMNA209 ◆

Students apply index laws to numerical expressions with integer indices.

ACMNA210 ◆

Students express numbers in scientific notation.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about index laws, we can challenge them to recognise the relationships between different representations and the numbers they describe, by ***asking*** questions.

To begin this activity, present the following scenario to students:

If your parents offered you two different options for receiving your pocket money, which of the following would you choose?

Option A: $200 per week

Option B: One cent paid on the first day of the month, two cents paid on the second day, 4 cents paid on the third day and so on, with the amount received each day doubling until the end of the month.

Then discuss with them, by asking:

- *What do you think?*
- *What might Option B be worth? What is an amount that is too big? What is an amount that is too small?*
- *How might you make your decision?*
- *Would it be enough to check the total at the end of the week? 2 weeks?* (You are getting paid **every day** so that might add to more at the end of the month.)
- *What if you only got paid the amount that is calculated on the last day?*

As the students are calculating some very large numbers, it is an opportunity to consider the different ways of representing them in terms of powers of 2 and in scientific notation. Consider the different representations in Figure 1 and ask students:

- *How are these representations the same? How are they different?*

This is a type of backwards problem which is counter-intuitive and challenges our exponential thinking. Ask students this sometime after the pocket money problem:

A lily pad grows so that each day it doubles its size (area). On the 20^{th} day of its life, it completely covers the pond. On what day of its life was the pond half covered?

Fast thinking suggests the 10^{th}. Asking students to show you or convince you, will reveal it is the 19^{th}. Let them change their mind by thinking more deeply about the problem.

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2^{28} 268435456 2.68×10^7

Figure 1

Example 3: Ferrari ride – determining rates to make a prediction

ACMNA208 ◆
Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about interpreting simple rates, we can challenge them to interpret rates to make predictions in context, by ***asking*** questions.

This is an Estimation 180 activity, it is in three acts.

The problem situation could be replicated by students using their own remote control car. This would also challenge students to consider how accurate information such as time, distance and length could be measured accurately. Students might consider altering the weight or length of the rope to see what factors might affect the rate.

Ask students:

- *What do you think? How might you solve this problem?*
- *How could you do it another way? How are the methods the same/different?*
- *What factors might affect the outcome? Is the car travelling at a constant rate?*

The **Ferrari ride** activity can be accessed at:
http://www.estimation180.com/ferrariride.html

Example 4: What were they thinking? – common misconceptions involving indices

ACMNA209 ◆

Students apply index laws to numerical expressions with integer indices.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about index laws, we can challenge them to recognise the common misconceptions when applying the laws, by ***asking*** questions.

Tackling misconceptions: Knowing what it isn't

We can challenge our students reasoning by asking '*Why can't I…? / Why is it not…? / What's wrong with…?*' questions. This type of question can be used to address common misconceptions. Students can be given problems which are answered incorrectly, without working out. They can be asked to give this (fictional student) feedback summarising the mistakes that they have made and how to avoid that mistake in the future.

Present the ideas in Figure 2, that Year 8 students had when simplifying expressions with indices.

$2^3 \times 2^4 = 4^7$	$2^7 \times 3^5 = 6^{12}$	$3^7 + 3^4 = 3^{11}$
$2^0 = 0$	$(2^3)\ 5 = 2^8$	$\frac{2^2}{2^1} = \frac{2^2}{2^1}$
$\frac{12^5}{3^4} = 4^1$		

Figure 2

Then ask students:

- *Do you agree with these answers?*
- *How might you check if their thinking is correct?*
- *What feedback might you give to each of these students about their thinking?*

It seems that they might be applying some of the laws of numbers that we use to simplify expressions, but maybe not in the appropriate way. Ask students:

- *What general advice might you give to these students so that they might think differently about simplifying expressions involving indices?* (Rather than applying a rule they have memorised, try to remember what the shorthand notation means. For example, 2^5 is shorthand for repeated factors of 2. There are no numbers with the value 5. Five is just a 'counter' for the number of repeated factors of 2.)

Example 5: Prime factorisation and index laws

ACMNA209 ◆
Students apply index laws to numerical expressions with integer indices.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about index laws, we can challenge them to recognise relationships between different representations and the numbers they describe, by ***asking*** questions.

For this activity, distribute or display the four question cards shown in Figure 3.

Then ask students:

- *What do you notice?* (Each of these products have the same result.)
- *Are there other cards that might belong to this set? Convince me.* (Students could evaluate the prime factorisation of the numbers in the other cards, such as 720 x 3150. They might also realise that they can create any combination of the prime factors that simplifies to (2^5x3^4x5^3) x 7, for example, (2^5x3^4) x (5^3x7).)
- *Would 90 be a factor of 226800? How could you check?* (You can divide 226800 by 90. You can also consider that 90 = 2x3^2x5, and as all these factors are in 226800, then 90 will be a factor.)
- *If 90 was a factor, what would the other factor be?* (The remaining factor would be 2^4x3^2x5^2 x 7. Note that it contains all the factors of 226800 that are not in 90.)
- *If you wrote 90 and the other factor in their prime factorisation what would you notice?*
- *What other numbers might be factors of 226800, and what would the remaining factor be? Is it possible to find them all?* (If a list was made systematically and identified the factors in pairs, it does not take as long as you think. A tree diagram is another effective way or counting.)

Extend students' understanding of prime factorisation and index laws using 'Example 9: Prime factorisation, index laws and number theory' on page 18 of this narrative.

Write this product in simplest index notation: 2^3 x 3 x 5^2 x 2^2 x 3^3 x 5 x 7	**Evaluate:** 600 x 3780
Write this product in simplest index notation: 2^4 x 3^2 x 5 x 2 x 3^2 x 5^2 x 7	**Write this product in simplest index notation:** 3^4 x 7 x 2^5 x 5^3

Figure 3

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 6–9)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 6: Filling the water tank Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.	ACMNA208 ◆
Example 7: Burning down – comparing two rates in context Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.	ACMNA208 ◆
Example 8: Finals week – ratio of caffeine in drinks Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.	ACMNA208 ◆
Example 9: Prime factorisation, index laws and number theory Students apply index laws to numerical expressions with integer indices.	ACMNA209 ◆

Example 6: Filling the water tank

ACMNA208 ◆
Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take themselves, by ***asking*** questions.

This activity is a Dan Meyer ***Three-Act Maths Task***. It can be presented to students along with the question, *What's the first question that comes to mind?*

While this is a rich measurement problem solving task, it also involves linear relationships between volume and time, depth of water and time filling. There are videos about filling and emptying the tank, as well as the filling of a 16oz jug. Note that rates of flow can be related to slopes. This task links all three content strands.

Rather than using the entire Three-Act Maths Task, interesting data can be collected from the videos to explore bivariate numerical data. Consider the three screen captures in Figure 4 as a source of data for students to estimate the time it will take to fill the tank.

How long will the tank take to fill?

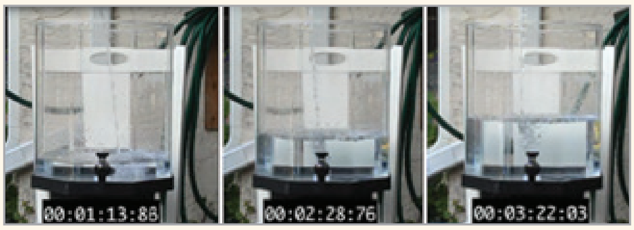


Figure 4

The video of filling the tank completely can be used to collect data about depth with time. Ask students:

- *How could we use this data?* (By using other information such as the height of the tank, an estimate could be made about the depth of the water. Plotting a scatterplot and fitting a linear graph will give a rate that the water will rise.)
- *Do you expect the time to fill the tank to be the same as the time to empty? Why? How could we check?*
- *Do you think the tank might empty at the same rate regardless of how much water is in the tank?* (There is a video of the tank emptying that can be used to explore these questions.)

The **Water tank** activity can be accessed at: http://mrmeyer.com/threeacts/watertank

(This activity also appears in the *Data representation and interpretation: Year 10/10A* narrative.)

Example 7: Burning down – comparing two rates in context

ACMNA208 ◆
Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take themselves, by ***asking*** questions.

This activity is from the NRICH website.

The link to the problem on the NRICH website is: http://nrich.maths.org/497

This activity challenges students to work with two simple rates within a problem. This is a great opportunity to ask students to solve the problem in as many ways as possible. They might use tables of values, graphs, algebraic methods or reasoning of their own.

The NRICH website shows a solution found by solving simultaneously. Students may approach this problem using a more conceptual and practical approach.

One approach is to consider the height of the candles at 9.30 pm to be a whole unit. Considering half hour intervals, it can be reasoned that the small candle will burn down $\frac{1}{3}$ every 30 minutes to be burnt out at 11 pm and the tall one will burn down $\frac{1}{4}$. Working backwards in time you can rebuild the candles to the time each was lit. The tall one is 3 units, and the short one is $2\frac{2}{3}$. This means that this $\frac{1}{3}$ difference in height equates to 3 cm, making the tall one 27 cm (3 units is $9\times\frac{1}{3}$ which is 9x3) and the shorter is 3 cm shorter, which is 24 cm.

Alternatively, using a graph of the height of the candle against time, the known information can be plotted using uniform non-standard units. Identifying that the slope of the line from 9.30 pm to burnout represents the rate of change in height, the line can be extrapolated back to the time when the candle was lit. The height difference between the two candles represents 3 cm and this information determines what the vertical scale represents, which can be used to determine the height of the candles.

Sharing different approaches such as these two, exemplifies that there are many ways to do problems. It fosters student dialogue as well as celebrating and encouraging creativity.

These solutions can be accessed at: http://nrich.maths.org/497/solution

Interpret

What question have you been asked to solve? What's an answer that's too big? What's an answer that's too small? What's a bit closer to the answer that you think you'll find? What do you need to show to answer that question? What information is helpful? What information is not helpful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Questions to be used only after students have grappled with the problem for a few minutes: Do you know which candle burns most quickly? If we know that they were the same height at 9.30 am, how could we use this to compare the rate that they burn? Would it help to start by thinking about two candles with particular heights? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

How will you record, summarise and display your findings? What do you think is the most important feature of your strategy? Is there another way that you could have solved this problem? Could you test your strategy in a different way?

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

An extension on this activity might be to consider why candles might burn at a different rate, or whether candles do actually burn at constant rates. Conjectures can be explored by collecting data, linking and connecting learning to other strands.

Example 8: Finals week – ratios of caffeine in drinks

ACMNA208 ◆
Students solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take themselves, by ***asking*** questions.

This activity is a Dan Meyer ***Three-Act Maths Task***. It can be presented to students along with the question, *What's the first question that comes to mind?*

While this is a rich ratio problem solving task, it also involves concepts of volume and best value for money.

The **Finals week** activity can be accessed at: http://threeacts.mrmeyer.com/finalsweek/

Interpret

What question have you been asked to solve? Do you have a question of your own? What do you need to show to answer that question? What information is helpful? What information is not helpful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Questions to be used only after students have grappled with the problem for a few minutes: Do you know which drink contains most caffeine? Is there anything else we should consider? Would it help to start by thinking about just two different drinks? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

How will you record, summarise and display your findings? What do you think is the most important feature of your strategy? Is there another way that you could have solved this problem? Could you test your strategy in a different way?

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Example 9: Prime factorisation, index laws and number theory

ACMNA209 ◆
Apply index laws to numerical expressions with integer indices.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take themselves, by ***asking*** questions.

If needed, an introduction to this thinking or an easy entry point to this problem solving can be found in 'Example 5: Prime factorisation and index laws'. If students struggle with their number facts and fluency, you may choose to free up their working memories using technology, for example: Calculator soup at http://www.calculatorsoup.com/calculators/math/prime-factors.php

For this activity, introduce the following to students:

Year 9 students found that they could use prime factorisation to understand numbers a lot better. After much exploration, each group reported back about the new knowledge they had by making a single statement. *Can your group see how this might be done?*

This is an opportunity to differentiate the exploration for groups in your class. Rather than give them the explanation at the start, provide enabling prompts that you can give to the group on request or when the struggle is no longer productive for them.

Students can explore one of the following group statements, or one of your own:

GROUP 1

We can just look at a prime factorisation and tell you how many 0s the number will end in when evaluated.

To end in 0, the number must have a factor of 10, that is, a factor of 2 and 5. So for the following, $2^5 \times 3^4 \times 5^3 \times 7$ will end in 3 0s because when you write it this way $(2^3 \times 5^3) \times 2^2 \times 3^4 \times 7$, you can see the most factors of 10 that can be made is 3. Check with your calculator.

Enabling prompts may begin with them working backwards and finding the prime factorisation of numbers that end in 0.

GROUP 2

We can look at the prime factorisation of two or more numbers and tell you their highest common factor.

For $2^2 \times 3^5 \times 7$ and $2^3 \times 3^4 \times 5^3$, consider what is the highest number of each of the factors they both have? The second number has 3 factors of 2, but the highest number they both have is **2 factors of 2**.

The first number has 5 factors of 3, but the highest number they both have **4 factors of 3**.

One has a factor of 7 but the other hasn't and same with the factors of 5.

Hence the highest common factor is $2^2 \times 3^4$. Check with your calculator.

Enabling prompts might start with simpler examples that build on their existing number facts.

GROUP 3

We can tell you their Highest Common Factor (HCF) and their lowest common multiple (LCM).

Multiples of a number must contain at least all of that number's factors.

To be a multiple of $2^2 \times 3^5 \times 7$, there must be at least 2 factors of 2, **5 factors of 3** and a **factor of 7**.

To be a multiple of $2^3 \times 3^4 \times 5^3$, there must be at least **3 factors of 2**, 4 factors of 3 and **3 factors of 5**.

To be a multiple of both, you must have **3 factors of 2**, **5 factors of 3**, **3 factors of 5** and a **factor of 7**, ie $2^3 \times 3^5 \times 5^3 \times 7$.

Enabling prompts might be to provide the answer to the above example without any explanation and continue to provide examples until the student recognises and can verbalise the pattern. Then ask 'Why does that work?'

GROUP 4

We can look at the prime factorisation of a number and tell you how many factors it has.

Consider 2^3. You can make up a factor by picking none, some or all of the factors of 2^3.

So 2^0, 2^1, 2^2, 2^3 so there are 4 factors. In general, 2^n would have n+1 factors.

For $2^3 \times 3^4$, there would be 4 options for picking how many factors of 2 you want in your number and for each of those there would be 5 options for picking how many factors of 3 you want.

That is 4 x 5 = 20 factors. $2^0 3^0$, $2^1 3^0$, $2^2 3^0$, $2^3 3^0$ and $2^0 3^1$, $2^1 3^1$, $2^2 3^1$, $2^3 3^1$, etc.

Enabling prompts might be to give the answers to these examples without explanation and ask them to see the pattern, or ask students to write prime factorisations of all the factors of 20 and then ask 'What do you notice?'

Connections between ‘Real numbers’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use real numbers as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 9	
Whilst working with Real numbers, connections can be made to:	**How the connection might be made:**
Students extend and apply the index laws to variables, using positive integer indices and the zero index. ACMNA212	Refer to: **Example 5: Prime factorisation and index laws** **Example 9: Prime factorisation, index laws and number theory**
Students calculate the surface area and volume of cylinders and solve related problems. ACMMG217	Refer to: **Example 6: Filling the water tank** **Example 8: Finals week – ratios of caffeine in drinks**
Students sketch linear graphs using the coordinates of two points and solve linear equations. ACMNA215	Refer to: **Example 7: Burning down – comparing two rates in context**
Students identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources. ACMSP228	Refer to: **Example 6: Filling the water tank**
Investigate very small and very large time scales and intervals. ACMMG219	Refer to: **Example 6: Filling the water tank**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Real numbers’ from Year 7 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Real numbers:

Recognise, describe and represent real numbers ◆
In Foundation to Year 3 students recognise, describe and represent fractions and decimals. In Year 10A students are mostly recognising and describing more abstract real numbers.

Compare and order real numbers ◆
In Year 4 to Year 5 students are also expected to compare and order fractions and decimals.

Convert and calculate using real numbers ◆
In Year 6 students mostly convert and calculate using fractions and decimals. In Year 7 students mostly convert and calculate using fractions, decimals and percentages.

Apply and solve problems using real numbers ◆
In Years 8 to Year 9 students are mostly solving problems using percentages, rates and ratios.

Year level	‘Fractions and decimals’ content descriptions from the AC: Mathematics: Year 1 to Year 6
Year 1 ◆	Students recognise and describe one-half as one of two equal parts of a whole. ACMNA016
Year 2 ◆	Students recognise and interpret common uses of halves, quarters and eighths of shapes and collections. ACMNA033
Year 3 ◆	Students model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole. ACMNA058
Year 4 ◆	Students investigate equivalent fractions used in contexts. ACMNA077
Year 4 ◆ ◆ ◆	Students count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line. ACMNA078
Year 4 ◆ ◆	Students recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation. ACMNA079
Year 5 ◆ ◆	Students compare and order common unit fractions and locate and represent them on a number line. ACMNA102
Year 5 ◆ ◆	Students investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator. ACMNA103
Year 5 ◆	Students recognise that the place value system can be extended beyond hundredths. ACMNA104
Year 5 ◆ ◆	Students compare, order and represent decimals. ACMNA105
Year 6 ◆ ◆	Students compare fractions with related denominators and locate and represent them on a number line. ACMNA125
Year 6 ◆ ◆	Students solve problems involving addition and subtraction of fractions with the same or related denominators. ACMNA126
Year 6 ◆	Students find a simple fraction of a quantity where the result is a whole number, with and without digital technologies. ACMNA127
Year 6 ◆	Students add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers. ACMNA128
Year 6 ◆	Students multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies. ACMNA129
Year 6 ◆	Students multiply and divide decimals by powers of 10. ACMNA130
Year 6 ◆	Students make connections between equivalent fractions, decimals and percentages. ACMNA131

Year level	‘Real numbers’ content descriptions from the AC: Mathematics: Year 7 to Year 10A
Year 7 ◆ ◆	Students compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line. ACMNA152
Year 7 ◆ ◆	Students solve problems involving addition and subtraction of fractions, including those with unrelated denominators. ACMNA153
Year 7 ◆	Students multiply and divide fractions and decimals using efficient written strategies and digital technologies. ACMNA154
Year 7 ◆	Students express one quantity as a fraction of another, with and without the use of digital technologies. ACMNA155
Year 7 ◆	Students round decimals to a specified number of decimal places. ACMNA156
Year 7 ◆	Students connect fractions, decimals and percentages and carry out simple conversions. ACMNA157
Year 7 ◆	Students find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. ACMNA158
Year 7 ◆ ◆	Students recognise and solve problems involving simple ratios. ACMNA173
Year 8 ◆	Students investigate terminating and recurring decimals. ACMNA184
Year 8 ◆	Students investigate the concept of irrational numbers, including π. ACMNA186
Year 8 ◆	Students solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies. ACMNA187
Year 8 ◆	Students solve a range of problems involving rates and ratios, with and without digital technologies. ACMNA188
Year 9 ◆	Students solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. ACMNA208
Year 9 ◆	Students apply index laws to numerical expressions with integer indices. ACMNA209
Year 9 ◆	Students express numbers in scientific notation. ACMNA210
Year 10A ◆ ◆	Students define rational and irrational numbers and perform operations with surds and fractional indices. ACMNA264
Year 10A ◆ ◆ ◆	Students use the definition of a logarithm to establish and apply the laws of logarithms. ACMNA265

Numeracy continuum: Using fractions, decimal, percentages, ratios and rates	
End Foundation	Recognise that a whole object can be divided into equal parts. Identify quantities such as more, less and the same in everyday comparisons.
End Year 2	Visualise and describe halves and quarters. Solve problems using halves and quarters.
End Year 4	Visualise, describe and order tenths, hundredths, 1-place and 2-place decimals. Solve problems using equivalent fractions for tenths, hundredths, 1-place and 2-place decimals.
End Year 6	Visualise, describe and order equivalent fractions, decimals and simple percentages. Solve problems using equivalent fractions, decimals and simple percentage.
End Year 8	Visualise and describe the proportions of percentages, ratios and rates. Solve problems using simple percentages, ratios and rates.
End Year 10	Illustrate and order relationships for fractions, decimals, percentages, ratios and rates. Solve problems involving fractions, decimals, percentages, ratios and rates.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

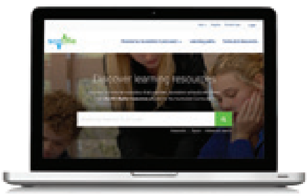
http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Scootle

https://www.scootle.edu.au/ec/p/home



This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.

Estimation 180

http://www.estimation180.com

Estimation 180 is a website with a bank of daily estimation challenges to help students to improve both their number sense and problem solving skills.

reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.