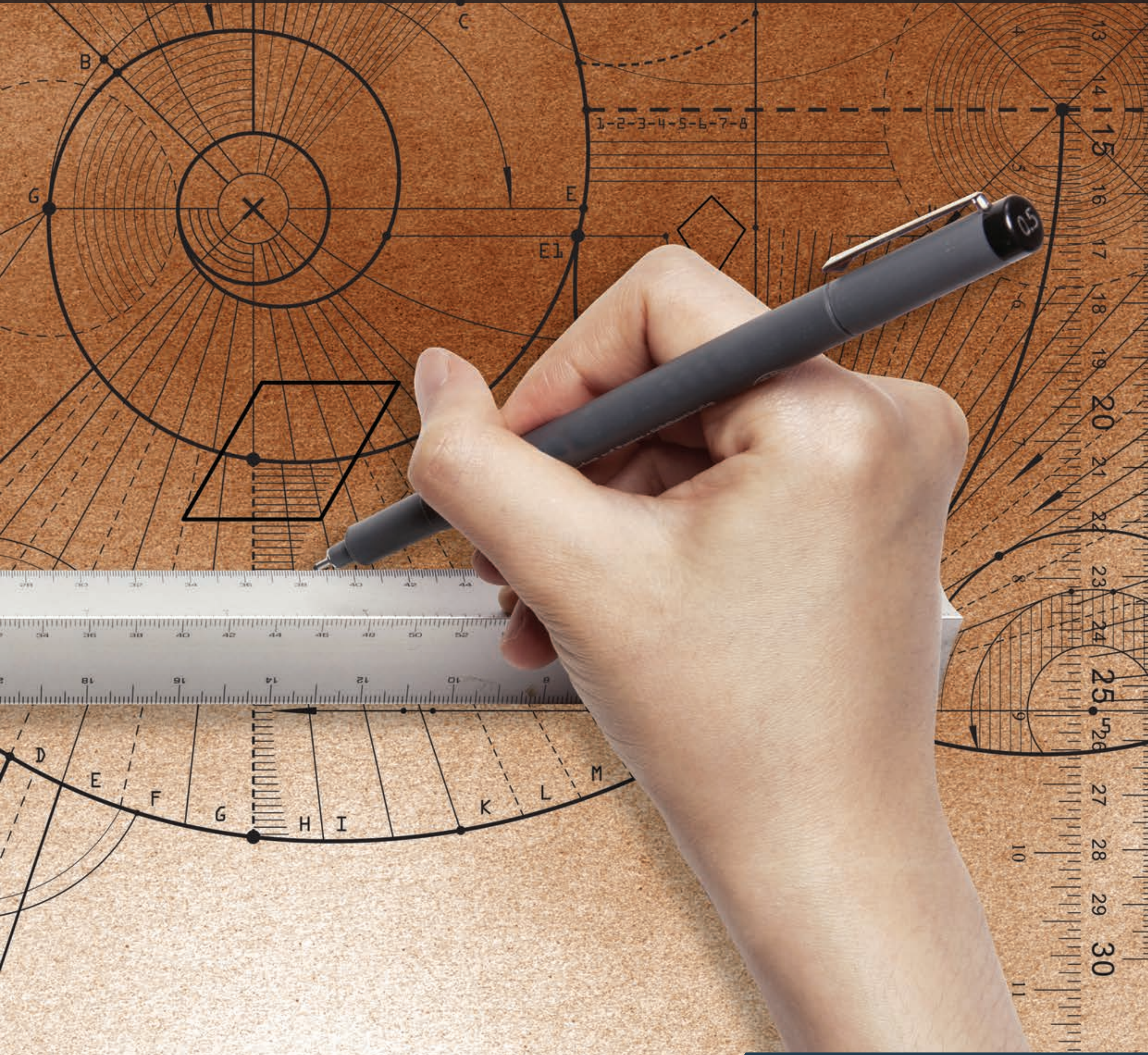
Using units of measurement: Year 10/10A

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

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Throughout this narrative—and summarised in **'Using units of measurement' from Foundation to Year 10A** (see page 25)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with units of measurement:

- Using informal units for direct or indirect comparisons
- Using standard metric units
- Establishing and applying formulae
- Estimating.

What the Australian Curriculum says about ‘using units of measurement’

Content descriptions

Strand | Measurement and geometry.

Sub-strand | Using units of measurement.

Year 10 ◆ | ACMMG242
Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Year 10A ◆ | ACMMG271
Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Year level descriptions

Year 10 ◆ | Students find unknowns in formulas after substitution.

Year 10 ◆ | Students use a range of strategies to solve equations.

Year 10 ◆ | Students calculate the surface area and volume of a diverse range of prisms to solve practical problems.

Achievement standards

Year 10 ◆ | Students solve surface area and volume problems relating to composite solids.

Year 10 ◆ | Students find unknown values after substitution into formulas.

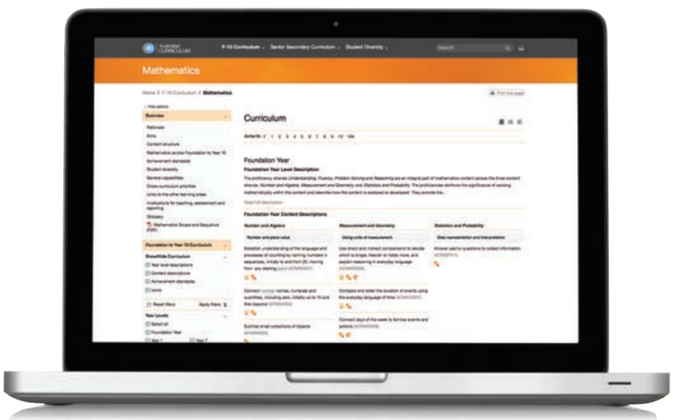
Numeracy continuum

Using measurement | **Estimating and calculating with whole numbers**

End of Year 10 ◆ | Students solve complex problems involving surface area and volume of prisms and cylinders and composite solids (Using measurement: Estimation and measure with metric units).

End of Year 10 ◆ | Students solve and model problems involving complex data by estimating and calculating using a variety of efficient mental, written and digital strategies (Estimating and calculating with whole numbers: Estimate and calculate).

Note: In the Australian Curriculum: Mathematics, the concept of ‘time’ is addressed in the sub-strand ‘Using units of measurement’, but in this resource, ‘time’ has its own narrative.



Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Working with units of measurement

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on in Year 10/10A 'using units of measurement'

In Year 9 the focus is on calculating areas and perimeters of composite shapes and surface areas of prisms. Students establish the formula for the volume of a cylinder and solve problems involving the volume of cylinders and prisms.

In Year 10 of the AC: Mathematics – calculations of surface area and volume of cylinders and prisms is extended to include problems involving composite solids. In **10A** problem solving involving surface area of pyramids, cones, spheres and composites of these solids is introduced.

- **Notice that, with the exception of the formula for calculating the volume of a sphere, cone and right pyramids, the content descriptions in this element of Year 10 are describing application of previously established formulae.** If students are unsure of the relevant formulae, refer to the Year 8 narrative for ideas about establishing this knowledge. To keep rigour in this element for Year 10 students, we challenge students to problem solve using this knowledge rather than practise routine calculation questions.
  - Present shapes in different orientations eg rectangles with sides that are not horizontal and vertical, triangles where no side is horizontal.
  - Give additional, unnecessary information, rather than just the information that is required, or require students to take measurements themselves.

  This challenges the student to think about the process that they need to use and reveals students who do not have conceptual understanding.

- **Fluency in measurement requires students to recall, choose and use formulae flexibly.** At this stage of development we would expect students to be confident in the recall of formulae for calculating the area of a rectangle, triangle, parallelogram, rhombus, trapezium, kite and circles. In relation to volume we would expect students to be fluent in working with formulae for right prisms, right pyramids, cylinders, cones and spheres.

- **Measurement is the use of number applied to a spatial context** but we do not need to develop the relevant number skills before we start. Measurement provides a context for developing and consolidating understanding of concepts within number, eg fractions, decimals, percentages, ratio and proportion. Measurement is also a great context for working with scale, enlargement, shape, angle and statistics. Connections made in this narrative can be found on page 24.

- **Measurement skills are frequently used in the context of estimating.** Although the AC: Mathematics content descriptions don't state that students should estimate measurements, the numeracy continuum does acknowledge the importance of developing a capacity to estimate measurements and make sensible judgements. The numeracy continuum states that by the end of Year 8, students solve complex problems by estimating and calculating using efficient mental, written and digital strategies and by the end of Year 10, students solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

- **We understand that estimating is reasoning, not guessing** and we can support students to know this and to notice their own reasoning. Reasoning may be based on applying a known fact or a prior experience to a new situation. For further details about methods students normally use to estimate distances, refer to page 6 'Developing an ability to estimate' in the *Using units of measurement: Years 5–7 – Mathematics Conceptual Narrative*.

  To be able to estimate at Year 10 level, students must be able to:
  - Make an estimate of their numerical calculation, so that they are confident in the value(s) that they produce.
  - Have a reasonable appreciation if their result is of the correct order of magnitude. To do this, they need to have an appreciation of actual size.

Engaging learners

Classroom techniques for teaching units of measurement

Harnessing students' fascination with scale

People are often fascinated with very large or very small items. We are particularly fascinated with large items that should be small, and small items that should be large. For an example of this fascination, follow the link below to the news story about giant marionettes in Perth, WA. An estimated 1.4 million people attended *'The Giants extravaganza'*!

http://www.perthnow.com.au/news/western-australia/giants-in-perth-day-three/story-fnhocxo3-1227220081679

Perth Now
February 2015
Picture: Stewart Allen

To use this story to engage learners we would play one of the news stories, without the audio (at first) and ask students:

What questions do you have?

There are films, such as *'The Borrowers'* and *'Gulliver's Travels'*, that play on our fascination with scale. Such films and images can be used to make connections between measurement, scale, enlargement and fractions.

Images of large amounts of money and movie scenes that involve the transaction of large amounts of cash in small bags or briefcases, provide another engaging context for working with units of measurement.

We can support our students to develop a disposition towards using maths in their lives, ie becoming numerate, not only through the use of 'real world' maths problems, but through fostering a disposition towards asking mathematical questions about everything they see. We develop this disposition in our students when we promote, value and share their curiosity and provide opportunities for them to develop their questions and explore solutions to their questions.

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–7)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the name 'Isosceles triangle'** for themselves. They need to receive this information in some way. However, it is possible for my students to be challenged with a question that will result in them identifying **a formula to calculate the volume of a cylinder**, so I don't need to instruct that information.

When we challenge our students to **establish formulae**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular formula during the current unit of work, then it probably is quicker to tell them the formula and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the formulae is a false economy of time.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Establishing the volume of a sphere Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG271 ◆
Example 2: Establishing the formulae for right pyramids and cones Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG271 ◆
Example 3: Growing squares Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.	ACMMG242 ◆
Example 4: Immersion – volume by displacement Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG271 ◆
Example 5: Packaging problems Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG271 ◆
Example 6: The concept of vertical height Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG271 ◆
Example 7: Volumes and surface areas of real objects Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids. Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG242 ◆ ACMMG271 ◆

Example 1: Establishing the volume of a sphere

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the formula for the volume of a sphere we can challenge students to adapt the known to the unknown, by ***asking*** questions.

The AC: Mathematics does not suggest we should establish the volume of a sphere. However by exploring the relationship between the radius of spheres and their volume in a practical way, students can develop rich conceptual understanding rather than simply applying a formula to solve problems.

At Year 10 we would want students to appreciate that any volume formula involves multiplying three length units (one from each of three dimensions) together.

Ask students to list all the volume formulae they know.

Then ask students:

- *What is the same/different about these formulae?* (The rectangular prism formula has three length units, but does the cylinder? $V = \pi r^2 h$ Students need to recognise that the radius is squared so there are still three dimensions and that pi is a number and not a dimension.)
- *Does that make sense to you?*
- *Which dimensions would you use for a sphere? Can you predict what might be in the formulae for the volume of a sphere? Why do you think that?*

Even though conventionally we express the volume of a sphere as $4/3\ \pi\ r^3$, it can of course be expressed as $1/6\ \pi\ d^3$, so students could choose either diameter of radius as their starting point for establishing the volume of a sphere. Even if the students are already familiar with the formula this investigation gives them the opportunity to verify it.

Ask students:

- *What connection do you see between your sphere, the diameter cube (Figure 1a) and the radius cube (Figure 1b)?* (Have real spheres available, not just diagrams.)

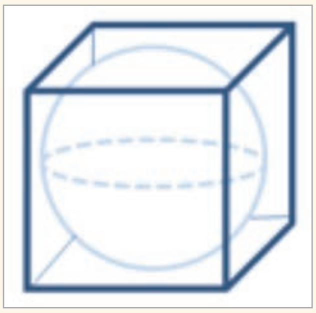


Figure 1a

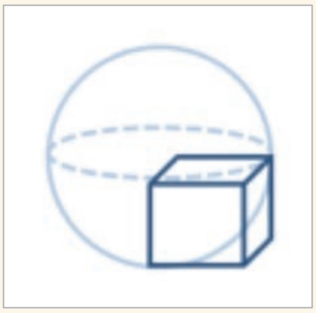


Figure 1b

- *Which would be larger, your sphere or the cube?* (When you consider the diameter cube, the sphere is smaller, when you consider the radius cube, the sphere is larger.)
- *What connection do you think you will find between the volume of the cubes and the volume of the sphere? If the diameter of the sphere is **d** metres, can you determine an expression for the volume of any of the solids accurately? For any of solids you can't, can you make any statement about their volume?* (We are looking for students to be able to make a statement such as 'the volume of the radius cube square is $1/8\ d^3$ and the diameter cube is d^3 so the volume of the sphere should be somewhere between the two.)
- *Do other groups have the same idea as you?*
- *What is the same/different about the results relating volume to diameter and the results relating volume to radius?*
- *How could you test your idea?* (Displacement.)

The experience of estimating the connection will be a prompt for students when required to recall the exact relationship described in the formula.

This is a good opportunity for students to collect data and fit a model using technology. Students can determine the radius/diameter and volume for a range of spheres, eg table tennis, tennis, baseball/cricket, softball, volleyball, netball, basketball balls.

Gather a collection of spheres, then ask students:

- *What different ways can be used to determine the radius/diameter of the spheres?*
- *Are some methods more appropriate for small/large spheres?* (Students could use calipers, place the balls between two parallel planes, determine the radius algebraically from the circumference etc. Measuring errors are more significant for smaller solids.)
- *How could the volume of the spheres be determined?*
- *Are some methods/designs more appropriate for small/large spheres?* (Ask students to design a procedure for a physical way to determine volume, such as displacement.)
- *Are the units of measurement important?* (If measuring volume in millilitres, (cm^3) then the radius must be in cm.)

Using Excel, enter the data for the radius and volume into a spreadsheet and draw a scatterplot and 'Fit a trendline' (Power function) and choose to display the equation and the R^2 value on the chart. The power should be approximately 3 (radius cubed, r^3) and the coefficient should be approximately $4/3\ \pi$. http://office.microsoft.com/en-au/training/results.aspx?qu=trendlines&ex=1&origin=RZ006107930

A graphics calculator can also be used to fit a power model to the data.

This equation can be compared to $V=4/3\ \pi r^3$, and can be used to predict the volume of a sphere that was not used when collecting data, eg a beachball. The volume can be verified by immersion but ensure the water that is used for the experiment is re-used purposefully.

'Think, Pair, Share': a process to support all students to think deeply

With a classroom brainstorm, students tend to share a range of rapid first responses. This may not allow all students to think more deeply about the problem. A practice called 'Think, Pair, Share' allows all students time to consider the problem individually as well as a safe way to discuss their thoughts.

'Think' time is when each student thinks silently about the problem.

'Pair' time is for students to discuss their ideas with one other student.

'Share' time is an opportunity for the teacher to facilitate students sharing, comparing and contrasting the ideas that they have had or they have heard.

Example 2: Establishing the formulae for right pyramids and cones

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the formula for the volume of pyramids and cones we can challenge students to adapt the known to the unknown, by ***asking*** questions.

The AC: Mathematics does not suggest we should establish the volume of right pyramids and cones. However by exploring the relationship between the dimensions of the solids and their volume in a practical way, students can develop rich conceptual understanding rather than simply applying a formula to solve problems.

The following information about displacement is an extract from page 21 'Example 5: Connecting volume and capacity' in the *Using units of measurement: Years 5–7 – Mathematics Conceptual Narrative*.

The metric system is beautifully constructed, such that there is a relationship between the different units. For example, 1 cm^3 is the same as 1 ml and for water this quantity has a mass of 1 g. At this stage of development it is only necessary for students to understand the connection between volume and capacity. For example: 1 ml = 1 cm^3.

Submerging MAB blocks in a measuring cylinder (like those shown in Figure 10) containing water and observing the effect on the water level, is a very easy way to establish this connection, without instructing students. We can ask students to submerge different quantities of MAB blocks in a measuring cylinder and ask:

What connection do you notice *between the number of centimetre cubes that you place in the measuring cylinder and the number of millilitres the water level rises by?*

Take care to use MAB cubes/lengths that have been cut to metric measurements. Cubes should be 1 cm x 1 cm x 1 cm and the 10 lengths should be 1 cm x 1 cm x 10 cm. It's worth checking your MAB blocks before using them in this way.

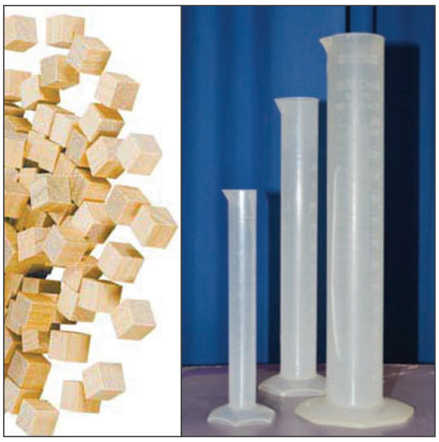


Figure 10

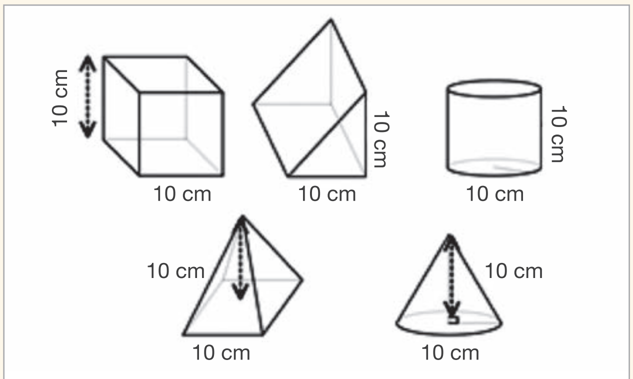


Figure 2

This understanding can be used to establish formulae for pyramids and cones.

Consider a set of solids as shown in Figure 2 that have comparable dimensions just as you would find in many boxed sets of 3D shapes. If you do not have the set, build them from nets or provide students with a set of 5 cards with a solid on each. Ask them to name them:

- *Which one might be a square-based prism? Why?*
- *How might you sort these solids?*

- *Can you predict the placement if they were ordered according to volume?*
- *If you filled each with water, how many mls would it take to fill the cube?*
- *Now that you know the formula for calculating the volume of a prism, how could you adapt that to apply to a pyramid?*
- *What's the same about the triangular prism and the triangular based pyramid?*
- *What's different about the triangular prism and the triangular pyramid?*
- *What connections do you see between the prism formula and the lengths on the prism?*
- *How could this be adapted to suit a pyramid?*
- *How could you test your idea?* (Displacement, as described in the extract from the Year 5–7 narrative, is one option.)

Maths class is time for talk

'Turn and talk' or 'think-pair-share' encourages students to voice their ideas. Giving them a minute or so to talk with a partner also makes them more likely to contribute to class discussion. Opportunities to develop language associated with measures of area, perimeter and volume must be intentionally designed. It is important for students to see, hear, speak and record this language.

('Marilyn Burns: 10 Big Maths Ideas' http://www.scholastic.com/teachers/article/marilyn-burns-10-big-math-ideas)

Example 3: Growing squares

ACMMG242 ◆

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the relationships between similar shapes/solids and their areas/volumes, we can challenge students to adapt the known to the unknown, by ***asking*** questions.

Show students Figure 3 and ask them:

- *What connections do you see between the squares in this pattern?*
- *What if this was three-dimensional? What connections would there be between the surface area of the boxes? What connections would there be between the volume of the boxes?*
- *Can you demonstrate your thinking numerically?*
- *What if the side of the smallest square was 3 units long would it still be the same?*
- *Is there a rule we could use to describe this connection?*
- *Can you prove your thinking algebraically?*

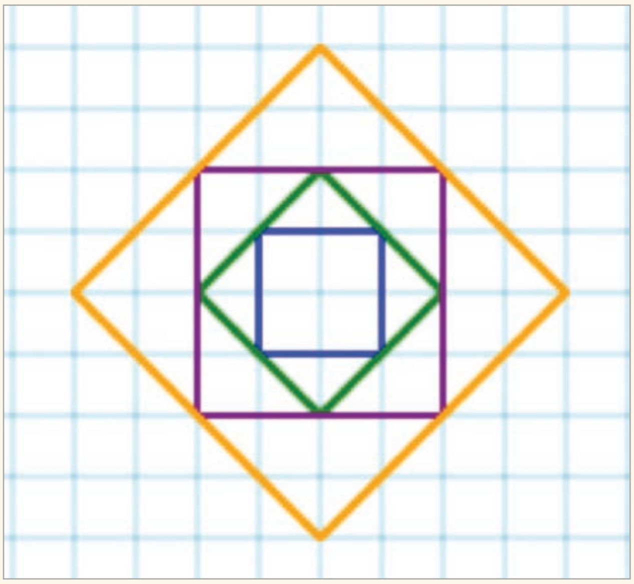


Figure 3

Example 4: Immersion – volume through displacement

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

What can you infer?

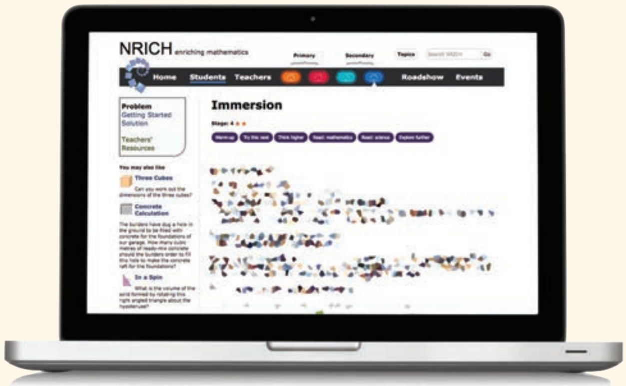
Instead of ***telling*** students about the relationship between volume and immersion we can challenge students to adapt the known to the unknown, by ***asking*** backwards questions.

This activity is from the NRICH website.

There is considerable reasoning required in a problem that is asking, '**What is the connection between** a given set of shapes and a graph representing the displacement when they are immersed in liquid?'

This problem is an example of a backwards problem because it starts with a graph and challenges the students to identify objects that could connect to the graph. For some students this backwards question would become a problem solving activity. The warm-up activity option for this task can prompt student thinking about these concepts.

The link to this problem on the NRICH site is: http://nrich.maths.org/6439



Example 5: Packaging problems

ACMMG271 ◆
Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you answer backwards questions?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students the formula for the volume of chocolate or the shape here we are ***asking*** students to decide and create in different ways.

Show students images of chocolate, with varying volumes and shapes, like the ones in Figure 4. Then ask:

- *Can you design the shape and dimensions for a chocolate moulding for one of these fixed volumes of chocolates?* (The student is beginning with the volume of the chocolate and is required to think about the dimensions of a possible sphere, right pyramid, cone or composite shape. This can be developed into a 'multiple representations' question.)
- *Is there another solution?* (As an extension or if students want a greater challenge they can compare the surface area of the different possibilities or consider creating a mould for an increased/decreased volume of chocolate.)
- *What if the mould needed to hold 12% more chocolate?*

Cadbury Chocolate are considering changing the size of their family block, ask students:

- *What questions come to mind?*

Figure 4

(A rectangular prism version of this activity is on page 21 of the Using units of measurement: Year 8 narrative and a cylinder version is on page 12–13 of the Using units of measurement: Year 9 narrative.)

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

Example 6: The concept of vertical height

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

In what ways can you prove it?

Instead of ***telling*** students how to identify the vertical height of shapes and solids, we can challenge them to construct their own knowledge, by ***asking*** questions.

Provide the students with a range of different geometrical solids such as those shown in Figure 5.

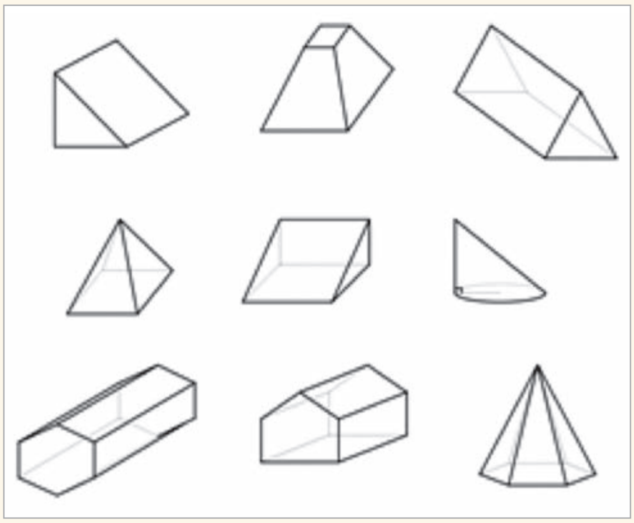


Figure 5

For each solid, ask students:

- *If you were asked the height of the solid, how might you measure it?*
- *Which one of the solids in Figure 5 is the odd one out? Why?*
- *Is there another way to measure height?*
- *For which solid(s) is the height measured along one of the edges?*
- *What if I turned it over, would it still be the same?*
- *What do you think is meant by 'vertical height'? Why?*
- *How does your thinking compare with the definition in a maths dictionary?*
- *What's wrong with this working out?*

Tackling misconceptions: Knowing what it isn't

We can challenge students' reasoning and address common misunderstandings by asking *'Why can't I …'*; *Why is it not…*; *What's wrong with…'* questions. Students can be given problems from fictional students which are answered incorrectly, without working out. They can be asked to give feedback, summarising the mistakes that the students have made and how to avoid that mistake in the future.

How can we measure vertical height when it is not a side or edge? ABCD is a rhombus as shown in Figure 6a and 6b.

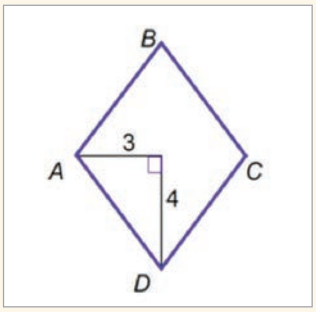


Figure 6a

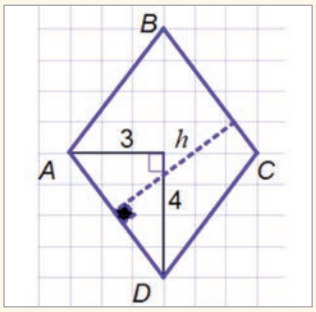


Figure 6b

- *What is the area of ABCD? Is there another way to calculate that?*
- *Was the answer the same? Does that make sense to you?*

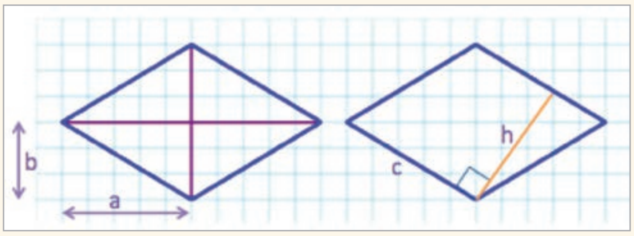


Figure 7

- *How is the rhombus in Figure 7 the same/different to the one in Figure 6a/6b?*
- *How can your thinking in finding the area in Figure 7 help you prove that?* $h = \frac{2ab}{\sqrt{a^2 + b^2}}$

Example 7: Volume and surface area of real objects

ACMMG242 ◆

Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Understanding proficiency:

Can you represent/ calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

Instead of ***telling*** students how the formulae can be used to measure the volume and surface area of real objects, we can challenge students to adapt the known to the unknown by ***asking*** questions such as the ones in this example.

This is an alternative to text book 'application of knowledge' questions.

The Year 10 AC: Mathematics content descriptions focus on composite shapes and the numeracy continuum at this stage emphasises the development of estimation skills in relation to complex problems. These skills can be developed together if we ask students to calculate the surface area or volume of real objects or images that can be approximated to a series of different 2D shapes or prisms, cylinders, pyramids and spheres. The important thing is that the student needs to make some assumptions and approximations and judgements about appropriate levels of accuracy to be able to make progress with this type of task.

For **volume**, ask students to build a collection of suitable items, eg a hammer (different prisms, depending on the type of hammer – see Figure 8); a screw driver (cylinders); or a banana an apple, a pear (a series of cylinders). Ask students to look for/bring suitable objects. This helps students to look at their world as a series of composite 3D solids, eg prisms or cylinders.

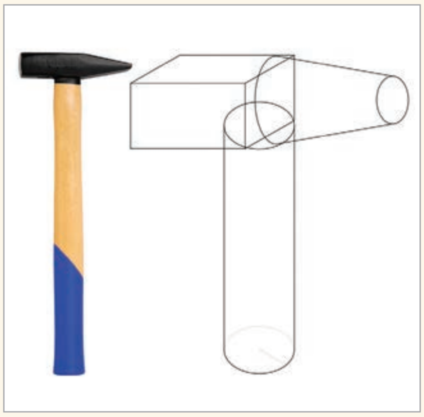


Figure 8

Figure 9

Ask students:

- *Why did you choose to use those prisms/cylinders in that way?*
- *Is there another way to model that object as a collection of prisms and pyramids etc?*
- *What is a way to check that?*

If possible, submerge the object in water to verify the volume; but getting the correct answer is less important than the process in this case.

For **area**, ask students to consider leaves from various plants and trees, pointing out they are composed of familiar shapes (like those in Figure 9). Ask students:

- *Why did you choose to use those rectangles/ trapezia/circles in that way?*
- *Is there another way to model that object as a collection of 2D figures etc?*
- *What is a way to check that?*

We can challenge students to:

- communicate reasons for their choices
- evaluate different approaches against different criteria (accuracy, efficiency, ease etc)
- look for relationships through graphing data
- compare distributions
- establish formulae.

The important common element of the questions in this section is that they challenge students to derive logical conclusions from something that they already know to be true. Even when students require support to make these connections, teachers are still modelling that mathematics is not a collection of disconnected ideas, but rather that it is a interrelated set of understandings and that we can construct new ideas from known facts.

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 8–15)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously; they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 8: Measuring tree volume Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.	ACMMG242 ◆
Example 9: Loads of money! – volumes/capacities involving money quantities Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.	ACMMG242 ◆
Example 10: Trashketball – paper balls in a cylindrical bin Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.	ACMMG242 ◆
Example 11: Pyramid of pennies Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids. Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG242 ◆ ACMMG271 ◆
Example 12: Square partitions Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.	ACMMG242 ◆
Example 13: Trapezium four Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.	ACMMG242 ◆
Example 14: From sphere to ellipsoid Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG271 ◆
Example 15: The water challenge – submerging objects to fill a container Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids. Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.	ACMMG242 ◆ ACMMG271 ◆

Example 8: Measuring tree volume

ACMMG242 ◆

Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

The measurement of tree volume is carried out for economic and scientific purposes, so this activity provides a real context for the application of volume calculations at this level.

Students could calculate the tree volume for trees on the school site, or make estimates and assumptions from photographs that they have taken. Further information about the measurement of tree volume can be found via this link: http://en.wikipedia.org/wiki/Tree_volume_measurement

This problem can be introduced by showing an image such as the one in Figure 10 and asking students:

- *What questions come to mind?*

This problem can be extended and connected to the use of timber in different contexts, eg paper production/paper usage in your school. Further information for this can be found via the following link: http://wiki.answers.com/Q/How_much_paper_does_one_tree_make#slide1

Ask students:

- *How much paper could be produced from this section of tree?*

Figure 10

Example 9: Loads of money! – volumes/ capacities involving money quantities

ACMMG242 ◆

Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

Present the following question to students:

- *How much money would fit in to a ……?*

Teachers or students can complete this sentence and decide on the parameters. Where students are creating the questions we will need to make the expected level of volume calculation clear to the students. Occasionally there are competitions that advertise the prize as 'a car full of money' or 'a …… full of money'. We could use advertisements such as these as the stimulus for the question.

Interpret

What information is helpful? What information is not useful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help to start by thinking about a smaller version of this pattern? Have you done a question like this one before? Why is this one harder? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

Questions to be used only after students have grappled with the problem for a few minutes:

Is there a rule that always works? How could you check that? Does that seem right to you? Do other people think that too?

Reflect

Ask students to pair up with someone who did it differently to discuss:

How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Example 10: Trashketball – paper balls in a cylindrical bin

ACMMG242 ◆
Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What ways can you communicate?
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is a ***Three-Act Maths Task*** from Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

This is an activity that challenges students to compare a theoretical and a practical answer to a problem and to discuss the difference.

The **Trashketball (cylinder)** activity can be accessed at: http://www.101qs.com/2008-trashketball-cylinder

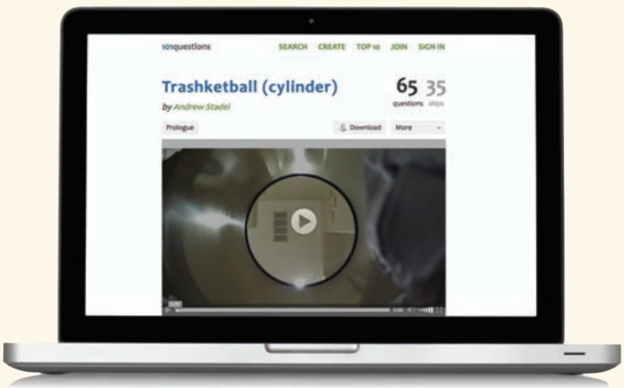


Figure 11 | Source: Dan Meyer 101 questions http://www.101qs.com

Identifying the question to solve

The group can share questions and sort them into mathematical and non-mathematical questions. Then, of the mathematical questions, students can sort their questions into those that cannot be answered with the given information and those that could be answered using the given information or additional information that could be inferred.

Dan Meyer has a technique, that we have seen many teachers adopt when generating and collecting questions from students. First, he asks students to individually write down questions that come to mind. Then, as he invites students to share their questions, he writes students' names next to the questions. He also asks if anyone else likes that question. *'Did you write it down, or if you didn't perhaps you still think that it's a good question.'* Through doing this, both Dan and his class get a sense of the questions that are of interest to the students.

Example 11: Pyramid of pennies

ACMMG242 ◆

Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is a ***Three-Act Maths Task*** from Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

This is an activity that provides an ideal opportunity for students to initially estimate and then develop a strategy to work out the number involved in a situation where it would be impractical to count.

The activity contains a short film that can be presented to students along with the questions such as:

- *What guess is too high/too low?*
- *What information will you need to work out the problem?*

The **Pyramid of pennies** activity can be accessed at: http://www.101qs.com/2011-pyramid-of-pennies

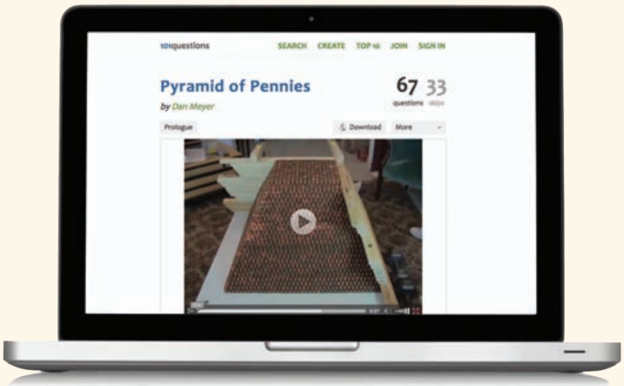


Figure 12 | Source: Dan Meyer 101 questions http://www.101qs.com

Example 12: Square partitions

ACMMG242 ◆
Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is a ***Three-Act Maths Task*** from Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

In this activity the question is suggested to the students, but they could extend the question by asking their own 'What if…?' question in relation to changing the original equally divided square. This problem is about area rather than surface area, but also the use of similarity in relation to solving problems involving 2D shapes.

The **Square partitions** activity can be accessed at: http://www.101qs.com/72-square-partitions

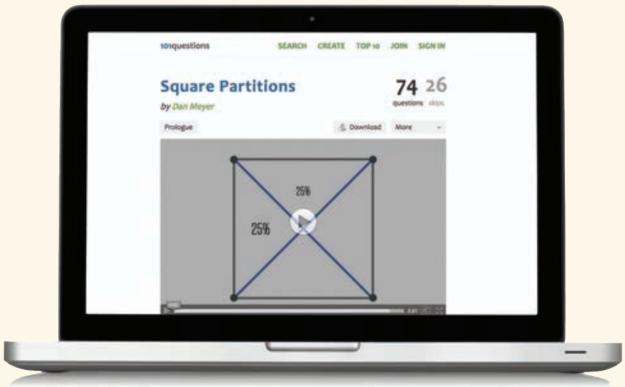


Figure 13 | Source: Dan Meyer 101 questions http://www.101qs.com

Example 13: Trapezium four

ACMMG242 ◆
Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

In this activity from the NRICH website, students are asked to consider whether the challenges posed about the 4 triangular areas created by the diagonals of a trapezium, are possible or not.

The link to this problem on the NRICH site is: http://nrich.maths.org/4960

Building resilience and positive learner identity: Using high challenge problems

We know that:

'Positive learner identity – is more readily built through succeeding at challenging tasks than experiencing 'dumbed down' activities.'

(David Price, Learning Futures, UK)

Example 14: From sphere to ellipsoid

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

Show students Figure 14 and ask them:

- *Given that* $4/3\ \pi r^3$ *is the formula for the volume of a sphere, how might the volume of an ellipsoid be calculated?* (The formula is somewhat intuitive in that the three dimensions of the solid are required for volume and that the sphere is a special ellipsoid.)
- *What everyday solids might be considered an ellipsoid?* (Solids such as eggs, melons and footballs all approximate an ellipsoid to varying degrees.)
- *How could these shapes be used to check the formula you proposed for the ellipsoid?* (Students could use the techniques used in **Example 1: Establishing the volume of a sphere** to verify a formula.)

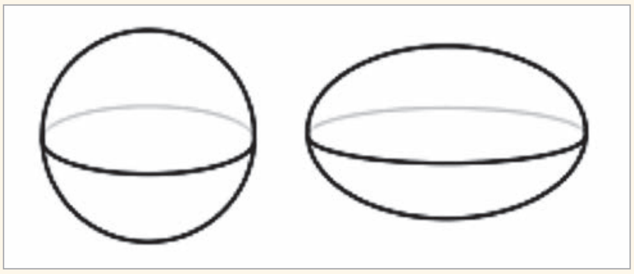


Figure 14

Example 15: The water challenge – submerging objects to fill a container

ACMMG242 ◆

Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.

ACMMG271 ◆

Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

Create challenges that involve taking the water to the top of an already partially filled container. This could be about submerging a number of items or partially submerging one large item in a particular orientation.

The challenge element comes through students needing to commit to the amount of items that they want to submerge, then testing their ideas. With larger objects, students can be challenged to identify how far they want to submerge the object and what orientation they want to use when submerging the object. The challenge could be extended to involve two different items that are submerged simultaneously.

The different combinations could be represented graphically. Using graphs like those drawn in **Example 4: Immersion – volume through displacement** for both items individually, students can plan the immersion levels by combining the rise in water levels. The data must be obtained from immersing the items in the same container as they will be tested in.

Bringing in two objects facilitates the students being challenged or challenging themselves to identify multiple solutions, asking:

- *Is there another way?*

Connections between ‘using units of measurement’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use units of measurement as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 10/10A	
Whilst working with units of measurement, connections can be made to:	**How the connection might be made:**
Investigate Pythagoras’ Theorem and its application to solving simple problems involving right-angled triangles (Year 9). ACMMG222	Refer to: **Example 3: Growing squares (Pythagoras’ Theorem is applied to compare sides).**
Define rational and irrational numbers and perform operations with surds and fractional indices (Year 10A). ACMNA264	Refer to: **Example 3: Growing squares (side ratios are irrational).**
Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes. ACMNA244	Refer to: **Example 12: Square partitions.**
Solve right-angled triangle problems including those involving direction and angles of elevation and depression. ACMMG245	Refer to: **Example 3: Growing squares (solving right-angled triangle problems).**
Use scatter plots to investigate and comment on relationships between two numerical variables. ACMSP251	Refer to: **Example 1: Establishing the volume of a sphere.** **Also consider the following:** **Plotting the relationship between the volume of an apple and its mass. Is this a linear relationship? Why would you expect/not expect that? Would it be different for different varieties of the same fruit/different fruits? Or the volume of fruit that is still growing on a tree, with data collected over time, or the volume of fruit growing on a tree in relation to the quantity of water that it is given or the number of hours of daylight that it is exposed to.**
Pythagoras’ theorem and trigonometry to solving three-dimensional problems in right-angled triangles (Year 10A). ACMMG276	Refer to: **Example 3: Growing squares.** **Example 5: Packaging problems.**
Use information technologies to investigate bivariate numerical data sets. ACMSP279	Refer to: **Example 1: Establishing the volume of a sphere.**

Making connections to other learning areas

There is great potential to make connections to science. One connection that can be used to remind students of the importance of recording units of measure, is this story about NASA losing a spacecraft because they had a miscommunication about units:

‘Mars Climate Orbiter team finds likely cause of loss’: http://mars.jpl.nasa.gov/msp98/news/mco990930.html

Another science connection can be made by comparing the distribution of the volume of fruit that is still growing on trees under different circumstances, eg trees in shady areas compared to full sunlight or young trees and older trees.

‘Using units of measurement’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with measurement:

Using informal units for direct or indirect comparisons ◆
From **Foundation to Year 2** students focus on **informal units of measurement.**

Using standard metric units ◆
From **Year 3 to Year 8** students develop their **understanding of metric units of measure.** This begins with the use of familiar metric units and extends to include a greater range of metric units and the flexibility to convert between different units.

Establishing and applying formulae ◆
From **Year 5 to Year 10** students **establish and use formulae** of increasing complexity relating to perimeter, area and volume.

Estimating ◆
Australian Curriculum references to **estimation** in relation to measurement lie entirely in the **Numeracy Continuum.**

Year level	‘Using units of measurement’ content descriptions from the AC: Mathematics
Foundation ◆	Students use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.
Year 1 ◆	Students measure and compare the lengths and capacities of pairs of objects using uniform informal units.
Year 2 ◆	Students compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units.
Year 2 ◆	Students compare masses of objects using balance scales.
Year 3 ◆	Students measure, order and compare objects using familiar metric units of length, mass and capacity.
Year 3 ◆	Students use scaled instruments to measure and compare lengths, masses, capacities and temperatures.
Year 4 ◆	Students compare objects using familiar metric units of area and volume.
Year 5 ◆	Students choose appropriate units of measurement for length, area, volume, capacity and mass.
Year 5 ◆	Students calculate the perimeter and area of rectangles using familiar metric units.
Year 6 ◆	Students connect decimal representations to the metric system.
Year 6 ◆	Students convert between common metric units of length, mass and capacity.
Year 6 ◆	Students solve problems involving the comparison of lengths and areas using appropriate units.
Year 6 ◆	Students connect volume and capacity and their units of measurement.
Year 7 ◆	Students establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
Year 7 ◆	Students calculate volumes of rectangular prisms.
Year 8 ◆	Students choose appropriate units of measurement for area and volume and convert from one unit to another.
Year 8 ◆	Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.
Year 8 ◆	Students investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Year level	'Using units of measurement' content descriptions from the AC: Mathematics *continued*
Year 8 ◆	Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
Year 9 ◆	Students calculate the areas of composite shapes.
Year 9 ◆	Students calculate the surface area and volume of cylinders and solve related problems.
Year 9 ◆	Students solve problems involving the surface area and volume of right prisms.
Year 10 ◆	Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.
Year 10A ◆	Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Numeracy continuum: Using measurement	
End Year 2 ◆	Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects.
End Year 4 ◆	Estimate and measure with metric units: estimate, measure and compare the length, temperature, volume, capacity and mass of everyday objects using metric units and scaled instruments.
End Year 6 ◆	Estimate and measure with metric units: choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems.
End Year 8 ◆	Estimate and measure with metric units: convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems.
End Year 10 ◆	Estimate and measure with metric units: solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer’s blog: 101 questions

http://www.101qs.com

Dan’s blog contains images and short films that can be presented to students along with the question: *What’s the first question that comes to mind?*

A spreadsheet of ***Dan Meyer’s Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by Office for Education and Early Childhood

Excluded from NEALS