## n-me  <br> Money and financial mathematics: Year 10

MATHEMATICS CONCEPTUAL NARRATIVE
Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together


## Contents

What the Australian Curriculum says about 'Money and financial mathematics' ..... 3Content descriptions, year level descriptions, achievement standards and numeracy continuum
Working with Money and financial mathematics ..... 4
Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum
Engaging learners ..... 5
Classroom techniques for teaching Money and financial mathematics
From tell to ask ..... 6
Transforming tasks by modelling the construction of knowledge (Examples 1-2)
Proficiency: Problem Solving ..... 10Proficiency emphasis and what questions to ask to activate it in your students (Examples 3-4)Connections between 'Money and financial mathematics' and other maths content13
A summary of connections made in this resource
'Money and financial mathematics’ from Year 1 to Year 10 ..... 14
Resources ..... 15

The 'AC' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The 'From tell to ask' icon indicates a statement that explains the transformation that is intended by using the task in that example
More information about 'Transforming Tasks': http://www.acleadersresource. sa.edu.au/index.php?page= into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The 'Bringing it to Life (BitL)' tool icon indicates the use of questions from the Leading Learning: Making the Australian Curriculum Work for Us resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics - these questions are for teachers to use directly with students.
More information about the 'Bringing it to Life' tool: http://www.acleadersresource sa.edu.au/index.php?page= bringing_it_to_life

Throughout this narrative-and summarised in 'Money and financial mathematics' from Year 1 to Year 10 (see page 14)-we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with money and financial mathematics:

- Recognise, order and count money
- Investigate and calculate with money
- Create plans and review financial decisions
-Solve problems relating to financial matters.


## What the Australian Curriculum says about 'Money and financial mathematics'

## Content descriptions

Strand | Number and algebra.
Sub-strand | Money and financial mathematics.
Year 10 | ACMNA229
Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.

## Year level descriptions

Year $10 \diamond$ | Students compare simple and compound interest in financial contexts.

## Achievement standards

Year $10 \leqslant$ | Students solve problems involving simple interest.

## Numeracy continuum

Estimating and calculating with whole numbers
End of Year 10 | Students identify and justify 'best value for money' decisions. (Use money)


Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

## Working with Money and financial mathematics

## Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

## What we are building on and leading towards in Year 9 'Money and financial mathematics'

In Year 8 students calculate percentage increases and decreases and solve problems involving profit and loss.
In Year 9 students solve problems using simple interest.
In Year 10 students bring together their knowledge of percentages and indices to develop an understanding of compound interest.

- While the creating and reviewing of financial plans in light of financial goals is not significantly emphasised in the AC content descriptions, notice it is a focus in the numeracy continuum. The references to 'solving problems' in real numbers and number and place value, that are evident in the Year level descriptions and the achievement standards, could certainly encompass learning opportunities for students in financial literacy.


## Engaging learners

Classroom techniques for teaching Money and financial mathematics

## MoneySmart Teaching

Young people (even if they are not money earners) are significant, if not critical, consumers in our financial society. Hence, there is an opportunity to explore a wide range of contexts that they are familiar with, but not necessarily informed about: credit cards, phone plans, internet purchases, currency conversions, etc.

ASIC's MoneySmart Teaching website is a good starting point to inform your learning design to support students in creating their own knowledge in real-life financial contexts: http://tiny.cc/SecondaryMoneySmart

## \$20 Boss

It is engaging to involve students in Business Enterprise programs like the one run by the Foundation for Young Australians (FYA), \$20 Boss.

Designed to make life easier for teachers and support them to bring the curriculum to life, $\$ 20$ Boss is a nationwide in-school challenge that aims to inspire and develop entrepreneurial skills and passion in young Australians. It can be delivered both through an online platform, or through traditional class-based activities:
https://www.fya.org.au/programs/20boss/


## From tell to ask

## Transforming tasks by modelling the construction of knowledge (Examples 1-2)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:
What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to create the terminology and symbolic representations relating to business calculations for themselves. They need to receive this information in some way. However, it is possible my students can be challenged with questions that will result in them identifying profitable situations and wise consumer decisions, so I don't need to instruct that information.

At this stage of development, students can foster an understanding of percentages as they relate to profit and loss. When teachers provide opportunities for students to make decisions about percentages in business contexts, they require their students to generalise from those decisions. Telling students which calculations to make, removes this natural opportunity for students to make conjectures, and verify connections that they notice when making good financial decisions.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular formula during the current unit of work, then it probably is quicker to tell them the formula and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent creator and user of mathematics, then telling students the formulae is a false economy of time.

When we challenge our students to establish theorems, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

## Curriculum and pedagogy links

The following icons are used in each example:


The 'AC' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.


The 'Bringing it to Life (BitL)' tool icon indicates the use of questions from the Leading Learning: Making the Australian Curriculum Work for Us resource.
The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu. au/index.php?page=bringing_it_to_life


The 'From tell to ask' icon indicates a statement that explains the transformation that is intended by using the task in that example.
This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the Leading Learning: Making the Australian Curriculum work for Us resource: http://www.acleadersresource. sa.edu.au/index.php?page=into_the_classroom


Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

## From tell to ask examples

## Example 1: Dating made easier - percentage increase/decrease

Students connect the compound interest formula to repeated applications of simple interest
using appropriate digital technologies.

ACMNA229
Example 2: Debt race
Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.

## Example 1: Dating made easier - percentage increase/decrease

ACMNA229
Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.


Questions from the BitL tool Understanding proficiency: What patterns/connections/ relationships can you see? Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised? What can you infer?


Instead of telling students about the effect of compounding percentage increases and decreases, we can challenge students to recognise the relationships for themselves, by asking questions.

This activity is from the NRICH website.
This task requires the students to explore how long it takes for an investment that increases or decreases by $10 \%$ each year, to double or halve (respectively), and explore why the answers are not the same.

To extend the problem into generalisation, you could ask students:

- What surprises you? (Students often expect the time to be the same.)
- Is this always the case?


The link to this problem on the NRICH site is: http://nrich.maths.org/5636

This example also appears in Money and financial mathematics: Year 9.

## Example 2: Debt race

## ACMNA229

Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.


Questions from the BitL tool
Understanding proficiency: What patterns/connections/ relationships can you see? Can you represent/calculate in different ways?
Reasoning proficiency: In what ways can your thinking be generalised? What can you infer?


Instead of telling students about the effect of lump payments for compound interest, we can challenge students to recognise the relationships for themselves, by asking questions.

This activity is from the NRICH website.
This problem requires students to calculate who will be the first to pay off the debt, algebraically. There is a possibility of differentiating this task by posing prompting questions and using technology (eg see Figure 1, Excel Spreadsheets using absolute values).

| 4 |  |  |
| :--- | :--- | :--- |
| 1 |  |  |
| 2 |  |  |
| 3 | 100000 | A |
| 4 | 14000 | Anount borrowed |
| 5 | 0.06 | Interest Rate |
| 6 | End of Year 1 | $-\$ A S 3^{*}(1+\$ A S 5) \cdot \$ A S 4$ |
| 7 | 2 | $-86^{*}(1+\$ A S 5)-S A S 4$ |



The link to this problem on the NRICH site is: http://nrich.maths.org/6088

All the borrowers planned to have repaid the debt by the end of the $10^{\text {th }}$ year. Ask students:

- Is this true for all borrowers? Prove it.
- If the borrowers decided to pay their annual interest and their lump sums in daily amounts, who would be the first to pay off their debt? Convince me.
- Would the result be the same if interest rates went up $1 \%$ for all borrowers in the $10^{\text {th }}$ year?
- Would the result be the same if all borrowers decided to increase their lump sum by $\$ 1000$ in the $10^{\text {th }}$ year?


## Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 3-4)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of all abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

## Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

## Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity
Unfamiliar problems tend to provoke a response of, 'I don't know', or 'l'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the beginning of a learning opportunity and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

## Proficiency: Problem Solving examples

## Example 3: To insure or not to insure

Students evaluate financial plans to support specific financial goals. (End Year 10: Use money)
NC LEVEL 6
Example 4: Fry's bank - compounding interest
Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.

## Example 3: To insure or not to insure

## NC LEVEL 6

Students evaluate financial plans to support specific financial goals. (End Year 10: Use money)

Questions from the BitL tool
Problem Solving proficiency: Interpret; Model and plan; Solve and check, Reflect.
Reasoning proficiency:
What can you infer?


Instead of telling students about financial considerations about insurance, we can challenge students to recognise the possible consequences for themselves, by asking questions.

This activity is from the NRICH website.
In this activity, the probabilities and risk associated with insuring possessions are explored collaboratively through simulation.

The link to this problem on the NRICH site is: http://nrich.maths.org/9598

This example also appears in Money and financial mathematics: Year 9.


## Example 4: Fry's bank - compounding interest

## ACMNA229

Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies.


Questions from the BitL tool
Problem Solving proficiency: Interpret; Model and plan; Solve and check, Reflect.
Reasoning proficiency: What can you infer?


Instead of telling students about financial considerations about insurance, we can challenge students to recognise the possible consequences for themselves, by asking questions.

This activity is a Dan Meyer Three-Act Maths Task. It can be presented to students along with the question, What's the first question that comes to mind?

This is an activity that challenges students to calculate how much money is in Fry's account using compound interest.

The Fry's bank activity can be accessed at: http://threeacts.mrmeyer.com/frysbank/

## Identifying the question to solve

The group can share questions and sort them into mathematical and non-mathematical questions. Then, of the mathematical questions, students can sort their questions into those that cannot be answered with the given information and those that could be answered using the given information or additional information that could be inferred.
Dan Meyer has a technique that we have seen many teachers adopt when generating and collecting questions from students. First, he asks students to individually write down questions that come to mind. Then, as he invites students to share their questions, he writes students' names next to the questions. He also asks if anyone else likes that question. 'Did you write it down, or if you didn't perhaps you still think that it's a good question.' Through doing this, both Dan and his class get a sense of the questions that are of interest to the students.


## Connections between 'Money and financial mathematics' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use Money and financial mathematics as a starting point.

Here are just some of the possible connections that can be made:

| Mathematics: Year 10 |  |
| :--- | :--- |
| Whilst working with Money and financial mathematics, <br> connections can be made to: | How the connection might be made: |
| Students use the language of 'if ... then', 'given', 'of', <br> 'knowing that' to investigate conditional statements and <br> identify common mistakes in interpreting such language. <br> ACMSP247 | Refer to: <br> Example 3: To insure or not to insure |
| Students use scatter plots to investigate and comment on <br> relationships between two numerical variables. ACMSP251 | Refer to: <br> Example 1: Dating made easy - percentage increase/ <br> decrease |
| Students explore the connection between algebraic and <br> graphical representations of relations such as simple <br> quadratics, circles and exponentials using digital technology <br> as appropriate. ACMNA239 | Refer to: <br> Example 2: Debt race |

## Making connections

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individuals, or small groups of students.

## 'Money and financial mathematics' from Year 1 to Year 10

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Money and financial mathematics:

## Recognise, order and count money

From Year 1 to Year 3 students recognise, order and count money.

## Investigate and calculate with money

From Year 3 to Year 10 students investigate and calculate with money.
Create plans and review financial decisions
From Year 5 to Year 7 students create simple financial plans.
Solve problems relating to financial matters
From Year 8 to Year 9 students solve problems relating to financial matters.

| Year level | 'Money and financial mathematics' content descriptions from the AC: Mathematics |
| :---: | :---: |
| Year 1 | Students recognise, describe and order Australian coins according to their value. ACMNA017 |
| Year 2 | Students count and order small collections of Australian coins and notes according to their value. ACMNA034 |
| Year $3 *$ | Students represent money values in multiple ways and count the change required for simple transactions to the nearest five cents. ACMNA059 |
| Year 4 | Students solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies. ACMNA080 |
| Year 5 | Students create simple financial plans. ACMNA106 |
| Year 6 | Students investigate and calculate percentage discounts of $10 \%, 25 \%$ and $50 \%$ on sale items, with and without digital technologies. ACMNA132 |
| Year 7 * | Students investigate and calculate 'best buys', with and without digital technologies. ACMNA174 |
| Year 8 | Students solve problems involving profit and loss, with and without digital technologies. ACMNA189 |
| Year 9 | Students solve problems involving simple interest. ACMNA211 |
| Year 10 | Students connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies. ACMNA229 |

Numeracy continuum: Estimating and calculating with whole numbers

| End Foundation | Use money: Recognise the different value of coins and notes in the Australian monetary system. |
| :--- | :--- |
| End Year 2 | Use money: Identify and use combinations of coins and notes for simple purchases. |
| End Year 4 | Use money: Estimate the change from simple purchases. |
| End Year 6 | Use money: Create simple financial plans, budgets and cost predictions. |
| End Year 8 | Use money: Identify and justify 'best value for money' decisions. |
| End Year 10 | Use money: Evaluate financial plans to support specific financial goals. |

[^0]
## Resources

## NRICH website

http://nrich.maths.org
In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.


Copyright © 1997-2018. University of Cambridge. All rights reserved. NRICH is part of the family of activities in the Millennium Mathematics Project.

## Scootle

https://www.scootle.edu.au/ec/p/home
This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.


## MoneySmart Teaching http://tiny.cc/SecondaryMoneySmart

MoneySmart Teaching is a comprehensive strategy to develop consumer and financial literacy capabilities in young Australians. MoneySmart Teaching uses real-life financial contexts for learning.

## Dan Meyer's blog: 101 questions

 http://www.101qs.comDan's blog contains images and short films that can be presented to students along with the question: What's the first question that comes to mind?

A spreadsheet of Dan Meyer's Three-Act Maths Tasks can be accessed at http://bit.ly/DM3ActMathTasks.


## Start Smart

http://www.startsmart.com.au/Secondary/ program-overview

The Start Smart Secondary program offers a suite of workshops developed for students in Years 7-11. Each workshop focuses on a different topic in depth and explores financial concepts such as understanding the impact of financial choices, getting your first job and managing your mobile phone as well as basic investment principles, savings goals and budgeting.

The Secondary program is an interactive learning experience that is designed to engage students and empower them to make confident financial choices.


Notes
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


[^0]:    Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

