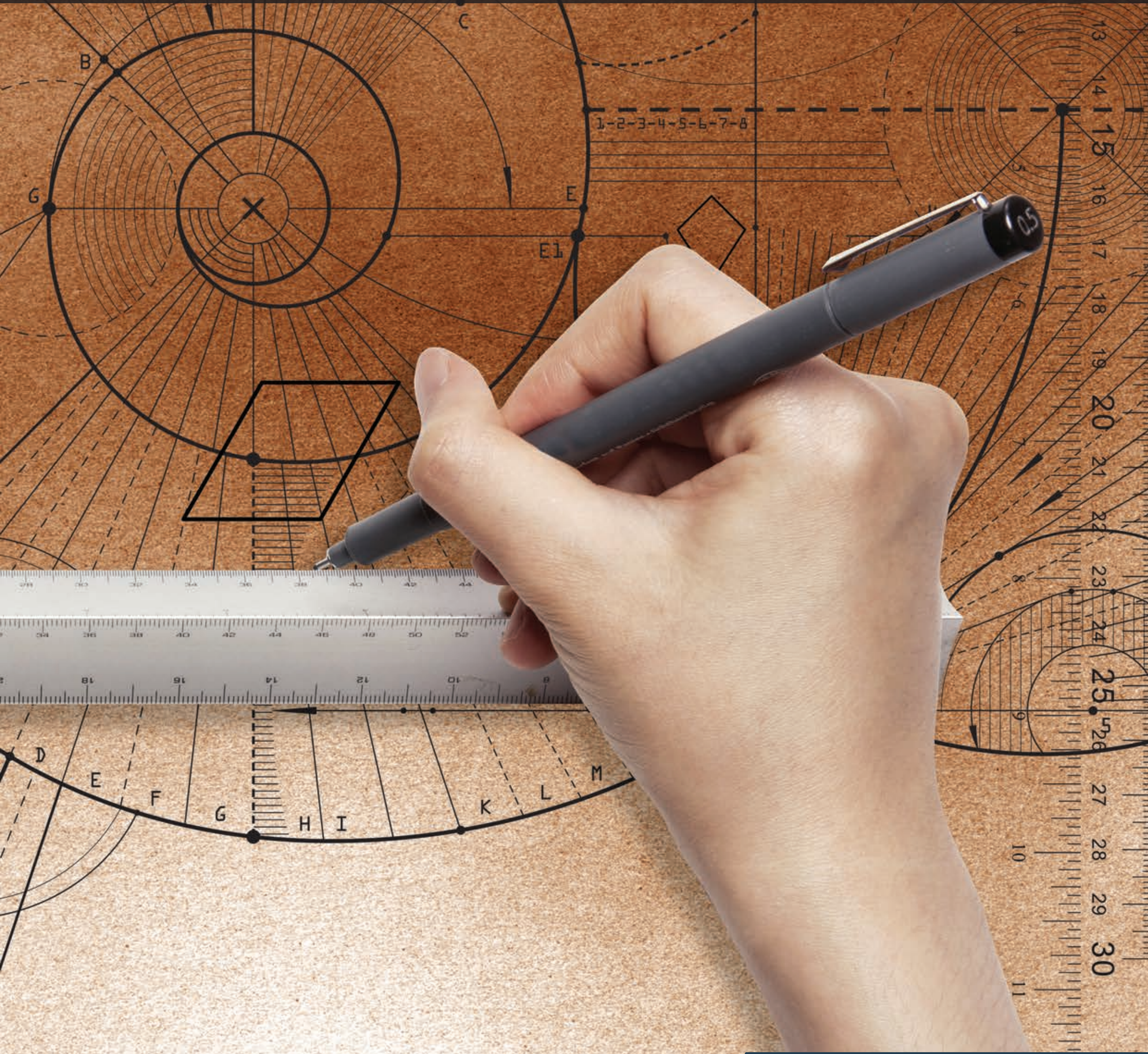
Using units of measurement: Year 9

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

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Throughout this narrative—and summarised in **'Using units of measurement' from Foundation to Year 10A** (see page 25)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with units of measurement:

- ◆ Using informal units for direct or indirect comparisons
- ◆ Using standard metric units
- ◆ Establishing and applying formulae
- ◆ Estimating.

What the Australian Curriculum says about ‘using units of measurement’

Content descriptions

Strand | Measurement and geometry.

Sub-strand | Using units of measurement.

Year 9 ◆ | ACMMG216
Students calculate the areas of composite shapes.

Year 9 ◆ | ACMMG217
Students calculate the surface area and volume of cylinders and solve related problems.

Year 9 ◆ | ACMMG218
Students solve problems involving the surface area and volume of right prisms.

Year level descriptions

Year 9 ◆ | Students calculate areas of shapes and surface areas of prisms.

Year 9 ◆ | Students formulate and model practical situations involving surface areas and volumes of right prisms.

Year 9 ◆ | Students follow mathematical arguments.

Achievement standards

Year 9 ◆ | Students calculate areas of shapes and the volume and surface area of right prisms and cylinders.

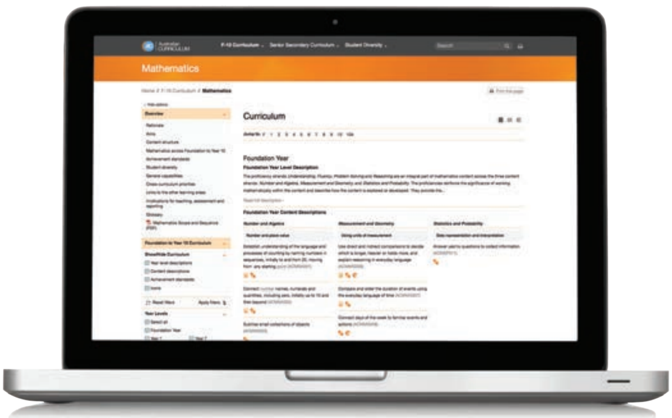
Numeracy continuum

Using spatial reasoning | **Using measurement**

End of Year 10 ◆ | Students visualise, describe and analyse the way shapes and objects are combined and positioned in the environment for different purposes (Using spatial reasoning: Visualise 2D shapes and 3D objects).

End of Year 10 ◆ | Students solve complex problems involving surface area and volume of prisms and cylinders and composite solids (Using measurement: Estimate and measure with metric units).

Note: In the Australian Curriculum: Mathematics, the concept of ‘time’ is addressed in the sub-strand ‘Using units of measurement’, but in this resource, ‘time’ has its own narrative.



Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Working with units of measurement

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 9 'using units of measurement'

In Year 8 area and perimeter calculations include trapezium and kites. Students *investigate* the area and circumference of circles and *they develop formulas* for calculating the volume of prisms. At this stage students convert between units of area and volume, such as changing from millimetres squared to metres squared.

In Year 9 the focus is on calculating areas and perimeters of composite shapes and surface areas of prisms. Students establish the formula for the volume of a cylinder and solve problems involving the volume of cylinders and prisms.

In Year 10 of the AC: Mathematics – calculations of surface area and volume of cylinders and prisms is extended to include problems involving composite solids. In **10A** problem solving involving surface area of pyramids, cones, spheres and composites of these solids is introduced.

- **Notice that, with the exception of the formula for calculating the volume of a cylinder, the content descriptions in this element of Year 9 are describing application of previously established formulae.** If students are unsure of the relevant formulae, refer to the Year 8 narrative for ideas about establishing this knowledge. To keep rigour in this element for Year 9 students, we challenge students to problem solve using this knowledge rather than practise routine calculation questions.
  - Present shapes in different orientations eg rectangles with sides that are not horizontal and vertical, and triangles where no side is horizontal.
  - Give additional, unnecessary information, rather than just the information that is required; or require students to take measurements themselves.

  This challenges the students to think about the process that they need to use and reveals students who do not have conceptual understanding.

- **Measurement is the use of number applied to a spatial context** but we do not need to develop the relevant number skills before we start. Measurement provides a context for developing and consolidating understanding of concepts within number, eg fractions, decimals, percentages, ratio and proportion. Measurement is also a great context for working with scale, enlargement, shape, angle and statistics. Connections made in this narrative can be found on page 24.

- **Measurement skills are frequently used in the context of estimating.** Although the AC: Mathematics content descriptions don't state that students should estimate measurements, the numeracy continuum does acknowledge the importance of developing a capacity to estimate measurements and make sensible judgements. The numeracy continuum states that by the end of Year 8, students solve complex problems by estimating and calculating using efficient mental, written and digital strategies and by the end of Year 10, students solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

- **We understand that estimating is reasoning, not guessing** and we can support students to know this and to notice their own reasoning. Reasoning may be based on applying a known fact or a prior experience to a new situation. For further details about methods students normally use to estimate distances, refer to page 6 'Developing an ability to estimate' in the *Using units of measurement: Years 5–7 – Mathematics Conceptual Narrative*.

  To be able to estimate at Year 9 level, students must be able to:
  - Make an estimate of their numerical calculation, so that they are confident in the value(s) that they produce.
  - Have a reasonable appreciation if their result is of the correct order of magnitude. To do this, they need to have an appreciation of actual size.

Engaging learners

Classroom techniques for teaching units of measurement

Harnessing students' fascination with scale

People are often fascinated with very large or very small items. We are particularly fascinated with large items that should be small, and small items that should be large. For an example of this fascination, follow the link below to the news story about giant marionettes in Perth, WA. An estimated 1.4 million people attended *'The Giants extravaganza'*!

http://www.perthnow.com.au/news/western-australia/giants-in-perth-day-three/story-fnhocxo3-1227220081679

Perth Now
February 2015
Picture: Stewart Allen

To use this story to engage learners we would play one of the news stories, without the audio (at first) and ask students:

What questions do you have?

There are films, such as *'The Borrowers'* and *'Gulliver's Travels'*, that play on our fascination with scale. Such films and images can be used to make connections between measurement, scale, enlargement and fractions.

Images of large amounts of money and movie scenes that involve the transaction of large amounts of cash in small bags or briefcases, provide another engaging context for working with units of measurement.

We can support our students to develop a disposition towards using maths in their lives, ie becoming numerate, not only through the use of 'real world' maths problems, but through fostering a disposition towards asking mathematical questions about everything they see. We develop this disposition in our students when we promote, value and share their curiosity and provide opportunities for them to develop their questions and explore solutions to their questions.

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–6)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the name 'Isosceles triangle'** for themselves. They need to receive this information in some way. However, it is possible for my students to be challenged with a question that will result in them identifying **a formula to calculate the volume of a cylinder**, so I don't need to instruct that information.

When we challenge our students to **establish formulae**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular formula during the current unit of work, then it probably is quicker to tell them the formula and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the formulae is a false economy of time.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Establishing the volume of a cylinder Students solve problems involving the surface area and volume of right prisms.	ACMMG218 ◆
Example 2: Enlargement and squared and cubed attributes Students solve problems involving the surface area and volume of right prisms.	ACMMG218 ◆
Example 3: Filling containers Students solve complex problems involving surface area and volume of prisms and cylinders and composite solids.	NC: End Year 10 (Estimate and calculate) ◆
Example 4: Packaging problems – redesigning a cylinder Students calculate the surface area and volume of cylinders and solve related problems.	ACMMG217 ◆
Example 5: Volume and areas of real life objects Students solve problems involving the surface area and volume of right prisms. Students calculate the areas of composite shapes.	ACMMG218 ◆ ACMMG216 ◆
Example 6: Creating prisms of equal volume Students solve problems involving the surface area and volume of right prisms.	ACMMG218 ◆

Example 1: Establishing the volume of a cylinder

ACMMG218 ◆

Students solve problems involving the surface area and volume of right prisms.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the formula for the volume of a cylinder we can challenge students to adapt the known to the unknown, by ***asking*** questions.

Select at least two graduated measuring cylinders. It is best to have groups using cylinders of different dimensions as it means there is more data to consider when students generalise their findings.

Instead of ***telling*** students the formula for the volume of a cylinder we can challenge students to ***adapt the known to the unknown***, by ***asking*** questions:

- *Can you predict the height of the water in the cylinders when you pour in 1 ml, 5 mls, 10 mls?*
- *How can you check?*
- *What do you notice? Are the heights the same or different for the same volume? Why?*

Fill the measuring cylinders to each graduation of volume and measure the height, eg 10 ml as shown in Figure 1.

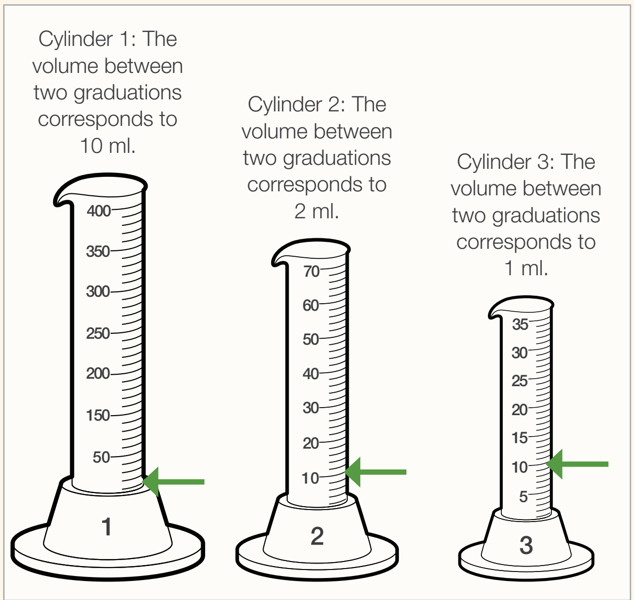


Figure 1

Then ask students:

- *What connections can you see?*
- *How are these cylinders the same/different to each other?*
- *Do other groups with different cylinders notice the same connections/patterns?*
- *Which measuring cylinder has the most volume and which has the least when the height is 1 cm? What can you infer?*
- *What other information would be useful?* (The size of the base affects the height.)

To explore this further ask students to find the area of the base of one of the cylinders. Find the volume of water needed for a depth of 1 cm, 2 cm, 3 cm, etc.

At this point, students may have noticed that the volume of water when the height is 1 cm is approximately equal to the area of the circular base of the measuring cylinder they are using. If not, we can encourage them to put the data in tables to identify patterns. As the area of the circular base affects the volume, it may help to include it in the table.

Ask students:

- *How might you represent that? Is there another way?*
- *What relationships can you see between the volume, height and area of the base?*
- *Do other groups with different cylinders notice the same connections/patterns?*
- *What can you infer?*
- *How is this the same/different to the formula we have for volume of prisms (eg rectangular prism, triangular prism, etc)?* (V = area of cross-section x H.)

Once students have identified that the Volume = area of the circular base x height, they can use this formula to check the volume of other cylindrical measures and everyday containers like cans and drink bottles. This is an opportunity for students to model practical situations and also to relate volume (in cm^3) to capacity (in ml).

Example 2: Enlargement and squared and cubed attributes

ACMMG218 ◆
Students solve problems involving the surface area and volume of right prisms.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students the effect of enlargements on measurements, we can challenge students to discover it for themselves by ***asking*** questions.

Instead of telling students the effect of enlargements on measurements, we can challenge students to discover it for themselves by asking questions.

For each of the pairs of images in Figure 2a and 2b, ask students:

- *What's the same about the shapes/objects?*
- *What's different about the shapes/objects?*
- *What connections can you see between their measurements?*

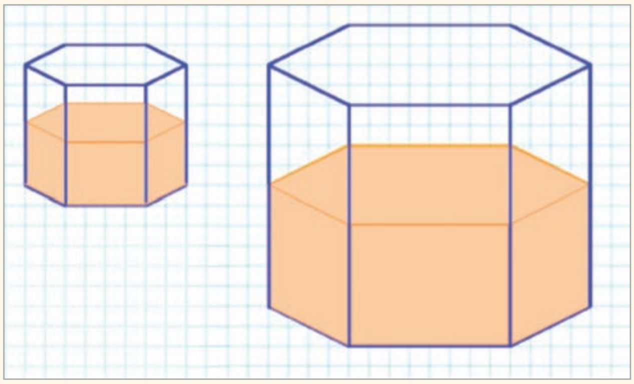


Figure 2a

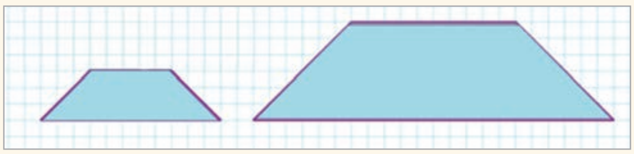


Figure 2b

In this example, each of the initial shapes have been enlarged with a scale factor of 2. A scale factor of 2 will mean that the surface area is 4 times the original (that's 2^2) and the volume is 8 times the original (that's 2^3).

Noticing connections between one shape or object and an enlarged version of it, opens a conversation where teachers can move students on from an understanding of congruence (identical in shape and size) to similarity (where the only difference between the shapes is scaling).

We can challenge students to generalise their findings, by asking:

- *What if you used a different scale factor, will the relationships be the same?*
- *Can you describe the relationship between scale factor and perimeter in words?*

For the trapezia in Figure 2b, a student described the relationship using symbols $P_{large} = 2 \times P_{small}$.

Ask students:

- *Can you explain what was meant? Why would a student write it this way? Does it make sense?*
- *How would this student record the relationship between the scale factor of 2 and the area?*
- *How would this student record the relationship between the scale factor of 2 and the volume?*

If we want to generalise further, and describe the relationship when the scale factor is *any* number, we could not use the specific scale factor of 2. Ask students:

- *Can you describe the relationship between scale factor (k) and perimeter in words?*
- *Can you write a rule (algebraically) to express the relationship between scale factor and perimeter?*
- *Can you write a rule (algebraically) to express the relationship between scale factor and area?*
- *Can you write a rule (algebraically) to express the relationship between scale factor and volume?*

Construction of knowledge: Building a bridge between the concrete and the abstract

When students have applied their conceptual learning to solve problems:

- ask them to explain their thinking verbally, to consolidate their understanding
- ask them to write it in words (possibly for someone who has not been involved in the learning) to organise their thoughts.
- ask them to record their reasoning using mathematical language and symbols to connect this abstract representation to their concrete experiences.

This is an example of how to scaffold the connection between the students' reasoning about a relationship they have observed and an algebraic equation. Interpreting the recording of another student, provides a connection to a more mathematical way of recording the relationship. While this is an important connection to be made, it must not be considered more important than conceptual understanding and being able to explain the concept in a way that makes sense to them. Encourage and reward the use of mathematical symbols but require all students to justify their thinking even if they need to write this in a worded response. This is time consuming and, in most cases, is sufficient motivation for them to adopt at least some of the mathematical language which simplifies their recording.

Example 3: Filling containers

NC: End Year 10 ◆

Students solve complex problems involving surface area and volume of prisms and cylinders and composite solids (Estimate and calculate).

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you prove it?

What can you infer?

Instead of ***telling*** students about the differing relationship between height and volume, we can challenge students to identify this for themselves, by ***asking*** questions.

Select a range of containers, like those in Figure 3, that can be filled at a constant rate (x-axis: time) or by a volume (x-axis: volume) in fixed increments. Make sure that when filled, some will have linear graphs and others have non-linear graphs when plotting the height of the water level on the y-axis (see Figure 4).

Figure 3

Figure 4

Instead of ***telling*** students about the differing relationship between height and volume, we can challenge students to **identify this for themselves**, by ***asking*** questions (before giving students the opportunity to experiment with filling the different vessels):

- *Can you predict how high the water levels might be if you added the same amount to each container?*
- *When might the height increase most quickly? Why?*
- *When might the height increase more slowly? Why?*
- *Which containers might have similar graphs? Why?*
- *Can you predict what the graph might look like for each container?*
- *How could you test that?*

Prediction supports the development of conceptual understanding

When asked to make predictions, students are challenged to think more deeply about the process they are about to observe. Asking what the height 'might be' rather than what it 'will be' invites thinking rather than 'seeking the correct answer'. Students are often motivated to experiment and take notice of the results to see how well the results match their predictions, but we can support a growth mindset through valuing their thinking rather than the accuracy of their predictions.

Example 4: Packaging problems – redesigning a cylinder

ACMMG217 ◆
Students calculate the surface area and volume of cylinders and solve related problems.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students what the base of the new can needs to be, we can ***ask*** them to decide.

Ask students to redesign this soft drink can in Figure 5 to be a different cylinder, containing the same amount of soft drink.

The important point here is that the student is beginning with the volume of the container and is required to think about the dimensions of a possible cylinder. This can be developed into a 'multiple representations' activity.

Figure 5

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

Instead of ***telling*** students what the base of the new can needs to be, we can ***ask*** them to decide.

Ask students:

- *What is the connection between the dimensions of the can and its volume?*
- *Can you predict what might happen if you increase/decrease the diameter of the can?*
- *How could you check that?*
- *Now that you know that the volume will increase/decrease when you change the diameter, can you work out what to do to keep the volume the same?*

Trial and error can be time consuming, ask students:

- *What is another way to check that?*
- *How could you use technology to do that?*

An Excel spreadsheet can be used to facilitate trial and error searches. Screenshots of a spreadsheet used by a Year 10 student to solve the problem, are shown in Figure 6.

	A	B
1	**Volume of a can**	
2	**Radius (cm)**	Fixed radius
3	**Height (cm)**	
4	Initial height	=PI()*B2^2*A4
5		Fill Down
6		

	A	B
1	**Volume of a can**	
2	**Radius (cm)**	3
3	**Height (cm)**	
4	10	282.7
5	11	311.0
6	12	339.3
7	13	367.6
8	12.5	353.4

Figure 6

Ask students:

- *Can you explain to me how this works?*
- *How are these two spreadsheets the same/different?*
- *What is the connection between the equation they have in B4 and the formula we use?*

Using formulae with an absolute cell reference (B2) and the Fill Down function, students can fix a radius (3 cm) and an initial height (10 cm) and search for a height which produces a volume of 350ml.

http://office.microsoft.com/en-au/training/get-to-know-excel-enter-formulas-RZ006107930.aspx?section=13

Ask students:

- *Now that you know these answers, can you work out a can with the same volume?*
- *What is another way to re-design the can?*

It can be extended to require **reasoning** (about surface area and the amount of material to make the can, about the relationship between the circumference of the can and the comfortable grip of a hand, the height of the can and how easy it is to tip over). Ask students:

- *Why might the actual dimensions have been chosen? Are there benefits to changing the dimensions to your design?* (The width of the can is usually designed based on the comfortable grip of the user. Compare the width of a child's juice bottle to a standard soft drink can. Other considerations might be strength of the container, its ease for stacking and storing, practicality of the pouring spout, etc.)

This type of question could easily incorporate percentage increases and decreases, by asking students to create a container for the soft drink company, if they decide to increase/decrease the amount of soft drink by a particular percentage.

(A rectangular prism version of this activity is on page 21 of the *Using units of measurement: Year 8* narrative.)

Example 5: Volumes and areas of real life objects

ACMMG218 ◆

Students solve problems involving the surface area and volume of right prisms.

ACMMG216 ◆

Students calculate the areas of composite shapes.

Questions from the BitL tool

Understanding proficiency:

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

Instead of ***telling*** students how a certain shape can be approximated by prisms and/or cylinders, we can ***ask*** them to determine ways of doing this themselves.

The Year 9 AC: Mathematics content descriptions focus on composite shapes and the numeracy continuum at this stage emphasises the development of estimation skills in relation to complex problems. These skills can be developed together if we ask students to calculate the surface area or volume of real objects or images that can be approximated to a series of different 2D shapes or prisms and cylinders.

For **volume**, ask students to build a collection of suitable items, eg a hammer (different prisms, depending on the type of hammer – see Figure 7); a screw driver (cylinders); or a banana, an apple, a pear (a series of cylinders). Ask students to look for/bring suitable objects. This helps students to look at their world as a series of composite 3D solids, eg prisms or cylinders.

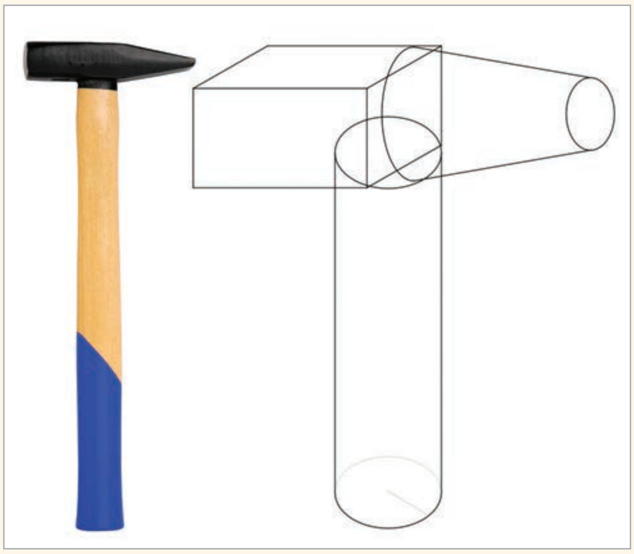


Figure 7

Ask students:

- *Why did you choose to use those prisms/cylinders in that way?*
- *What is another way to work that out?*
- *What is a way to check that?*

If possible, submerge the object in water to verify the volume; but getting the correct answer is less important than the process in this case.

For **area**, consider leaves from various plants and trees, cartoon images etc as being composed of familiar shapes. Ask students:

- *Why did you choose to use those rectangles/ trapezia/circles in that way?*
- *What is another way to work that out?*
- *What is a way to check that?*

Example 6: Creating prisms with equal volume

ACMMG218 ◆

Students solve problems involving the surface area and volume of right prisms.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you prove it?

What can you infer?

Instead of ***telling*** students how a certain shape can be approximated by prisms and/or cylinders, we can ***ask*** them to determine ways of doing this for themselves.

Instead of ***telling*** students how a certain shape can be approximated by prisms and/or cylinders, we can ***ask*** them to determine ways of doing this for themselves by setting them a challenge:

Your challenge (in a group of three) is to make three different prisms, all of which have a volume of 80 cm^3. The prisms must be the same height as each other, but the shape of the cross section must be different for each one.

Ask students:

- *What different types of prisms do you know?*
- *What is the same/different about these prisms?*
- *If the area of the base was 20 cm^3 what would the height be?*
- *If your base was a rectangle, a square, a triangle, what would the dimensions be to have an area of 20 cm^3?*
- *Is there another way?*

Once students have made their prisms, ask:

- *Whose prism might have the largest surface area? What makes you think that? Prove it!*

Value diverse thinking: Utilise peer tuition

This type of group task lends itself to mixed ability grouping. For some Year 9 students this will become a problem solving task, for others, who are more confident in working with the relevant formulae, this question would feel like a slightly challenging application question. This is a perfect opportunity for peer tutoring to be used.

As an extension, or if students want a greater challenge, change the problem by giving different rules about the height, for example:

- *Prism 'A' must be the tallest, Prism 'B' half the height of Prism 'A' and Prism 'C' half the height of 'B'.*
- *The prisms must increase in size in the order 'A', 'B' then 'C'. The difference in height between 'A' and 'B', must be the same as the difference between 'B' and 'C'.*

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 7–13)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously; they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 7: Measuring a cube Students solve problems involving the surface area and volume of right prisms.	ACMMG218 ◆
Example 8: Volume of a trapezoidal prism Students solve problems involving the surface area and volume of right prisms.	ACMMG218 ◆
Example 9: Volume of a cylinder Students calculate the surface area and volume of cylinders.	ACMMG217 ◆
Example 10: Can-struction – volume of a composite solid Students calculate the surface area and volume of cylinders.	ACMMG217 ◆
Example 11: Area of a circle – with a curvy twist Students calculate the areas of composite shapes.	ACMMG216 ◆
Example 12: Gutter problem Students calculate the areas of composite shapes.	ACMMG216 ◆
Example 13: Hexagonal prism Students solve problems involving the surface area and volume of right prisms.	ACMMG218 ◆

Example 7: Measuring a cube

ACMMG218 ◆

Students solve problems involving the surface area and volume of right prisms.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

What ways can your thinking be generalised?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

Present the following problem to students:

The distance between the centre of two adjacent sides of a cube is 18cm. What is the surface area and volume of the cube?

Interpret

What information is helpful? What information is not useful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help to start by thinking about a smaller version of this pattern? Have you done a question like this one before? Why is this one harder? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

Questions to be used only after students have grappled with the problem for a few minutes:

Is there a rule that always works? How could you check that? Does that seem right to you? Do other people think that too?

Reflect

Ask students to pair up with someone who did it differently to discuss:

How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Example 8: Volume of a trapezoidal prism

ACMMG218 ◆
Students solve problems involving the surface area and volume of right prisms.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is a ***Three-Act Maths Task*** from Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

The **Coffee traveler** videos can be accessed at: http://www.101qs.com/880-coffee-traveler

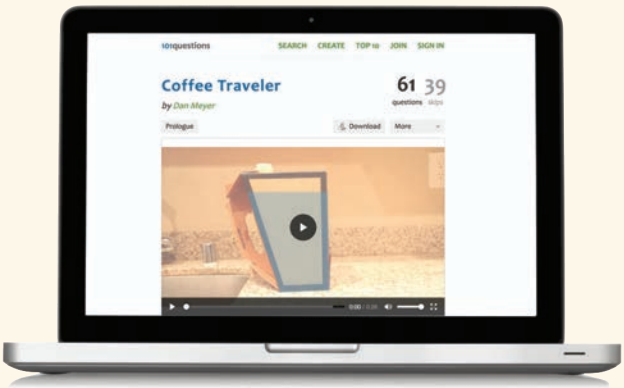


Figure 8 | Source: Dan Meyer 101 questions http://www.101qs.com

Join Dan Meyer's blog to download some resources and view what questions other teachers have asked their students about this problem.

Identifying the question to solve

The group can share questions and sort them into mathematical and non-mathematical questions. Then, of the mathematical questions, students can sort their questions into those that cannot be answered with the given information and those that could be answered using the given information or additional information that could be inferred.

Dan Meyer has a technique, that we have seen many teachers adopt when generating and collecting questions from students. First, he asks students to individually write down questions that come to mind. Then, as he invites students to share their questions, he writes students' names next to the questions. He also asks if anyone else likes that question. *'Did you write it down, or if you didn't perhaps you still think that it's a good question.'* Through doing this, both Dan and his class get a sense of the questions that are of interest to the students.

Interpret

What question have you selected? What might you need to work out to answer that question? What information is helpful? What information is not useful? What extra information do you need? What information can you reasonably infer? (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? Would it help if you thought about a similar problem for a rectangular prism first? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

What's a level on the container that's too high? What's a level on the container that's too low? What's a bit closer to the level on the container that you think you'll find?

Questions to be used only after students have grappled with the problem for a few minutes:

Could you split the problem in to smaller parts or split the trapezium into shapes that you are more comfortable working with? What formulas do you know that may help? Does that seem right to you? Do other people think that too?

Reflect

Ask students to pair up with someone who did it differently to discuss:

How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same, or a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Keeping control of the question

If students' questions sit outside of the area that we want them to work in, we have some choices to make. We can always add our own question to the list and ask for that question to be solved. But we probably want to minimise that, as students may lose interest in generating possible questions if they know that we'll always replace their questions with our own.

Students may have generated variations on the question that we had intended and if we can see that they will still use the concepts that we had intended, we could either let them answer their own question or ask them to answer our question and then reflect on whether or not their question was also answered in the process.

Example 9: Volume of a cylinder

ACMMG217 ◆

Students calculate the surface area and volume of cylinders and solve related problems.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is a ***Three-Act Maths Task*** from Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

The **Popcorn picker** short film can be accessed at: http://www.101qs.com/868-popcorn-picker

Students are asked to consider:

Which container holds more popcorn?

Figure 9 | Source: Dan Meyer 101 questions http://www.101qs.com

This activity invites students to give an intuitive answer to a relationship between surface area and volume and then investigate the situation more fully.

Join Dan Meyer's blog to download some resources and view what questions other teachers have asked their students about this problem.

Example 10: Can-struction – volume of a composite solid

ACMMG217 ◆

Students calculate the surface area and volume of cylinders and solve related problems.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is a ***Three-Act Maths Task*** from Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

This is a practical problem that challenges students to estimate and approximate through considering the construction as a whole or in terms of its parts.

The **Can-struction** activity can be accessed at: http://www.101qs.com/1189-canstruction

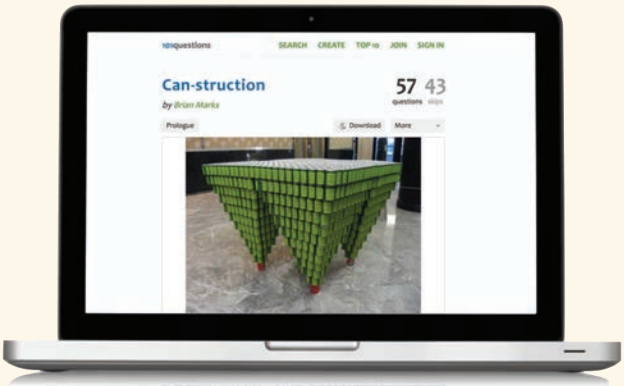


Figure 10 | Source: Dan Meyer 101 questions http://www.101qs.com

Example 11: Area of a circle with a curvy twist

ACMMG216 ◆
Students calculate the areas of composite shapes.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is from the NRICH website.

A series of images require students to look more closely at the composition of the shape to identify components that are familiar to them.

When presenting challenging tasks it is important for us to have thought about 'enabling prompts' that we can release to students as necessary. Such prompts could include:

- *What shapes do you recognise in this image? Which ones are unfamiliar to you?*
- *If you looked at one of the curvy areas, say the red one, could you divide it into parts that we might recognise?*
- *If I told you that the shape could be divided into semicircles, could you show me how that could be done?*

The link to this problem on the NRICH site is:
http://nrich.maths.org/6468

Building resilience and positive learner identity: Using high challenge problems

We know that:

'Positive learner identity – is more readily built through succeeding at challenging tasks than experiencing 'dumbed down' activities.'

(David Price, Learning Futures, UK)

Example 12: Gutter problem

ACMMG216 ◆
Students calculate the areas of composite shapes.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity is from the NRICH website.

This is a practical problem that asks students to explore different shapes created from the same materials, fostering their problem solving skills and developing mathematical reasoning and a notion of proof.

The link to this problem on the NRICH site is: http://nrich.maths.org/5673

Example 13: Hexagonal prism

ACMMG218 ◆
Students solve problems involving the surface area and volume of right prisms.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

Show students Figure 11 and state the following:

I think there is twice as much orange in the large container compared to the small container.

Then ask them:

- *What do you think? Convince me.*
- *What's wrong with this idea? Convince me.*

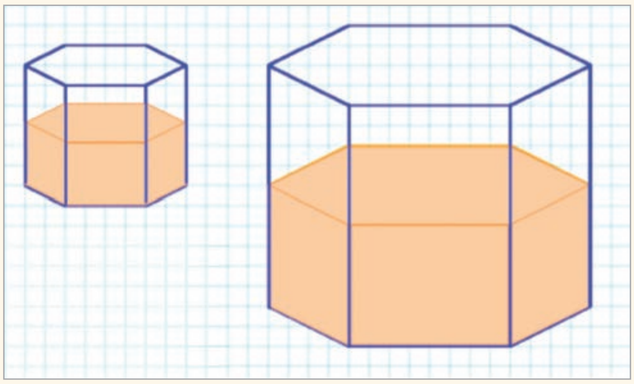


Figure 11

Tackling misconceptions: Knowing what it isn't

We can challenge students' reasoning and address common misunderstandings by asking *'Why can't I …'*; *Why is it not…*; *What's wrong with…'* questions. Students can be given problems from fictional students which are answered incorrectly, without working out. They can be asked to give feedback, summarising the mistakes that the students have made and how to avoid that mistake in the future.

When presenting challenging tasks it is important for us to have thought about 'enabling prompts' that we can release to students as necessary. Such prompts could include:

- *Is a cylinder a prism?*
- *What are the conditions for an object to be described as a prism?*
- *Does a cylinder meet those conditions?*
- *How is a cylinder similar/different to a prism?*
- *How would you work out the volume of a cylinder?*

Even though a cylinder is not a prism, because a prism must be a polyhedron (which means the cross section will be a polygon/have straight sides), we can still apply the general formula for calculating the volume of a prism, ie Volume = Area of cross section x Height.

Connections between ‘using units of measurement’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use units of measurement as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 9	
Whilst working with units of measurement, connections can be made to:	**How the connection might be made:**
Explore the relationship between graphs and equations corresponding to simple rate problems. ACMNA208	Refer to: **Example 1: Establishing the volume of a cylinder.**
Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software. ACMNA214	Refer to: **Example 7: Measuring a cube.**
Sketch linear graphs using the coordinates of two points and solve linear equations. ACMNA215	Refer to: **Example 1: Establishing the volume of a cylinder.**
Graph simple non-linear relations with and without the use of digital technologies. ACMNA296	Refer to: **Example 3: Filling containers.**
Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar ACMMG210	Refer to: **Example 2: Enlargement and squared and cubed attributes.**
Solve problems using ratio and scale factors in similar figures. ACMMG211	Refer to: **Example 2: Enlargement and squared and cubed attributes.**
Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles. ACMMG212	Refer to: **Example 7: Measuring a cube**
Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources. ACMSP228	Refer to: **Example 4: Packaging problems – redesigning a cylinder [data could be collected and processed on hand measures relating to comfortable grip of a can].**

Making connections to other learning areas

There is great potential to make connections to history when working with units of measurement. Many of the imperial units that students will hear being used, particularly by older relatives, will have stories that explain the use of a word. For example: pints and gallons. At this stage, when students are learning to convert between metric units of measurement, it can be useful to look at the alternative (the imperial system) and the challenges involved in changing between units in that system.

There are also many connections to science. One connection that can be used to remind students of the importance of recording units of measure, is this story about NASA losing a spacecraft because they had a miscommunication about units:

‘Mars Climate Orbiter team finds likely cause of loss’: http://mars.jpl.nasa.gov/msp98/news/mco990930.html

‘Using units of measurement’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with measurement:

Using informal units for direct or indirect comparisons ◆
From **Foundation to Year 2** students focus on **informal units of measurement.**

Using standard metric units ◆
From **Year 3 to Year 8** students develop their **understanding of metric units of measure.** This begins with the use of familiar metric units and extends to include a greater range of metric units and the flexibility to convert between different units.

Establishing and applying formulae ◆
From **Year 5 to Year 10** students **establish and use formulae** of increasing complexity relating to perimeter, area and volume.

Estimating ◆
Australian Curriculum references to **estimation** in relation to measurement lie entirely in the **Numeracy Continuum.**

Year level	‘Using units of measurement’ content descriptions from the AC: Mathematics
Foundation ◆	Students use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.
Year 1 ◆	Students measure and compare the lengths and capacities of pairs of objects using uniform informal units.
Year 2 ◆	Students compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units.
Year 2 ◆	Students compare masses of objects using balance scales.
Year 3 ◆	Students measure, order and compare objects using familiar metric units of length, mass and capacity.
Year 3 ◆	Students use scaled instruments to measure and compare lengths, masses, capacities and temperatures.
Year 4 ◆	Students compare objects using familiar metric units of area and volume.
Year 5 ◆	Students choose appropriate units of measurement for length, area, volume, capacity and mass.
Year 5 ◆	Students calculate the perimeter and area of rectangles using familiar metric units.
Year 6 ◆	Students connect decimal representations to the metric system.
Year 6 ◆	Students convert between common metric units of length, mass and capacity.
Year 6 ◆	Students solve problems involving the comparison of lengths and areas using appropriate units.
Year 6 ◆	Students connect volume and capacity and their units of measurement.
Year 7 ◆	Students establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
Year 7 ◆	Students calculate volumes of rectangular prisms.
Year 8 ◆	Students choose appropriate units of measurement for area and volume and convert from one unit to another.
Year 8 ◆	Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.
Year 8 ◆	Students investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Year level	'Using units of measurement' content descriptions from the AC: Mathematics *continued*
Year 8 ◆	Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
Year 9 ◆	Students calculate the areas of composite shapes.
Year 9 ◆	Students calculate the surface area and volume of cylinders and solve related problems.
Year 9 ◆	Students solve problems involving the surface area and volume of right prisms.
Year 10 ◆	Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.
Year 10A ◆	Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Numeracy continuum: Using measurement	
End Year 2 ◆	Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects.
End Year 4 ◆	Estimate and measure with metric units: estimate, measure and compare the length, temperature, volume, capacity and mass of everyday objects using metric units and scaled instruments.
End Year 6 ◆	Estimate and measure with metric units: choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems.
End Year 8 ◆	Estimate and measure with metric units: convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems.
End Year 10 ◆	Estimate and measure with metric units: solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by Office for Education and Early Childhood