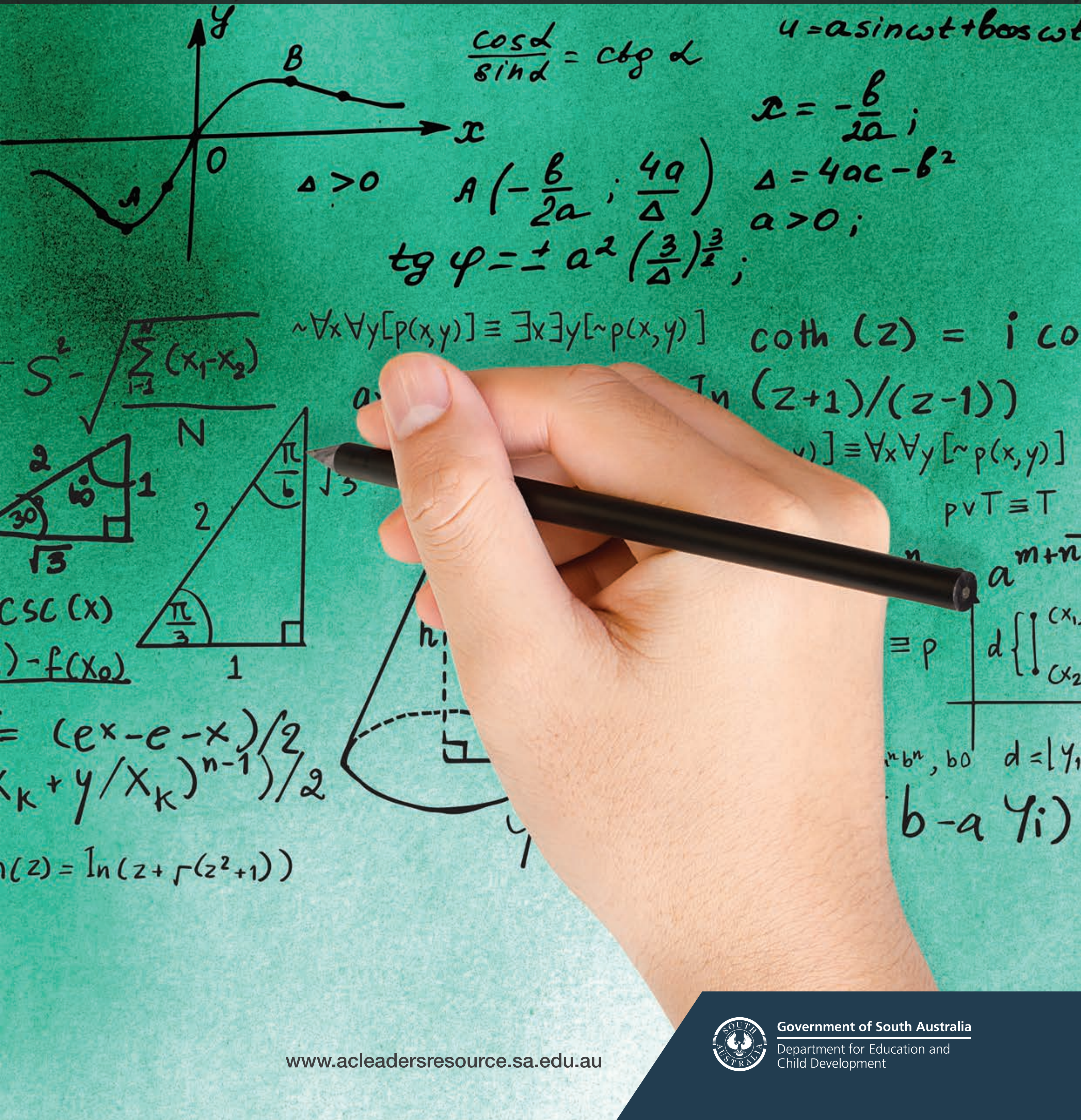
Pythagoras and trigonometry: Year 10/10A

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key

The ‘**AC**’ icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The ‘**From tell to ask**’ icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about ‘Transforming Tasks’:
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The ‘**Bringing it to Life (BitL)**’ tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the ‘Bringing it to Life’ tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **‘Pythagoras and trigonometry’ from Year 9 to Year 10A** (see page 25)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with Pythagoras and trigonometry:

- Investigating and understanding similarity, Pythagoras’ Theorem and/or trigonometry
- Using and applying similarity, Pythagoras’ Theorem and/or trigonometry to solve problems.

What the Australian Curriculum says about ‘Pythagoras and trigonometry’

Content descriptions

Strand | Measurement and geometry.

Sub-strand | Pythagoras and trigonometry.

Year 10 ◆ | ACMMG245
Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.

Year 10A ◆ ◆ | ACMMG273
Students establish the sine, cosine and area rules for any triangle and solve related problems.

Year 10A ◆ ◆ | ACMMG274
Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

Year 10A ◆ | ACMMG275
Students solve simple trigonometric equations.

Year 10A ◆ | ACMMG276
Students apply Pythagoras’ Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.

Year level descriptions

Year 10 ◆ | Students find unknown lengths and angles using applications of trigonometry.

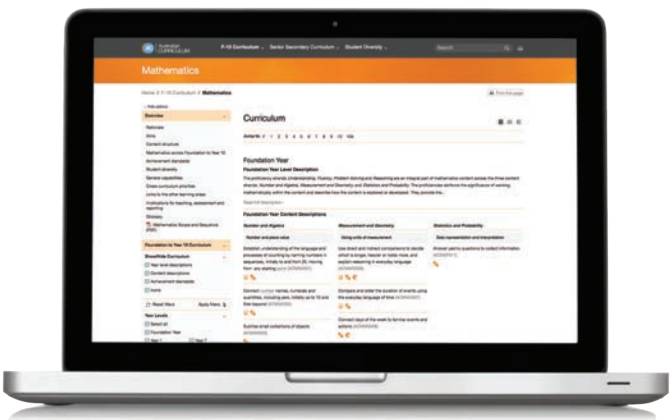
Achievement standards

Year 10 ◆ | Students use trigonometry to calculate unknown angles in right-angled triangles.

Numeracy continuum

Using spatial reasoning

End of Year 10 ◆ | Students visualise, describe and analyse the way shapes and objects are combined and positioned in the environment for different purposes (Using spatial reasoning: Visualise 2D shapes and 3D objects).



Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Working with Pythagoras and trigonometry

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 10/10A 'Pythagoras and trigonometry'

In Year 9 Pythagoras' Theorem and its applications are introduced. Students apply their understanding of similarity to establish the relationship between the corresponding sides of similar right-angle triangles. This is used to investigate the sine, cosine and tangent ratios in these triangles. The mathematical terminology 'opposite', 'adjacent' and 'hypotenuse' are introduced and students select and apply the appropriate trigonometric ratio to solve problems about angles and side lengths of right-angle triangles.

In Year 10 the terminology 'angles of elevation' and 'angles of depression' are introduced and students solve right-angle triangle problems applying a combination of Pythagoras' Theorem and trigonometry.

In Year 10A students solve problems in right-angle triangles in a three-dimensional context. Students graph trigonometric functions and establish their symmetrical and periodic properties. The use of sine and cosine rules for any triangle is also explored in 10A.

- **Solving word problems or problems from images, is NOT necessarily problem solving.** One element of fluency is choosing and using mathematics flexibly. If the student has knowledge of Pythagoras' Theorem, trigonometry and bearings, then applying that knowledge, even within a context, is expected. Becoming familiar and comfortable with the application of these concepts is useful, but it's important that teachers do not mistake this for problem solving.
- **Students can create their own understanding of even abstract concepts**: but an activity on its own does not ensure conceptual understanding.
  Some important questions we should ask:
  - Does the learning connect to and build on the student's prior learning?
  - Do the students have the opportunity to do their own thinking?
  - Are the students required to explain their thinking even if they are right?
  - Do students have the opportunity to regularly reflect on and record their learning?
  - Are there strong ongoing connections between the student's understanding of the solution and the appropriate formal mathematical language and notation?
  - Is there an expectation that all students will make sense of what they do?
- **Working in the context of direction, angles of elevation and depression** is the application of previously established understandings in working with right-angled triangles and angles on parallel lines. This *does not* mean that we need to re-teach these geometric skills to the whole class before we begin. Ask students to work in groups to record everything they know about Pythagoras' Theorem and trigonometry. Sharing this in class discussion gives us and the students an insight into what they remember. By using questions from the Understanding section of the BitL tool, we can identify where students may need additional support from us or from their peers when they encounter this content in solving problems.
- **Year 10A content descriptions extend Pythagoras and trigonometry to more abstract levels.** Despite this we can still support students in the development of conceptual understanding, if we design learning activities where students construct their own knowledge through exploring patterns, and avoid telling students what we want them to learn.
- **The new learning is in interpreting the information about directions, angles of elevation and depression to draw and label a right-angled triangle that can be used to solve the problem.** Students generally find this very challenging. Even if they understand the basic concepts, they cannot always apply them in context and so our learning design must address this. When we ask students to solve written problems with diagrams provided, we assume that they will automatically see the connections between the text with the factual information (given and/or implicit) and the two-dimensional diagram with mathematical shapes and labels. We need to encourage students to read the text for meaning and require them to identify where this is represented in the diagram, through questions that interrogate its features. When we ask them to answer these questions, we are providing scaffolding that will support them in drawing diagrams for themselves when necessary.
- **Practice on its own is not enough.** Often, we think students will get better at this if we just give them enough practice, but we need to be more deliberate and account for it in our learning design. Rather than giving students a large number of worded situations to represent in diagrams, consider giving them one labelled diagram and asking them to make up different contexts and describe the features of the scenario in different ways. Sharing different descriptions encourages students to focus on what is really taking place and what they want to find. We can ask students to act it out or even visualise the action as a film clip in their head.
- **Keep the rigour for Year 10 students.** We must be particularly aware of the need to challenge students to problem solve using this knowledge, rather than practise routine calculation questions. If students are unsure of the pre-requisite knowledge, refer to the Year 9 Narrative for ideas about establishing this understanding.

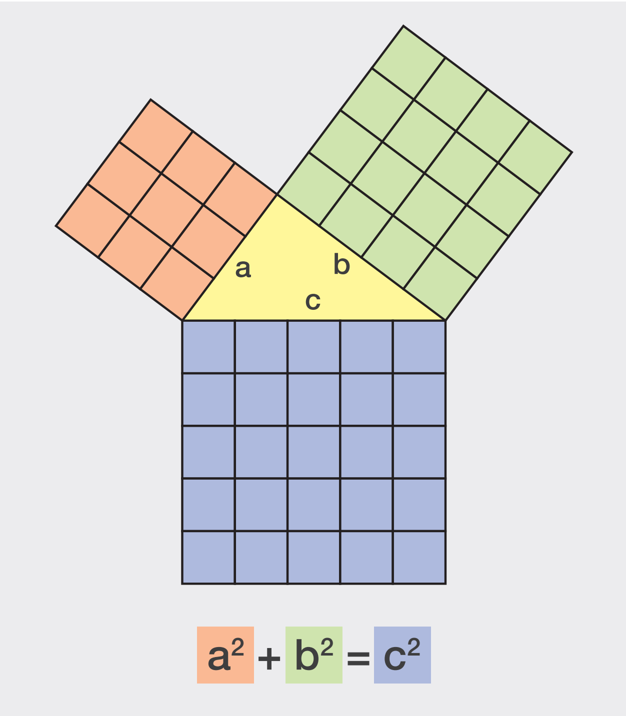
Engaging learners

Classroom techniques for teaching Pythagoras and trigonometry

Supporting learning with manipulatives

Idea six of Marilyn Burn's 10 Big Maths Ideas is to support learning with manipulatives and this is for the older students as well – http://www.scholastic.com/teachers/article/marilyn-burns-10-big-math-ideas.

It is possible to develop a conceptual understanding of Pythagoras' Theorem and trigonometry by drawing, cutting out, manipulating and measuring, both with concrete materials and digital objects. It enables all students access to this learning.



Real life contexts

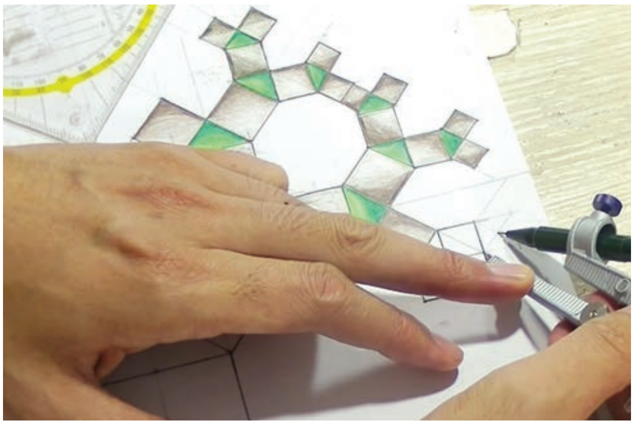
Our natural and built environments provide multiple practical and real life contexts for students to personalise, connect and apply their learning. Architectural applications and 'the knotted rope' example in the Year 9 Narrative, demonstrate the power and simplicity of these geometrical concepts.

Pythagorean tree

For students who enjoy artistic and visual approaches to their learning, they will be engaged and intrigued by drawing a Pythagorean tree.

Watch the video at https://www.youtube.com/watch?v=9UtdjVWSluo, then ask students:

- *How many geometric principles can you see in this construction?*
- *Why is it a Pythagorean tree?*



Source: Dearing Wang (2014) *How To Draw Fractal Tree*, accessed at https://www.youtube.com/watch?v=9UtdjVWSluo

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–13)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the name for sine or cosine ratios** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged with questions that will result in them identifying **constant ratios for right-angled triangles**, so I don't need to instruct that information.

At this stage of development, students can develop an understanding of Pythagoras and trigonometry through conducting their own experiments. When teachers provide opportunities for students to predict, identify, describe and represent the outcomes of the experiments and compare them to theoretical expectations, they require their students to generalise. Telling students the laws of trigonometry removes this natural opportunity for students to make conjectures and verify connections that they notice.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular formula during the current unit of work, then it probably is quicker to tell them the formula and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the formulae is a false economy of time.

When we challenge our students to *establish* theorems, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Bearings golf Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.	ACMMG245 ◆
Example 2: NRICH bearings problem Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.	ACMMG245 ◆
Example 3: Trevor's stuck in the tree (angles of elevation and depression) Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.	ACMMG245 ◆
Example 4: Establishing Pythagorean triads Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.	ACMMG245 ◆
Example 5: Unit circle Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.	ACMMG274 ◆ ◆
Example 6: Unit circle data Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.	ACMMG274 ◆ ◆
Example 7: Area of a triangle Students establish the sine, cosine and area rules for any triangle and solve related problems.	ACMMG273 ◆ ◆
Example 8: Periodic behaviour Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.	ACMMG274 ◆ ◆

Example 1: Bearings golf

ACMMG245 ◆

Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students features of true bearings, we can challenge students to recognise these for themselves, by ***asking*** questions.

Bearings are an abstract concept for students to understand, particularly when most problems they encounter are posed and solved by drawing small diagrams on paper; representative but not usually to scale. We can give students an opportunity to understand how bearings can be used to locate points and describe their positions relative to each other, by getting them to navigate their way through a 'golf course'. This activity requires pegs, rope, a compass (indicating North and an angle). Whiteboard protractors can measure angles accurately as well.

In an open space, set up 2 pegs: one indicating Hole 1 and the other the tee-off point on your golf course. Then ask students:

- *Can you describe where Hole 1 is from the tee-off position as accurately as possible? Is there another way?*

With no equipment at this stage, students will need to estimate distances in metres or in terms of paces, but are likely to describe directions in different ways eg 15 paces towards the big tree. If students have used angles or compass directions in some way, use this as an introduction to true bearings. If not, encourage them to consider more accurate methods through questioning:

- *How likely is a person to find the exact location of Hole 1 using your instructions? What instruments might we need to be more accurate in our description?*

Students are unlikely to 'discover' true bearings as a way of locating the hole by themselves. Drawing on the suggestions that students have used earlier in the activity, teachers can introduce bearings as a method all mathematicians use so that they can locate points using a convention that everyone accepts and understands: 'It uses compass directions, and angles and distances just like you suggested'.

Show students Figure 1 and organise them into 6 different groups. A green keeper has designed another golf course using these directions called **true bearings**:

Hole 1 from the tee-off :	057°T – 224 m
Hole 2 from Hole 1:	127°T – 312 m
Hole 3 from Hole 2:	252°T – 397 m
Hole 4 from Hole 3:	292°T – 337 m
Hole 5 from Hole 4:	53°T – 220 m

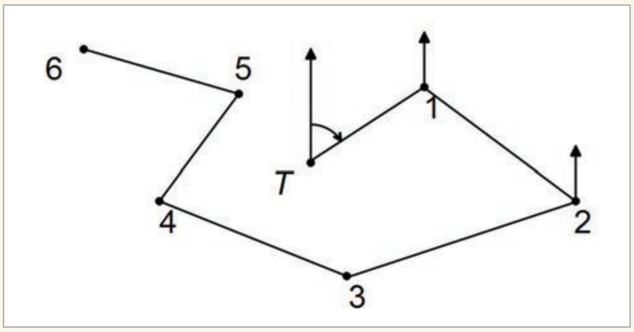


Figure 1

Ask students:

- *What do you notice? What is similar/different about these directions?*
- *What do you think the rules might be in giving a true bearing?*
- *What do you think the bearing for Hole 6 from Hole 5 on this golf course might be?*
- *What do you think the bearing of the tee-off from Hole 6 might be? Explain why.*
- *If you know that the bearing of Hole 6 from the tee-off is 297°T, what would the bearing of tee-off from Hole 6 be?*
- *Are there different ways you know to describe the same directions?*

Having identified that for true bearings they face North and turn clockwise, they could relate (120°T) to compass bearing (S60°E):

- *Why are compass bearings taken from North and South, but not East and West?* (Compasses use the Earth's magnetic field and only point North or South.)
- *What advantages/disadvantages are there to using true bearings?* (Turning 300° clockwise is 'the long way around'. It would be shorter to go 60° anticlockwise, but with true bearings a direction can be indicated using 3 digits only and there is little chance of confusion.)

Once students have established the rules for true bearings, ask the groups to improve their description for the location for Hole 1 on our golf course, using the true bearings. Place a hoop around the peg and state that any group whose bearing lands in the hoop, has a hole-in-one. Allow each group to use the equipment (a compass, board protractor, length of rope or tape measure) to find the location based on their estimate. This is also a way to make sure all students can use the equipment. If their estimate is not located in the loop, ask them to set up North again and give a bearing for their second shot.

Support and encourage – rather than rescue

Rather than always taking the lead, we should encourage students from different groups to help each other. Assist by asking for any clarifying questions, rather than instructing ie 'What would you like to know?' Do not rescue. If students reply with 'I don't get it', you could ask *'What are you trying to measure? What do you think the compass reads, if the rope goes out at this angle?'*

While it's quicker and easier to tell, student learning is more enduring and conceptual if they come to the understanding by themselves. While they may be resistant at first, students soon realise you are requiring them to think for themselves.

Allocate a hole to each group and ask them to set up a tee-off and hole. Instruct them to determine 'the true bearing of the hole from the tee-off'. Use this language. When the first group has finished, model checking the bearings by:

- facing the North and physically turning clockwise to the given direction, then
- moving the given distance to mark the hole.

This group can then use the same process to check the bearings determined by other groups for their hole.

Ask challenging questions:

- *Would 'the true bearing of the tee-off from the hole' be the same or different as the bearing from the hole to the tee-off?*
- *Is there a way to work out 'the true bearing of the tee-off from the hole' without measuring? What do you think the reverse bearing should be for Hole 1? Check it.*
- *What do you notice? Is there a pattern? Why?*

This requires students to recall what they know about angles on parallel lines (Year 7 ACMMG163).

As an indoor activity, a golf course could be designed on an old sheet and students have to estimate their shots on the greens using true bearings.

Example 2: NRICH bearings problem

ACMMG245 ◆

Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about common misconceptions, we can challenge students to recognise these for themselves, by ***asking*** questions.

This activity is from the NRICH website.

This problem requires an application of the understanding of bearings without a diagram provided. As an additional activity, offer a range of student-drawn diagrams (not necessarily correct) to compare and contrast like the ones in Figure 2. Organise students into groups and ask:

'Six different groups of students who tackled this NRICH problem got confused. All started by drawing diagrams to help them. Can you check if they have represented the information correctly and give them some feedback on how to correct and make the diagrams more useful?'

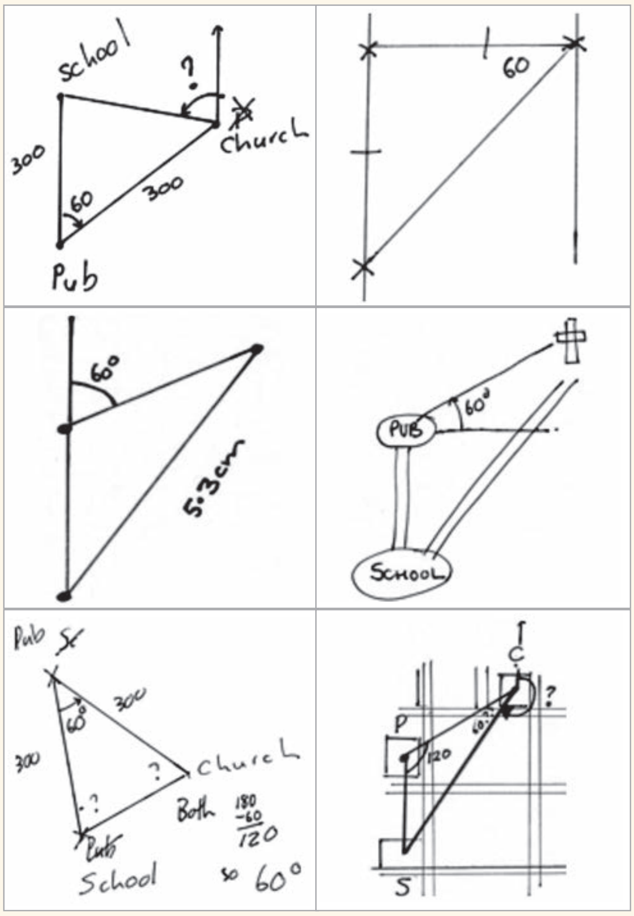
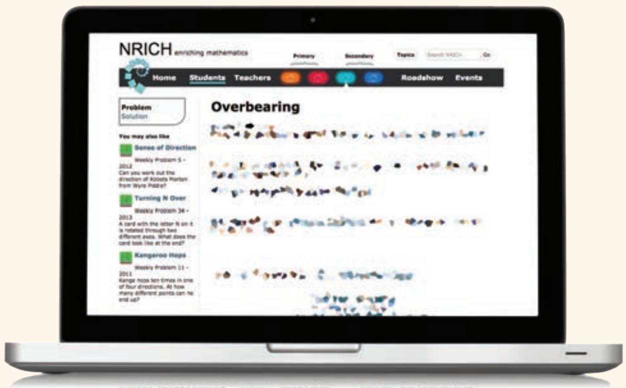


Figure 2

These examples illustrate common misconceptions students have when interpreting information about bearings. When the groups share their feedback, they are demonstrating reasoning and also identifying misconceptions that other groups may not have noticed.

Tackling misconceptions: Knowing what it isn't

This type of question can be used to address common misconceptions about bearings. Students can be given problems where information is missing or not represented in an accurate or useful way. Students often feel more confident to appraise the work of other students (even if fictitious), alerting themselves to the possible pitfalls of drawing diagrams for bearing problems, without us telling them.



The link to this problem on the NRICH site is: http://nrich.maths.org/6263

Example 3: Trevor's stuck in the tree (angles of elevation and depression)

ACMMG245 ◆

Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the hypotenuse is the longest side in a right-angle triangle, we can challenge students to recognise this for themselves, by ***asking*** questions.

Working in authentic contexts: Through creating connected learning experiences

As teachers, we can engage students to be more active in their learning through storytelling and acting out scenarios. Rather than telling students or defining what angles of elevation and depression are, we can allow them to physically experience naturally looking forward, and having to raise or lower their eyes to see an object. This experience helps to reinforce that the angles must be taken from the horizontal which can avoid the misconception that they can be taken from the vertical or any other 'line in a diagram'.

In this activity, instead of telling students about the hypotenuse, we can challenge them to recognise this for themselves, by presenting the following:

'I was standing in the shade of a tree just outside a block of flats looking out across the park. I heard Trevor the cat meowing and looked up to see he was stuck in the tree. A friend was sitting on the balcony of a third floor flat, heard Trevor at the same time, and looked down to see where the cat was stuck. Later, after the cat had been rescued, we were arguing about how high Trevor was in the tree and thought we should be able to work it out using maths.'

Ask students:

- *Do you have an idea about how to start?*
- *What information would you need to work out how high Trevor was?*

A photo taken at the time indicated the two angles of sight when looking at the cat were:

Angle of elevation: 75°

Angle of depression: 30°

- *What do you think this means? Why? Could you draw the photo that was taken of Trevor's dilemma?*
- *What do you understand about angles of elevation and depression? How could you explain it to a Year 9 student?*

Students can relate the word *elevation* to the everyday word *elevated*, meaning raised and *depression*, meaning *down*. But where is the angle taken from? The neutral stance that most people have would be looking straight ahead, towards the horizon or horizontally. A common error is taking the angle from the vertical, so this is a good opportunity to ask:

- *Why couldn't it be like this?*
- *What would you say to a Year 9 who drew it like this?*
- *What do these angles tell you about where Trevor was? Can you tell who was closer to the cat?* (You cannot deduce which person is standing closer to the cat, just because the angle is greater, unless you are on the same level.)
- *Who thinks I was closer? Who thinks my friend was closer? Who thinks the distance is the same? Can you draw how this is possible?*

A second photo taken later during the rescue, suggests that my angle of elevation of sight had changed to 85°. Ask students:

- *What does that tell you about how the situation had changed?*
- *What further information do you need to work out the height Trevor is above the ground?* (Knowing the horizontal distance that either person is from the cat, is enough to work out the height, relative to their eye level. To find the cat's height off the ground would require either the height of the person on the ground or, for the person on the third floor; their height and the height of the third-floor balcony. Students can either agree to use the same values for these measurements, or take different scenarios and compare their results.)
- *Did anyone else do it differently? How did your answers compare? Which is more likely to be accurate? What assumptions have you made?*

Example 4: Establishing Pythagorean triads

ACMMG245 ◆

Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about Pythagorean triads or triples, we can challenge students to recognise this for themselves, by ***asking*** questions.

In this activity, instead of telling students about the hypotenuse, we can challenge them to recognise this for themselves. We can introduce this idea by showing students Figure 3 and stating:

'These right-angle triangles are special because the length of all three sides is a whole number.'

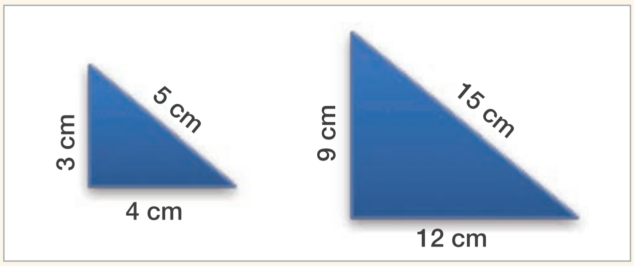


Figure 3

Then asking questions such as:

- *Can you see any connections between these triangles?*
- *Now that you have noticed a relationship, could you suggest another right-angle triangle with all three sides of a whole number value?*
- *How could you check that you have produced another right-angle triangle?*
- *Is there a rule that you could use to describe how you worked out another possibility?*
- *Is there another triangle that you could produce using your rule? How many could you make?*

You have identified a relationship between the lengths of the sides of the triangles. These special triangles are called Pythagorean triads (or Pythagorean triples).

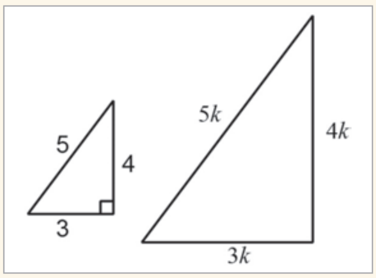


Figure 4

Show students Figure 4, then ask:

- *Can you prove that a right-angle triangle with sides that are multiples of 3, 4, and 5 will be right-angled also?*
- *Is it possible to have a Pythagorean triple that is not in this ratio? Convince me.* (This question models to students that when they see a possible connection, they should try to extend their thinking to other cases to see if the connection continues.)

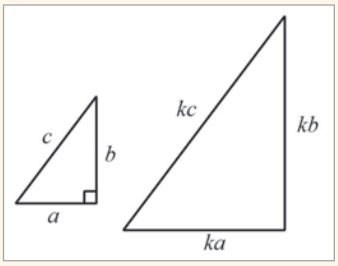


Figure 5

Show students Figure 5, then ask:

- *Can you prove that if a triangle with sides a, b and c is right-angled, then a triangle with sides ka, kb, kc will be right-angled also?*

Constructing knowledge through formal proof: From information to understanding

In this questioning sequence, we have moved from noticing a possible pattern, through to using that information to suggest another possibility; checking that possibility, generalising the relationship, and finally into a formal mathematical proof about Pythagorean triads.

To establish that the larger triangle would be right-angled, you need to prove that Pythagoras' Theorem would work for that triangle as well. You must prove that $(ka)^2 + (kb)^2 = (kc)^2$.

Two possible proofs are shown below:

LHS (Left hand side) $= (ka)^2 + (kb)^2$
$= k^2 a^2 + k^2 b^2$
$= k^2 (a^2 + b^2)$
$= k^2 c^2$
$= (kc)^2$

OR

Given $a^2 + b^2 = c^2$
$k^2 a^2 + k^2 b^2 = k^2 c^2 \quad k \neq 0$
$(ka)^2 + (kb)^2 = (kc)^2$

Simple examples of this flawed logic, show that statements are not always true in both directions:

- If Gertie is a goat, then Gertie is an animal. But if Gertie is an animal, it is not necessarily true that she is a goat.
- If $x = -2$, then $x^2 = 4$ is a true statement. But if we start with what we are trying to prove … if $x^2 = 4$, is it always true that $x = -2$? What other possibility is there?

(Another version of this activity is on pages 11–12 of the Pythagoras and trigonometry: Year 9 narrative.)

Example 5: Unit circle

ACMMG274 ◆ ◆

Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students that the unit circle can be used to find trigonometric ratios, we can challenge students to recognise this for themselves, by ***asking*** questions.

This activity is from the NRICH website.

In this activity, the students are asked to collect data about the sine and cosine for triangles using the unit circle.

The link to this problem on the NRICH site is: http://nrich.maths.org/5615/index

State the following:

'A Year 11 student claims that they can find the trigonometric ratios for angles without using a calculator.'

Then ask students:

- *Do you know any trigonometric ratios for any angles without using your calculator? How?* (Students may know that the sine of 30° is ½, or even that sine of 0 is 0 and the sine of 90° is 1. Ask students to check using their calculator and explain what it means and how that might be useful to us.)

At this point, ask students to keep a record of all the values they work out in a table – either as class or individual data – so that you can use this valuable information for Example 6: Unit circle data.

State the following:

'A Year 11 student claims to be able to find an approximate value for the sine of any angle using a scale diagram of a circle with radius 1 and centre (0,0), or the unit circle.'

Then ask students:

- *Why do you think it was called that?* (The circle has a radius of a single unit.)

Show students Figure 6, then state the following:

'I asked her to show me how she would find sine 57° and she drew the following and answered 0.84.'

Then ask students:

- *How do you think she arrived at this answer? Does it make sense to you? Check using your calculator.*
- *Why does it work? How accurate was it? Why?*

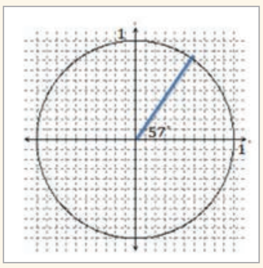


Figure 6

- *How do you think she would find the sine of another angle? What about the cosine of an angle?*
- *What does it make you wonder?*
- *How could you check this?*

If students need some support, ask them to use the unit circle. Draw a similar diagram for 30° and see what they notice about the coordinates of the point on the unit circle.

Once students notice that the x coordinate is the cosine of the angle and the y coordinate is the sine of the angle, teachers can encourage students to check other angles to verify the relationship. Challenge them to prove it in the general case by asking:

- *Would this always be the case? Convince me. Prove it.*

Students might:

- draw up the right-angled triangle as shown in Figure 7 and then link the legs (sides that are not the hypotenuse), to the *x* and *y* coordinates
- draw in the *x* and *y* lengths and realise they have created a right-angled triangle.

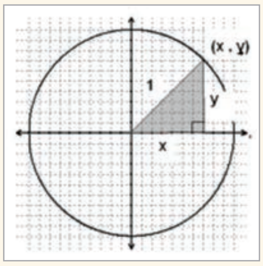


Figure 7

Using basic right-angled trigonometry, students can deduce that:

$\sin 30° = \frac{y}{1}$

$y = \sin 30°$

Ask students to use their calculator to find the sin123°, then ask:

- *Have you ever calculated this before when you were working with right-angled triangles in Year 9? Why not?*

Inform students that angles in right-angled triangles are between 0 and 90 degrees, then ask:

- *How do you think the Year 11 student might have worked that out? Can you check that?*

Ask students to use their calculator to find the cos123°, then ask:

- *What do you wonder? What else do you notice? How might you use the unit circle to find these values?* (Students may recognise that it is the same as sin57, or that cos123 will be negative. The coordinates of the unit circle are still (cos123°, sin123°). It is important that students note that the angle of 123° is taken from the positive x-axis.)

You can explore this a little further with students by:

Using a sheet of coloured paper, accurately construct and cut out a right-angled triangle with hypotenuse 1 unit (10 cm) and $\theta = 30°$. Place it on the unit circle and using the scale, estimate the sin30° and cos30° (refer to Figure 8).

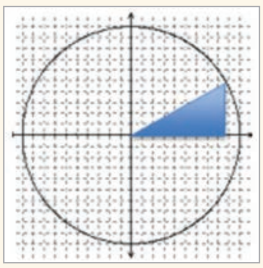


Figure 8

Reflect the triangle in the *y*-axis, mark the point on the circle and find its coordinates. Then ask students:

- *Why might this triangle be useful for finding sin150 and cos150? Check using your calculator. What surprises you?* (Students have not had negative trigonometric ratios – they have only been working with the acute angles in right-angled triangles where it is not possible to have a 150° angle. By understanding how trigonometry relates to the unit circle, we are able to consider the sine and cosine of angles bigger than 90°.)
- *What other angles can you find the cosine and sine of?* (Refer to Figure 9 – Students can find the sin and cos of 210° (180 + 30) and 330° (360 – 30) by reflecting the triangles in the x-axis.)

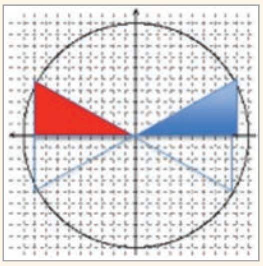


Figure 9

- *How might you use this triangle and the unit circle to find the sin60° and cos60°?* (Place the triangle shape so the 60° angle is at the origin instead of the 30°.)
- *Would this work for other triangles? What has to be true for the triangle to be used on the unit circle? Can you draw a triangle of your choice and use it to find the cosine and sine of a range of related angles?* (The triangle needs to have a hypotenuse of length 1 unit.)

We can also ask students to write an explanation for other Year 10's about what they have understood about the unit circle and what information they could gain from it.

Further tasks that support students to create their own understanding of the content of Year 10A Mathematics can be found in the Mathematical Association of South Australia publication '**Rich Tasks for 10A Mathematics**' available at http://www.masanet.com.au/contact-us/.

Example 6: Unit circle data

ACMMG274 ◆ ◆

Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the shape of the sine graph, we can challenge students to recognise this for themselves, by ***asking*** questions.

During the exploration in *Example 5: Unit circle*, students were asked to collect data about the sine and cosine ratios for the triangles they have used.

Prompt students to consider the table of information they collected about the unit circle and ask them to:

- *List as many different interesting things you notice about this data as you can.*
- *Is there another way other than a table to represent your information?*

'Think, pair, share': A process to support all students to think deeply

With a classroom brainstorm, students tend to share a range of rapid first responses. This may not allow all students to think more deeply about the problem. A practice called 'Think, pair, share' allows all students time to consider the problem individually, as well as a safe way to discuss their thoughts.

'Think' time is when each student thinks silently about the problem.

'Pair' time is for students to discuss their ideas with one other student.

'Share' time is an opportunity for the teacher to facilitate students sharing, comparing and contrasting the ideas that they have had or they have heard.

Graphing is a good way to help us see trends in data. Students can plot their sine and cosine values against the angle and compare and contrast the scatter plots. Excel spreadsheets or a graphics calculator can be used for students to input their data and observe the trends. Students might notice the maximum and minimum values of 1 and (–1).

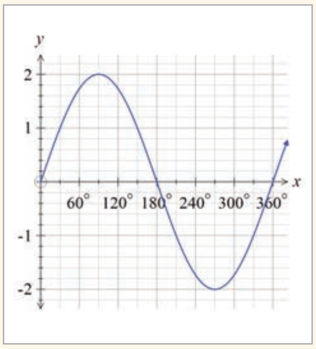


Figure 10

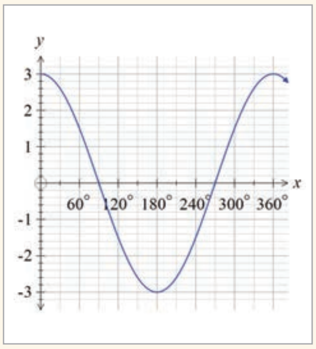


Figure 11

Show students the graphs in Figures 10 and 11, then ask:

- *How do you think this data might have been created?*

Figure 10 is similar to the $y = \sin x$ graph but with a maximum of 2. The graph of $y = 2\sin x$ can be created using a circle of radius 2 on a coordinate axes.

Figure 11 is most like the $y = \cos x$ graph with a maximum of 3. The graph of $y = 3\cos x$ can be created with a circle of radius 3.

A free graphing calculator is available at this website: https://www.desmos.com/calculator
A short tutorial supports the user to set up the function with or without sliders so students can explore trigonometric functions.

Ask students to generate and print a range of trigonometry graphs, and post them up for students to identify and recreate using the graphing calculator.

Further tasks that support students to create their own understanding of the content of Year 10A Mathematics can be found in the Mathematical Association of South Australia publication '**Rich Tasks for 10A Mathematics**' available at http://www.masanet.com.au/contact-us/.

Example 7: Area of a triangle

ACMMG273 ◆ ◆

Students establish the sine, cosine and area rules for any triangle and solve related problems.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the formula for finding the area of any triangle, we can challenge students to recognise this for themselves, by ***asking*** questions.

Inform the students that you want them to construct a particular triangle accurately on their page.

Not supplying all relevant information

Students identify the information which is needed to solve the problem. When setting a task for students, try giving them insufficient information and require them to get the detail they will need by asking questions. Often, we give students all the essential information they need and no extra. When we do this, we deprive them of the opportunity to determine what is important and what is irrelevant when solving a problem. We can also tend to take a long time explaining in detail what we want them to do. This does not necessarily mean that they will be better prepared to complete the task, as they will often stop listening when we do all the talking and all the thinking.

Ask students:

- *If I ask you to draw a particular triangle, what questions would you ask before you started the task?*

Start with an acute-angled triangle with dimensions that will fit on A4 paper, but at least ½ page so that any inaccuracies due to measurement are less significant.

In this case, it is best to be unhelpful in the sense that you require specific, thoughtful questions. On some occasions you might even choose to give only yes or no answers:

- 'How long are the sides?' I don't know the length of all the sides.
- 'Do you know the length of one of them?' Yes.

If the students ask for the size of one of the angles, give the measure but not the fact that it is the angle between the two sides. If it is placed in another position, it will produce a different triangle and this idea connects with conditions necessary for congruent triangles that students explored in Year 8.

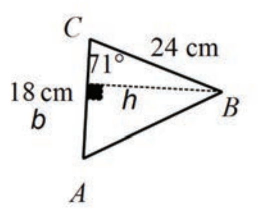
Give them the information that two of the sides are 18 cm and 24 cm, and one of the angles is 71°.

Ask them to determine the area of the triangle in as many different ways as possible, then ask:

- *Is there another way? Is the answer the same?*

Share the different methods the students used, ensuring that feedback is in order of increasing abstraction eg counting squares, drawing a square around the triangle, using the formula with the measured vertical height, and using the formula and the vertical height, calculated using trigonometry (see Figure 12). Ask students:

- *Which method is the easiest/quickest/most accurate? Is this always the case?* (Sometimes the less abstract methods can be easier or quicker and if we don't require an exact answer, like when we are ordering pavers to the nearest square metre, the result will be sufficiently accurate.)



$$\sin 71 = \frac{h}{24}$$
$$h = 24.\sin 71$$
$$h \approx 22{\cdot}7$$
$$A = \frac{1}{2} b \times h$$
$$= \frac{1}{2} \times 18 \times 22{\cdot}7$$
$$= 204{\cdot}3 \text{ sq units}$$

Figure 12

Ask students to draw a triangle of their choice and use a method that uses trigonometry. If students are not fluent in using this method, ask them to choose another method to verify their answer. Allow them to ask the student who used trigonometry initially, to help them in their solution if they struggle. In this way, we can share the role of expert with our students:

- *When would we be able to use A = ½ b x h without measuring or trigonometry? What information would we need to know?*

- *Would we always be able to work out the height using trigonometry for finding the area of a triangle? What information would we need to know?* (We would need two sides and the angle in between. Students should consider having 2 sides and not the included angle, which is not possible—unless right-angled—and an obtuse-angled triangle with an external height, which is possible.)
- *How could you generalise? Can you repeat the same steps you did with the numbers, but with pronumerals instead?*
- *How could you check that this works? Convince me. Did you prove or verify the rule?* (See Figure 13.)

Students need to understand that the algebraic process was **proving the new rule for area**, based on the fact that A = ½ b x h.

When we go back to a previous problem and substitute the numbers from the triangle into ½ absinC to see if we get the same answer as before, we were **only verifying the rule, A = ½ absinC**.

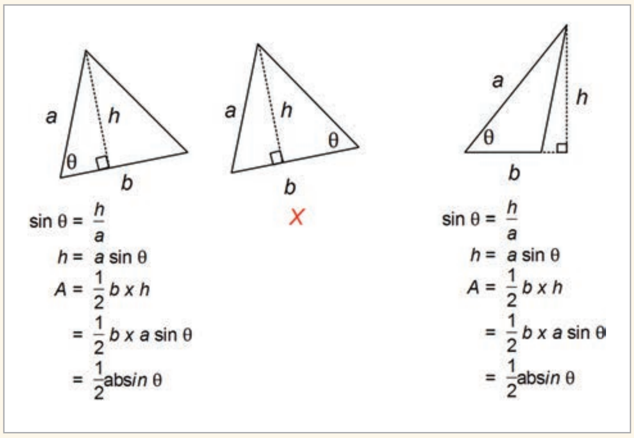


Figure 13

Construction of knowledge: Building a bridge between the particular and the general cases

When students have repeatedly applied the same method to solve problems:

- Ask them to explain their method verbally, to consolidate their understanding
- Ask them to write the general procedure in words (possibly for someone who has not been involved in the learning) to organise their thoughts
- Ask them to record their reasoning using mathematical language and symbols, to connect this generalised reasoning to their particular experiences.

This is an example of how to scaffold the connection between the students' reasoning about a particular case they have solved and the general solution to develop a formula.

'If we think this will be a useful method that we are likely to use more in the future, we could perform the process using pronumerals instead of numbers. Using pronumerals helps us track the calculations all the way to the answer, so that next time we need to calculate the area of a triangle, we can substitute the values into the algebraic answer at the end of the process (into the formula).'

Example 8: Periodic behaviour

ACMMG274 ◆ ◆

Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about periodic behaviour and when it is likely to occur, we can challenge students to recognise this for themselves, by ***asking*** questions.

Figure 14

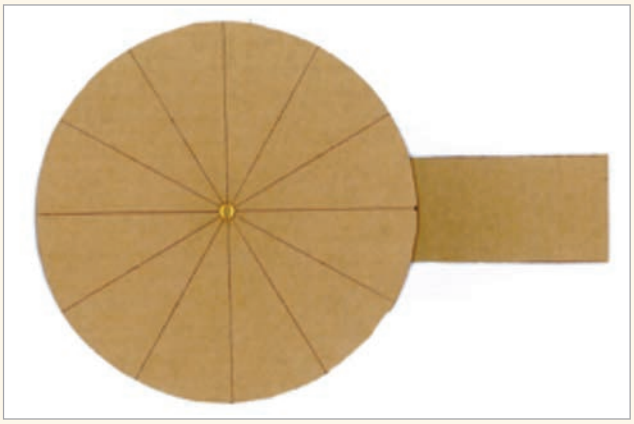


Figure 15

The Melbourne Star, in Figure 14, is a viewing wheel in Melbourne that:

- has 7 spokes
- is 120 metres tall
- does one revolution in 30 minutes.

A virtual tour is available via the *Melbourne Star sneak peek* YouTube clip at https://www.youtube.com/watch?v=I1_EtyweaK0.

Propose the following problem:

'Can you work out how high you will be above the ground at any minute during the ride?'

Then ask students:

- *How high will you be after 1 minute? How high might you be after 10 minutes? When will you be at that height again?* (Often students assume there is a linear relationship between the height and time, and that they will be 10 times as high at 10 minutes. There is often also an assumption that the height above the ground at the start is zero. While there is not ready information about the initial height, students could estimate this height from photos and clips, using ratios.)
- *How could you test your ideas?*

Scale diagrams can be used, but a physical model like the one in Figure 15, helps students test multiple situations in time.

If students do not suggest that they might use a physical model, we can provide a suggestion that a student last year used this thing made from cardboard and a split pin:

- *How do you think this might have helped? Do you think it would give us information that would be accurate enough? What concerns do you have? How could we improve this model?*

Discussions about the need to consider the scale of the wheel and its height off the ground, and how far the wheel would turn every minute, supports students in constructing a model which can generate useful data:

- *How did your results compare with your estimate? What did you notice? What will happen after 30 minutes? What do you wonder?*
- *How might you represent your results in a different way? Can you predict what a graph of height over time might look like?*

Graphing data helps us recognise patterns, and in this case the periodic nature of the data becomes obvious and can be explained easily in terms of the context of the circular motion of the wheel.

The second tallest building in Adelaide is Telstra House, which is 105 metres tall. Ask students:

- *At what stage of the ride will you be higher than this building?*

Students can use a graph of their own data from the model, or input the data into a graphics calculator and fit a sine model to the data. Use the model of this equation to verify the results found earlier. Explain how the values in the equation might relate to the wheel.

Further tasks that support students to create their own understanding of the content of Year 10A Mathematics can be found in the Mathematical Association of South Australia publication '**Rich Tasks for 10A Mathematics**' available at http://www.masanet.com.au/contact-us/.

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 9–12)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 9: The box challenge Students use Pythagoras' Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.	ACMMG276 ◆
Example 10: Trevor's stuck in a tree Students solve right-angled triangle problems including those involving direction and angles of elevation and depression. Students use Pythagoras' Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.	ACMMG245 ◆ ACMMG276 ◆
Example 11: Tilted squares Students use Pythagoras' Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.	ACMMG276 ◆
Example 12: Diagonals of area Students establish the sine, cosine and area rules for any triangle and solve related problems.	ACMMG273 ◆ ◆

Example 9: The box challenge

ACMMG276 ◆

Students use Pythagoras' Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students to apply Pythagoras' Theorem to solve the problems we can challenge them to identify them for themselves, by ***asking*** questions.

Present a cardboard box to your students and ask them to cut the longest piece of rod that will fit into this box. Inform the students that they can't measure the box, but they can know its length, width and height. They get to cut and try the rod just once, so students need be sure that they are convinced about their solution before they make the cut.

Teachers or students could extend the problem into generalisation by asking:

- *Can I create a rule that will work for any rectangular prism?*

Interpret

What have you been asked to calculate? What information is helpful/no use? What additional information would you like? (Establish that the student is aware where the rod will need to be placed. Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? What could you try? What mathematics could help you with this problem? Is there more than one way that you could solve the problem? Would it help if you drew a sketch? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

Does that length seem about right to you? How could you check? Do other people think that too?

Questions to be used only after students have grappled with the problem for a few minutes:

How could you extend this problem? Could you create a rule that would work for any rectangular prism? What if you needed to position the rod in a different prism, for example, in a triangular, pentagonal or hexagonal prism (where the cross-section is a regular polygon). Would you be able to calculate the length of the rod if you were only given the length of the edges of the polygon and the depth of the prism?

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any values?

Example 10: Trevor’s stuck in the tree

ACMMG245 ◆

Students solve right-angled triangle problems including those involving direction and angles of elevation and depression.

ACMMG276 ◆

Students use Pythagoras’ Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students to apply Pythagoras’ Theorem to solve the problems we can challenge them to identify them for themselves, by ***asking*** questions.

Watch the commercial where the cat, Trevor, is stuck in the tree. **Estimate how high you think the cat might be above the ground.** Use the knowledge you gained in *Example 3: Trevor’s stuck in the tree* and the commercial video to check your estimate.

The **AAMI Trevor’s in the tree** commercial video can be accessed at https://vimeo.com/126248929.

Example 11: Tilted squares

ACMMG276 ◆

Students use Pythagoras’ Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the hypotenuse is the longest side in a right-angle triangle, we can challenge students to recognise this for themselves, by ***asking*** questions.

This activity is from the NRICH website.

Tasks such as this one, allow students to investigate interesting geometrical connections using digital objects. The digital object asks the students to notice a pattern about the area of tilted squares on a grid. Students can make a conjecture and then verify the conjecture with multiple examples. Question students as to whether this constitutes proof; and establishing that it does not, consequently challenge them to prove it.

The NRICH site has a collection of ways to engage students in proof of Pythagoras’ Theorem: http://nrich.maths.org/6553

Provide an intellectual challenge by asking students to consider the volume of a tilted cube on a 3D grid.

The link to this problem on the NRICH site is: http://nrich.maths.org/2293

Example 12: Diagonals for area

ACMMG276 ◆
Students use Pythagoras' Theorem and trigonometry to solve three-dimensional problems in right-angled triangles.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the hypotenuse is the longest side in a right-angle triangle, we can challenge students to recognise this for themselves, by ***asking*** questions.

Problems such as this one give students the opportunity to create new knowledge from the known. Having developed a new formula to find the area and with knowledge about the trigonometry ratios for obtuse angles; students can investigate, make conjectures and attempt formal proof.

Propose the following problem:

'Can your new knowledge about the area of triangles be used to know more about quadrilaterals?'

Then ask students:

- *What do you wonder?*
- *Does it tell you anything about some of the special quadrilaterals?*
- *Is there a new rule that works for all quadrilaterals?*

The link to this problem on the NRICH site is: http://nrich.maths.org/437

Connections between ‘Pythagoras and trigonometry’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use Pythagoras and trigonometry as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 10/10A	
Whilst working with Pythagoras and trigonometry, connections can be made to:	**How the connection might be made:**
Factorise algebraic expressions by taking out a common algebraic factor. ACMNA230	Refer to: **Example 4: Establishing Pythagorean triads**
Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes. ACMMG244	Refer to: **Example 4: Establishing Pythagorean triads** **Example 5: Unit circle** **Example 7: Area of a triangle** **Example 9: The box challenge** **Example 11: Tilted squares** **Example 12: Diagonals of area**
Substitute values into formulas to determine an unknown. ACMNA234	Refer to: **Example 4: Establishing Pythagorean triads** **Example 7: Area of a triangle** **Example 9: The box challenge** **Example 12: Diagonals of area**
Use scatter plots to investigate and comment on relationships between two numerical variables. ACMSP251	Refer to: **Example 6: Unit circle data** **Example 8: Periodic behaviour**
Investigate and describe bivariate numerical data where the independent variable is time. ACMSP252	Refer to: **Example 8: Periodic behaviour**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Pythagoras and trigonometry’ from Year 9 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Pythagoras and trigonometry:

Investigating and understanding similarity, Pythagoras’ Theorem and/or trigonometry ◆ In **Year 9 to Year 10** students investigate for conceptual understanding and establish the trigonometric rules for finding areas of any triangles
Using and applying similarity, Pythagoras’ Theorem and/or trigonometry to solve problems ◆ In **Year 10 to Year 10A** students apply their knowledge of Pythagoras and trigonometry to solve increasingly complex problems in increasingly abstract situations.

Year level	‘Pythagoras and trigonometry’ content descriptions from the AC: Mathematics
Year 9 ◆	Students investigate Pythagoras’ Theorem and its application to solve simple problems involving right-angled triangles. ACMMG222
Year 9 ◆	Students use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles. ACMMG223
Year 9 ◆	Students apply trigonometry to solve right-angled triangle problems. ACMMG224
Year 10 ◆	Students solve right-angled triangle problems including those involving direction and angles of elevation and depression. ACMMG245
Year 10A ◆ ◆	Students establish the sine, cosine and area rules for any triangle and solve related problems. ACMMG273
Year 10A ◆ ◆	Students use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies. ACMMG274
Year 10A ◆	Students solve simple trigonometric equations. ACMMG275
Year 10A ◆	Students apply Pythagoras’ Theorem and trigonometry to solve three-dimensional problems in right-angled triangles. ACMMG276

Numeracy continuum: Using spatial reasoning	
End Foundation	Visualise 2D shapes and 3D objects: sort and name simple 2D shapes and 3D objects.
End Year 2	Visualise 2D shapes and 3D objects: identify, sort and describe common 2D shapes and 3D objects.
End Year 4	Visualise 2D shapes and 3D objects: visualise, sort, identify and describe symmetry, shapes and angles in the environment.
End Year 6	Visualise 2D shapes and 3D objects: visualise, sort, describe and compare the features of objects such as prisms and pyramids in the environment.
End Year 8	Visualise 2D shapes and 3D objects: visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects.
End Year 10	Visualise 2D shapes and 3D objects: visualise, describe and analyse the way shapes and objects are combined and positioned in the environment for different purposes.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

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