Using units of measurement: Foundation–Year 2

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together

www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key


This teacher will raise questions, answer students' questions and share some of her classroom practice.


This teacher will give you his top pedagogy tips.


These students will raise questions and model student thinking.

Bringing it to Life (BitL): key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

What the Australian Curriculum says about 'using units of measurement'

Content descriptions

Strand | Measurement and geometry

Sub-strand | Using units of measurement

Foundation | ACMMG006
Students use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.

Year 1 | ACMMG019
Students measure and compare the lengths and capacities of pairs of objects using uniform informal units.

Year 2 | ACMMG037
Students compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units.

Year 2 | ACMMG038
Students compare masses of objects using balance scales.

Year level descriptions

Foundation | Students compare the lengths of objects.

Foundation | Students explain processes for indirect comparison of length.

Year 1 | Students explain direct and indirect comparisons of length using uniform informal units.

Year 2 | Students use informal units iteratively to compare measurements.

Achievement standards

Foundation | Students compare objects using mass, length and capacity.

Foundation | Students group objects based on common characteristics and sort shapes and objects.

Year 1 | Students order objects based on lengths and capacities using informal units.

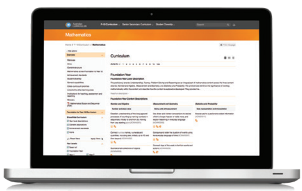
Year 2 | Students order shapes and objects using informal units.

Numeracy continuum

End of Foundation | Measure by comparing objects and indicate if these measurements are the same or different.

End of Year 2 | Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects.

In the Australian Curriculum: Mathematics, the concept of 'time' is addressed in the sub-strand 'Using units of measurement', but in this resource, 'time' has its own narrative.



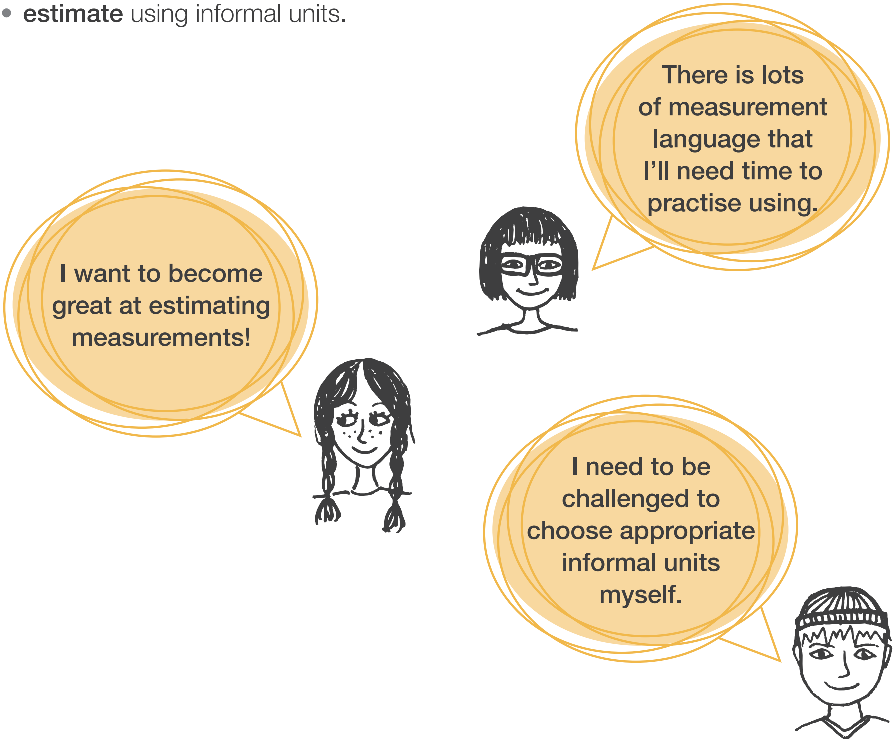
Source: ACARA, Australian Curriculum: Mathematics, Version 8.0

Working with units of measurement

Important things to notice

When we design learning about measurement, it is easy to think solely about 'using measuring instruments' (either informal or metric). It is also important to design opportunities for students to:

- **select** appropriate informal units and tools
- **identify** measurable attributes
- **develop language** associated with measurement
- **estimate** using informal units.



Building on learning from past years

A quick look at the Early Years

In South Australian preschools, educators refer to *Belonging, Being & Becoming – The Early Years Learning Framework for Australia*, and *Reflect, Respect, Relate* as the basis for the design and implementation of learning experiences.

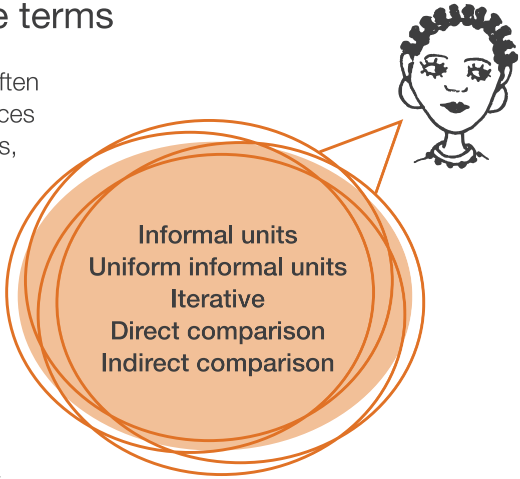
From the beginning of 2016, preschool teachers will also be required to use the numeracy and literacy indicators to inform their planning and teaching, to monitor children's numeracy and literacy development, and to inform the *Statement of learning* for discussion with, and reporting to, families. The information will also be used to support the transition of children from preschool to school.

The numeracy indicator relating to measurement is, **'I measure and compare my world'**. The key elements of this indicator are:

- **Use measurement to compare objects, events and space.**
- **Notice objects, events and space have measurable attributes.**
- **Choose and use the appropriate tool and strategy for the attribute.**
- **Use comparative language.**
- **Recognise that the principles of measurement do not change.**

Australian Curriculum descriptions

Getting clear about the terms



- **Informal units** of measure are often human related units, such as paces or fingers, but can also be blocks, paper clips, sticks etc. Informal units are sometimes referred to as non-standard units of measure.
- **Uniform informal units** are informal units that are of the same size as each other. For example the blocks used are all the same as each other.
- **Using informal units iteratively** means the repeated use of the same unit. For example, moving one paper clip along the length of a shape 10 times, rather than using 10 paper clips in a line.
- **Direct comparison** is when the two (or more) items that are being compared, are brought together. For example, the pen is laid next to the pencil to compare their lengths. The ball is placed on one side of the balance and the rock is placed on the other side.
- **Indirect comparison** is when an intermediary is used to make the comparison. Initially the intermediary could be another item. For example, a piece of string could be used to compare the height of the school tables with the height of the table in a child's home. The two items can't be brought together for direct comparison, but they can be compared to each other via the piece of string.

In the early stages of using units of measurement I challenge my students to make direct comparisons. I support students to understand the need for a *baseline starting point for direct comparison*. In other words, students learn to line up the items being compared. I find it useful to talk about 'the start' and 'the end' of the object.

When we begin to work on indirect comparison, I have my students use a single informal unit, so they can focus on using a baseline starting point for measuring (lining up the start of the object with the start of the string), *without* needing to consider gaps and overlaps.

For single informal units I use items such as a length of string, a strip of paper or a measurement against themself.

Then I challenge students to use *multiple* informal units for indirect comparison. For example, using many blocks, many paper clips or many paces. When we do this, I design tasks so that my students understand the need for informal units to be *uniform in size, with no gaps and no overlaps*.

As my students are learning to choose and use appropriate tools for measuring, I challenge them to notice the **relationship between the size of the unit and the quantity required.** For example, that measuring an item using smaller units would require more of those units and vice versa. Refer to **Examples 1**, **2** and **6.**

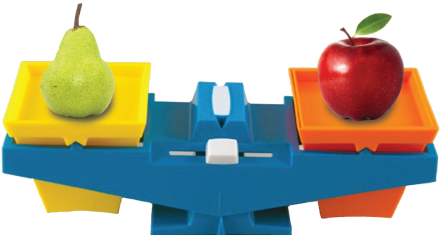
Sometimes I challenge students to **describe, more accurately, the relationship between the size of the unit and the number required.**

For example, when measuring length, observing that it will take twice as many if the unit is half as long.

I challenge my students to begin to use informal units iteratively by restricting the number of items available for measuring with.

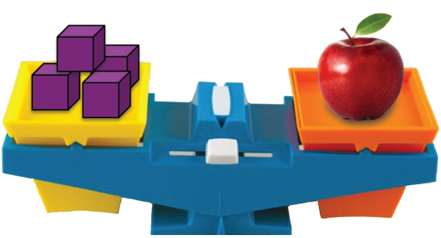
We have lots of fun calibrating and using our own measuring devices using informal units. I do this to prepare students for using standard metric measuring devices. Refer to **Example 19.**

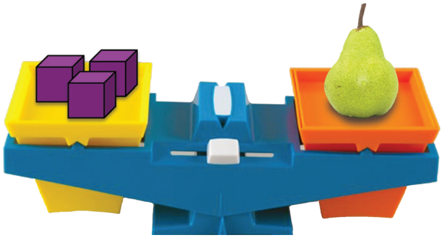
Direct comparison of an apple and a pear



Indirect comparison of an apple and a pear

(Using multiple informal units – blocks)



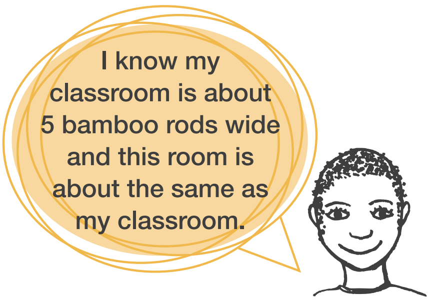


Developing an ability to estimate

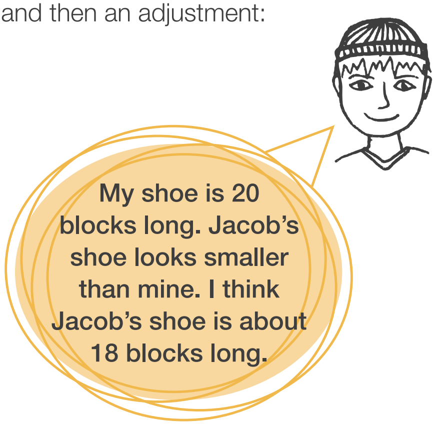
Different ways to reason when estimating length

Typically we estimate distance in one of the following ways:

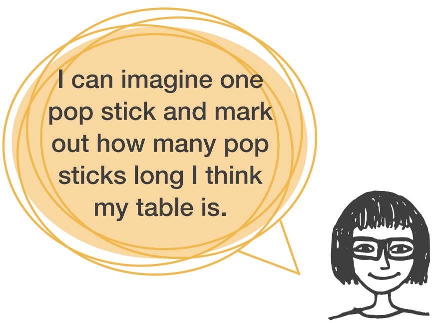
A comparison to another known distance:



A comparison to another length and then an adjustment:



Visualise/mark out the unit distance and count:



Engaging learners

Harnessing students' fascination with scale

People are often fascinated with very large or very small items. We are particularly fascinated with large items that should be small and small items that should be large. For an example of this fascination, follow the link below to the news story about giant marionettes in Perth, WA. An estimated 1.4 million people attended *'The Giants extravaganza'*!

http://www.perthnow.com.au/news/western-australia/giants-in-perth-day-three/story-fnhocxo3-1227220081679

Perth Now
February 2015
Picture: Stewart Allen

To use this story to engage learners we would play one of the news stories, without the audio (at first) and ask students:

What questions do you have?

There are films, such as *'The Borrowers'* and *'Gulliver's Travels'*, that play on our fascination with scale. Such films and images can be used in relation to informal units, as well as with formal metric units of measurement.

For the marionette example from *'The Giants extravaganza'* (left), the height of the men in the photograph can be compared to the height of the marionette.

We can support our students to develop a disposition towards using maths in their lives, ie becoming numerate, not only through the use of 'real world' maths problems, but through fostering a disposition towards asking mathematical questions about everything they see. We develop this disposition in our students when we promote, value and share their curiosity and provide opportunities for them to develop their questions and explore solutions to their questions.

! PAGES 12/13 ARE REALLY IMPORTANT – you will be tempted to skip them in your enthusiasm to get to the examples. You only need to READ THIS ONCE EVER.

Embedding the Australian Curriculum: Mathematics proficiencies

Pedagogy supporting you to embed the proficiencies

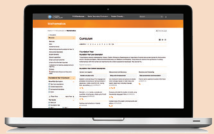
AC: Mathematics proficiencies

The verbs used in the four Mathematics proficiencies from the Australian Curriculum (AC: Proficiencies) describe the actions in which students can engage when learning and using mathematics content.

To embed the AC: Proficiencies in students learning experiences, we need to ask questions that activate those actions in students. But what questions will achieve this? The AC: Proficiencies describe the actions, but not the questions that can drive those actions.

There are four proficiency strands in the Australian Curriculum: Mathematics:

- Understanding
- Problem Solving
- Reasoning
- Fluency.



Bringing it to Life tool

The Bringing it to Life (BitL) tool was developed by the South Australian Teaching for Effective Learning (TfEL) team, to support teachers to bring the AC: Proficiencies to life in the classroom. The BitL tool models questions that can be used to drive the actions described in the AC: Proficiencies.

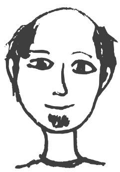
The Bringing it to Life tool is located in the *Leading Learning: Making the Australian Curriculum Work for Us* resource (www.acleadersresource.sa.edu.au), in the section *Bringing it to Life – essence meets content*.

Beware of the old paradigm

There is a prevalent assumption that we should instruct our students with processes and develop their understanding before we challenge them to problem solve and reason. In this paradigm students will only gain problem solving experience at the end of the unit of work, assuming they get through all of the practice questions quickly enough.

The pedagogy shift of innovative educators across the world acknowledges that new understanding and the ensuing fluency are not simply a resource for problem solving and reasoning, but a product of problem solving and reasoning.

Students can construct new understanding when we support them, as they grapple with questions that challenge them to reason with, and extend their current understanding. Instruction can be used at the point of need.



Why does this resource look at each proficiency separately, when they are intertwined skills?

We acknowledge that the proficiencies intertwine and that it is possible to experience a range of proficiencies within one particular problem. However, we have used the BitL questions to organise the examples into categories that emphasise each particular proficiency. The intention in doing this is to support teachers to understand the emphasis of each proficiency deeply in order to be able to intertwine them as appropriate.

You will find less of an emphasis, in this resource on fluency examples, as many textbook and worksheet resources already provide this.

Examples modelling embedding the proficiencies using BitL questions:

- **Understanding**
  Examples 1–9
- **Problem Solving**
  Examples 10–13
- **Reasoning**
  Examples 14–18
- **Fluency**
  Examples 19–22

It is intended that teachers will select/adapt and sequence examples that are appropriate for their students. These examples have been grouped by proficiency, not learning sequence.

Proficiency: Understanding

Proficiency emphasis and what questions to ask to activate it in your students (Examples 1–9)

The AC: Mathematics defines the proficiency of 'Understanding' as:

What the AC says	**Proficiency: Understanding** Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.
BitL tool	There are three BitL questions associated with this proficiency. They reflect the student actions as described in the AC: Mathematics. The three questions are: **Q1 What patterns/connections/relationships can you see?** **Q2 Can you answer backwards questions?** **Q3 Can you represent or calculate in different ways?**
Q1 Examples overview	**What patterns/connections/relationships can you see?** The intent of this question is to promote learning design that intentionally plans for students to develop a disposition towards looking for patterns, connections and relationships. **Example 1: How many do you need?** Relationship between the size of an informal unit and the quantity required **Example 2: Hand span** Relationship between the size of an informal unit and the quantity required **Example 3: Weighted numbers** Conservation of quantity in the context of measurement **Example 4: What could we measure?** Identifying measurable attributes **Example 5: Comparing collections (or pairs)** Identifying measurable attributes

Q2 Examples overview

Can you answer backwards questions?

The intent of this question is to promote learning design that intentionally plans for students to develop flexibility in the way that they can work with a concept.

Example 6: My left shoe
Relationship between the size of the unit and the quantity required

Example 7: Finding collections (or pairs)
Identifying measurable attributes and measuring using informal units

Example 8: Counting squares
Identifying measurable attributes and measuring using informal units

Q3 Examples overview

Can you represent or calculate in different ways?

The intent of this question is to promote learning design through which students experience multiple representations and create multiple approaches. We encourage teachers to look for opportunities to:

- present information/problems in a range of ways
- ask the questions:
  Is there another possibility?
  Is there another way?

Example 9: Smallest to largest
Identifying measurable attributes and measuring using informal units

Example 1: How many do you need?

Relationship between the size of an informal unit and the quantity required

Figure 1

1 Have students measure one item using a range of different informal units, like those shown in Figure 1.

2 Ask students to order their units from smallest to largest and record next to each unit, how many they needed to measure their item, then ask:

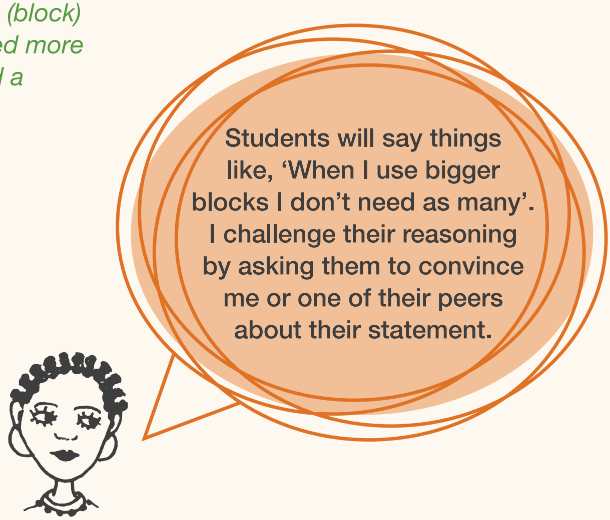
What connections can you see between the size of the unit and the amount that you needed?

When you used a bigger unit (block) to measure with, did you need more or fewer than when you used a smaller unit (block)?

3 Support students to make general statements (reason) through providing sentences such as:

The larger the block (or similar unit) the .... the amount that I needed to measure with.

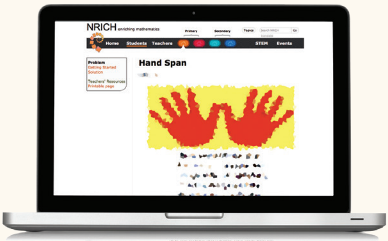
The smaller the block (or similar unit) the .... the amount that I needed to measure with.



Example 2: Hand span

Relationship between the size of an informal unit and the quantity required

This task provides a great opportunity to work collaboratively and to take maths outside the classroom. Involving people with larger hands than junior primary students would add value to this activity. This makes it perfect for involving parents or perhaps an older ‘buddy class’.



The link to this problem on the NRICH site is http://nrich.maths.org/209

Example 3: Weighted numbers

Conservation of quantity in the context of measurement

This interactive task is great for establishing conservation of quantity. For example, that ‘6 units’ will balance with ‘two lots of 3 units’ or ‘three lots of 2 units’ etc.

The link to this problem on the NRICH site is http://nrich.maths.org/4726

Example 4: What could we measure?

Identifying measurable attributes

Figure 2

Figure 3

We can show a collection of objects/ shapes and ask:

What could we measure about all of these objects/shapes? What else? What else?

For the collection in Figure 2, students could identify that it's possible to measure:

- various lengths (for example, the height)
- how heavy it is (the mass)
- how much it holds (the capacity)
- how much table space it covers when we draw around it.

But, if we remove the bottle and replace it with a chocolate (Figure 3), we can ask:

What if we make this change, can we still measure the same things for this collection? If not, why not?

Example 5: Comparing collections (or pairs)

Identifying measurable attributes

We can show students collections (or pairs) of shapes/objects that are:

- the same length as each other, but different mass
- the same height as each other, but different capacities
- the same mass as each other, but different length/width/volume
- the same capacity as each other, but different heights

Then we can ask:

What measurement(s) might be the same for this collection/pair?

What measurement(s) might be different for them?

Convince me/convince someone who thinks differently to you.

When we ask these questions we challenge students to look for attributes that connect different shapes or objects.

This will help to heighten their awareness that the same object can have different measurable attributes.

Example 6: My left shoe

Relationship between the size of the unit and the quantity required

This example has been written for length, but could be adapted for mass and capacity. It is a slight twist on *Example 1: How many do you need?*. In this example students have been given a quantity of units to aim for and their challenge is to identify an appropriate informal unit (for example, blocks, paper clips, fingers etc).

1 Establish something for the student to measure. For example, ask students to draw around their left shoe and then draw a straight line from the heel to the toe. This is the line that we will be measuring.

2 The challenge is to complete as many sentences as possible on the sheet, 'My left shoe' (Figure 4).

3 Make it really clear to students that it might not be possible to find a unit that works for each of the challenges on the sheet.

4 Engage students in predicting how many they think they will be able to find. Explain that you aren't interested in how many they actually find, but you *are* interested in the mathematical thinking they do while they are trying to look for possibilities. Then introduce a sheet such as the one shown in Figure 5, or if appropriate, for your students, discuss how they will record great thinking that didn't lead to an answer on the sheet, 'My left shoe'.

My left shoe!
One ....is the same length as my left shoe.
Two ....is the same length as my left shoe.
Three ....is the same length as my left shoe.
Four ....is the same length as my left shoe.
Five ....is the same length as my left shoe.
Six ....is the same length as my left shoe.
Seven.......................................is the same length as my left shoe.
Eight ....is the same length as my left shoe.
Nine ....is the same length as my left shoe.
Ten ....is the same length as my left shoe.

Figure 4

Great maths thinking doesn't always lead to an answer that works!
Let's use this board to collect examples of great thinking that didn't work out.

I tried to use ....
(draw a picture or name the unit)

.... was too many and
.... was not enough
(Eg 5 was too many and 4 was not enough)

Figure 5

5 Support students to reflect as a group on what they learned through doing this activity. Asking questions about the strategies that they used for finding units of just the right size.

This activity provides the teacher with an opportunity to identify if students have noticed connections between the size of the unit and the quantity used. It also provides the teacher with the opportunity to support students to make observations that will develop this understanding.

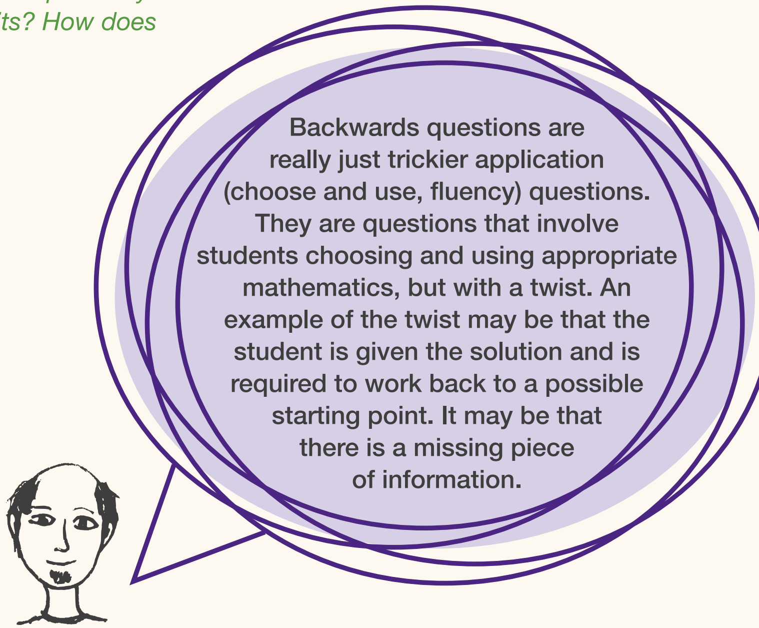
We can use questions, such as the following, to establish if the student understands the relationship between size and quantity:

I notice that you used six paper clips. Will you need something that is bigger or smaller than a paper clip when you are looking for ten units? How does that work?

Can you see a connection between the size of the unit and the amount of units that you need?

If appropriate, we can ask students what connections they notice between the size of units for which they have used half/double the amount. For example, we can say:

I notice that you needed only 3 blocks, but you needed 6 paper clips. Let's put the paper clip and the block next to each other. What do you notice?



Example 7: Finding collections (or pairs)

Identifying measurable attributes and measuring using informal units

We can reverse the question shown in *Example 5: Comparing collections (or pairs)*, and ask students to make the collections.

- Find a pair of objects that you can compare the capacity of (Foundation).
- Find a collection of objects that you can measure and compare the capacity of (Year 1 and 2).
- Find a pair of objects that you can compare length of (Foundation).
- Find a collection of objects that you can measure and compare the length of (Year 1 and 2).
- Make a collection of objects that you can measure their mass and length, but not their capacity (Year 1 and 2).

VALUE MATHEMATICAL THINKING, NOT JUST THE ANSWER

For most children, **Example 8** will demand some problem solving skills. They will need to try out some different ideas. They may, for example, want to compare the length of the shapes first. We must take care to support students to realise that trials, which lead to them to say what the answer is not, also involve successful mathematics.

There are many ways for us to show that mathematical thinking is valued and that it's not always, all about the answer. During the whole class reflection for example, we could ask:

Who tried to find out if these shapes had been ordered by length? Who else tried this? What did you find out? What did you do? Is anyone convinced that Claire didn't order these shapes by length? Thanks for sharing. You have convinced us that it's not length. That's helpful thinking.

Example 8: Counting squares

Identifying measurable attributes and measuring using informal units

We can provide students with a collection of shapes that have been ordered from smallest to largest with a particular attribute in mind, and then we can ask them to identify the attribute that has been used. For example, we could share the collection of shapes shown in Figure 6 or Figure 7 and say:

Claire said that she has arranged these shapes from smallest to largest. This is the order that she put the shapes in. What do you think she might have been measuring when she put them in this order?

In this example the student needs to consider what attributes are measurable for these shapes. The shapes can be cut out, rotated, compared directly or indirectly.

Students could compare:

- how long/wide the shapes are
- how much space the shapes cover (area)
- how far it is around the outside edge of the shape (the perimeter).

Some children may observe instantly that the space covered by the shapes (area) gets bigger and bigger. For children who have this insight the teacher can say:

Really? Convince me.

Is there another way that you could convince someone?

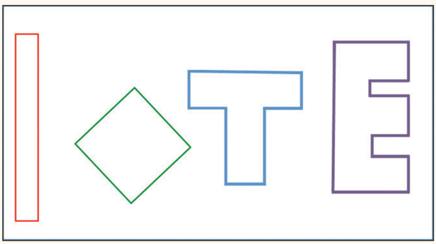


Figure 6

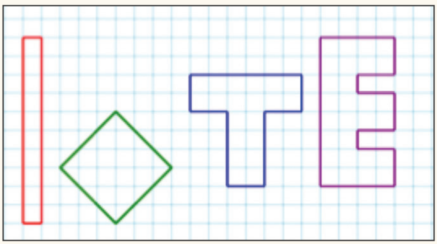


Figure 7

Example 9: Smallest to largest

Identifying measurable attributes and measuring using informal units

We can put together a collection (or just a pair) of shapes or objects, such as those in Figure 8 and 9, and ask students:

Can you order these shapes/objects from smallest to largest?

Can you order them from smallest to largest, but in a different way?

We can activate the students' reasoning and ask:

Can you estimate the order first?
Why did you make those decisions?

Can you prove to yourself/convince me that this is the order the shapes/objects should be in?

If the student is reluctant to prove their idea, just suggest that you believe the order to be different and ask them:

Why can't it be like this? You'll have to convince me this is the wrong order!

Figure 8

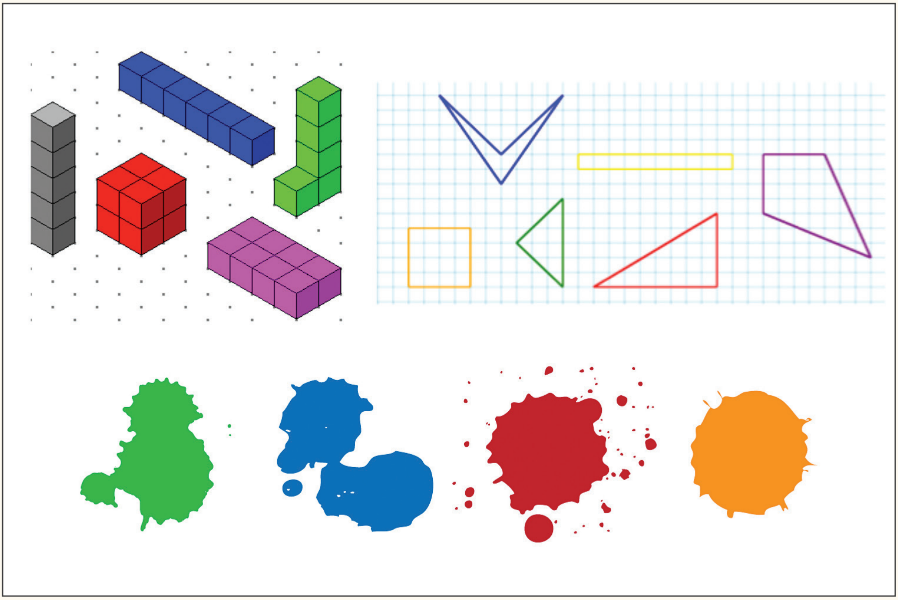


Figure 9

Things to consider when putting collections together

1 **Use objects of different densities.** Using a large piece of sponge and a small piece of dense rock in the same collection can mean that the ordering from smallest to largest changes significantly depending on the attribute that is being measured. This can challenge students' assumptions about size and mass.

2 **Use containers of different heights, but similar capacities.** Using containers that are tall and thin alongside containers that are short and wide, can challenge students' assumptions about height and capacity.

3 **Students can order the same collection of objects in different ways.** For example, by height, mass and capacity. The Australian Curriculum does not introduce perimeter until Year 5, but at that stage the emphasis is about finding efficient ways to calculate perimeter using metric units. Younger children (approximately Year 2) could be introduced to the idea of, 'the distance around the outside edge of a flat shape'. This can be measured using string or other appropriate informal units. The term perimeter could be introduced once the concept is established.

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 10–13)

The AC: Mathematics defines the proficiency of ‘Problem Solving’ as:

What the AC says

Proficiency: Problem Solving

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

BitL tool

There are four BitL questions associated with this proficiency. The four questions are:

How can you interpret?

In what ways can you model and plan?

In what ways can you solve and check?

Reflect?

Examples overview

Interpret; Model and plan; Solve and check; Reflect

The Problem Solving section of the BitL tool works differently to the rest of the maths BitL tool—the sequence of four questions reflect the problem solving process. Therefore, unlike the other three proficiencies, we have not provided examples categorised under each question.

Example 10: Fruit punch and party cups
Estimating and measuring using informal units

Example 11: Money sticks
Estimating and measuring using informal units

Example 12: Gulliver’s chair
Estimating and measuring using informal units

Example 13: Design and build a bridge
Estimating and measuring using informal units

What's the difference between meaningful and unfamiliar problems?

Meaningful problems

Meaningful problems are those in which the mathematics and strategy being applied is familiar to the student. They have solved a problem similar to it before, so they can use the same strategy. The student understands the required content.

Unfamiliar problems

Unfamiliar problems include:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach/develop a strategy.
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having re-combined that knowledge to form a new piece of understanding.

BUILDING RESILIENCE

Unfamiliar problems tend to provoke a response of, 'I don't know' or 'I'm not sure'. Students respond differently to this feeling. Some students shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move they need to make on the journey to *finding out more* is to ask, 'What could I do to work this out?'

Example 10: Fruit punch and party cups

Estimating and measuring using informal units

Provide the students (groups of 4) with a simple recipe for a fruit punch. To do this, the teacher will need to adapt any recipe that is given in metric units to suit informal measures that are available to students. Students make the punch and place in the fridge to chill.

Provide each group of 4 students with 4 cups of different sizes. While the punch is chilling in the fridge, ask the students:

How can you make sure that everyone in you group gets the same amount of fruit punch?

Experiment with water, so that you know just what to do when the punch is ready.

Interpret

What have you been asked to do? Why is this tricky? What information do you need to know?

Model and plan

What could you try? What equipment might be helpful? Have you had this problem before? Speak to someone who you think is being a good problem solver today and ask them to show you what they are trying.

Solve and check

Does that seem right to you? Does it look like the same amount in each cup? Why? Why not? How could you check? Did other people agree? Is there another way that you could solve this problem? Could you put any extra in each of the cups? Will there be enough fruit punch for your idea to work?

Reflect

Did other people do it differently? Is there something that you would do differently next time?

Example 11: Money sticks

Estimating and measuring using informal units

This example connects to the context of money, by using coins as the informal measuring unit.

Through showing a picture, as in Figure 10, and building a relevant scenario around the problem, we can present students with a decision, such as:

You can choose to have:

- *one (red) stick length of 20c coins, or*
- *two (red) stick lengths of 10c coins.*

Which would you choose to have (which would be worth the most money)?

Figure 10

Example 12: Gulliver's chair

Estimating and measuring using informal units

Photographs, such as this one in Figure 11 by Statler Hilton from 'Dan Meyer's 101 questions' (http://www.101qs.com/434-pleasetake-a-seat-gulliver) can be used to engage students in asking their own questions.

In response to this image, children at this stage of development could ask questions such as:

Is that table/chair taller than our classroom?

Would that table/chair fit in to our gym?

How many people could sit on that chair?

To answer questions such as this, students would need to consider:

How many adults tall/wide is the table/chair?

How many children tall/wide is the table/chair?

Figure 11

Students would also need to consider which adult/child to use as their measure. Could they use any? How will they make a measure for that adult/child (out of string/out of paper/find an object the same size etc). Make sure the copy of the image is large enough for students to work with.

When I use an image like this to engage my students, I ask, 'What questions do you have?'. We sort them into mathematical and non-mathematical questions. Sometimes I identify a question that I want all of the students to solve and other times I get them to choose. I think it's important for students to learn to ask questions, not just answer them.

Example 13: Design and build a bridge

Estimating and measuring using informal units

Challenge students to:

Make a bridge that is:

smaller than this (shoe box)

tall enough to push this (toy car) under

strong enough to hold this (bottle of soft drink) for 1 minute.

Students and teachers could negotiate the range of materials that are used. You might want to add other constraints, such as:

use at least 3 different materials, or

use only one material

use no more than ...... pop sticks etc.

I use construction activities as an opportunity to really focus on developing students' comparative language. For example, using words such as taller, shorter, longer, thinner, wider and narrower. We create word walls of the language that we use as we are making our construction.

Proficiency: Reasoning

Proficiency emphasis and what questions to ask to activate it in your students (Examples 14–18)

The AC: Mathematics defines the proficiency of ‘Reasoning’ as:

What the AC says	**Proficiency: Reasoning** Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.
BitL tool	There are four BitL questions associated with this proficiency. They reflect the student actions as described in the AC: Mathematics. The four questions are: **Q1 In what ways can you prove?** **Q2 In what ways can you communicate?** **Q3 In what ways can we generalise?** **Q4 What can you infer?**
Q1 Examples overview	**In what ways can you prove?** The intent of this question is to promote learning design in which students justify their (mathematical) actions in order to develop an increasingly sophisticated capacity for logical thought and action. You can look for opportunities to ask students to prove their thinking to you, to themselves, or to their peers. Whether an answer is right or wrong, you can be ready to say: *Are you sure? Convince me, convince someone who thinks differently to you.* *Convince me*, often builds on from a sequence of thinking. Many examples in this resource provide appropriate opportunities for you to say, ‘Convince me’.

Q2 Examples overview

In what ways can you communicate?

The intent of this question is to promote learning design that intentionally plans for students to compare, contrast and evaluate different ideas and approaches and to explain their reasoning using increasingly sophisticated mathematical terminology and conventions.

We must intentionally plan for communication to be in different modes, including written, verbal, symbolic and descriptive.

Example 14: Estimate and explain
Estimating measurement through reasoning

Example 15: Why can't I?
Measuring using informal units

Q3 Examples overview

In what ways can we generalise?

The intent of this question is to promote learning design that intentionally plans for students to look for generalisations. As students learn to generalise, they see mathematics as a set of connected ideas, rather than lots of separate rules and processes.

Example 16: Unit-size and quantity
Understanding the relationship between unit-size and quantity

Example 17: Actions of a good 'measurer'
Measuring length using informal units

Q4 Examples overview

What can you infer?

The intent of this question is to promote learning design in which students think about the implications of information they have, and develop flexibility to refine their thinking as more information is used.

Example 18: Tricky comparisons
Transitivity in the context of measurement

Example 14: Estimate and explain

Estimating measurement through reasoning

Earlier in this resource, we said that it is important for students to develop an ability to estimate measurements and to appreciate that estimating is reasoning, not guessing. We can support students to become aware of their reasoning, through challenging and supporting them to explain their thinking. Students also benefit from hearing each other's reasoning. This example can be used as a quick group activity. This example is written in relation to length, but could be adapted for mass, using balance scales.

1. Show students one length that you would like them to estimate. This can be as simple as a line on a piece of paper.
2. For pairs or small groups allocate an informal unit that you'd like the students to use for their estimation.
3. Students could be engaged in this process through making a 'lucky dip' selection from cards showing images of units. Examples of cards are shown in Figure 12.

   The image on the card should not represent the actual size of the unit, but simply act as a prompt for the students to know what unit they will be using. More than one group could have the same unit.
4. Provide some time for students to estimate the length of the line in their chosen unit.
5. Collect estimates form the different groups and record them on the white board.
6. Select students (in their pairs) to explain how they decided on their estimate.

Figure 12

7. Ask students if they would like to change their initial estimate and change the recording on the board to reflect their new thinking.
8. Establish a range from the collective wisdom of the group. For example, we have estimated that the line is somewhere between: 10 and 16 counters long, 3 or 4 pegs long etc.
9. Use this opportunity to support students to make the connection between the size of the unit and the amount required. For example, ask why you would need more counters than pegs.

10 This activity can be concluded by:

A Checking the measurements and reflecting on how they could improve their estimating skills in future, or

B Congratulating students for the thinking they did in making their estimates and *not* measuring to check. Not checking leaves a level of uncertainty for the students. Learning to feel comfortable with uncertainty is challenging and activities such as this can help to develop a disposition towards making a decision for which nobody is able to verify if you are right or wrong.

I support students to become comfortable with not measuring to check, by occasionally setting a clear success target, such as:

Today, success on this activity is making an estimate that you are happy with and being confident enough not to measure afterwards.

Example 15: Why can't I?

Measuring using informal units

Why can't I…? questions can be a good way of hearing students' reasoning and challenging and developing their understanding. For example, you could ask:

Why can't I…?
Then demonstrate an incorrect measuring technique, such as overlapping, or leaving gaps, or not lining up the items that are being directly compared.

Students can then demonstrate a correct technique. Students creating photographs or videos of 'how to measure' and 'how not to measure' can be a good way of capturing this learning. The children can 'act' both parts of the video.

Example 16: Unit-size and quantity

Understanding the relationship between unit-size and quantity

Students can be encouraged to create statements about the relationship between unit-size and quantity, through challenging them to think about sentences such as:

- *The ……(smaller)….. the unit the….(more)…. that I use.*
- *The ……(larger)……. the unit the….(fewer)…. that I use.*

Activities that lead to the opportunity to make this type of statement are shown in **Examples 1, 2 and 6**.

Example 17: Actions of a good 'measurer'

Measuring length using informal units

Students can be encouraged to create statements about 'being a good measurer of length'.

A good measurer of length will:

- always measure from the start to end of an object
- always make sure that there are no spaces or overlaps between the units
- always use the same unit for the whole length of the measure
- always use the same unit to make a comparison.

Teachers can support students to generalise further, by challenging them to consider:

Which rules still make sense when we are measuring capacity, or mass, or area, or volume?

What do our rules mean when we are measuring capacity, or mass, or area, or volume?

In general the following questions can be useful in eliciting student's thinking:

Can you show me how that works?

Why did you choose to…?

Can you explain that to (a peer)…?

When working with measurement there are many opportunities for photographs of work to be annotated. Taking photographs of the development of the thinking, rather than the end point is a good way to show students their understanding changing over time.

Example 18: Tricky comparisons

Transitivity in the context of measurement

The concept of transitivity (making comparisons that are not direct comparisons) can be developed in the context of measurement. For example, if Bob is older than Tom, and Tom is older than Clare, we know that Bob is older than Clare.

An example of this could be:

Jo's pencil is longer than Tom's and Pam's pencil is longer than Jo's. Choose three pencils (from this selection) that Tom, Jo and Pam could be using?

Is there another possibility?

What's the same about all of these different possibilities?

Problems such as this can be used to model the benefits of breaking a problem in to smaller parts. For example, just thinking about which pencils could belong to Jo and Tom, then thinking about Pam.

Variations on this can be created for mass, perimeter, area and volume.

Proficiency: Fluency

Proficiency emphasis and what questions to ask to activate it in your students (Examples 19–22)

The AC: Mathematics defines the proficiency of 'Fluency' as:

What the AC says	**Proficiency: Fluency** Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.
BitL tool	There are two BitL questions associated with this proficiency. They reflect the student actions as described in the AC: Mathematics. The two questions are: **Q1 What can you recall?** **Q2 Can you choose and use flexibly?**
Q1 Overview	**What can you recall?** The intent of this question is to promote learning design in which students develop their capacity to recall mathematical information and processes. Many worksheets are fluency driven exercises. Practice is a necessary part of developing fluency, but we should be purposeful about addressing the other proficiencies and be purposeful about developing fluency. There are many worksheets that require recall of measurement related information. Instead of examples, in this section we have provided guidance about the language, facts and connections that we can aim for students to have recall of by the end of Year 2.

Q2
Examples overview

Can you choose and use flexibly?

The intent of this question is to promote learning design that intentionally plans for students to experience contexts in which they can choose to use knowledge and skills that they have developed.

Example 19: Personalised measuring tools
Making and using personalised (length) measuring tools

Example 20: Small to large
Creating and ordering collections

Example 21: Look at that truck!
Asking measurement based questions

Example 22: Target measurement games
Measuring accurately using informal units of capacity

What can you recall?

By the end of Year 2 we are aiming for students to have recall of the measurable attributes they have worked with. For example, length, mass, capacity, area and volume.

We would also expect students to be able to choose and use familiar vocabulary associated with these attributes, including comparative and superlative vocabulary. For example, (but *not* limited to):

- **Length:** long, longer, longest, short, shorter, shortest, tall, taller, tallest, wide, wider, widest, width, high, height, deep, deeper, deepest
- **Mass:** heavy, heavier, heaviest, light, lighter, lightest
- **Capacity:** holds more, holds less
- **Area:** 2D space
- **Volume:** 3D space.

By the end of Year 2, we are aiming for students to be able to compare and order a collection of objects in relation to length, mass, capacity, area and volume. It's expected that the method of comparison by this stage is an iterative process using a suitable informal unit. Students should be able to identify a measurable attribute and both estimate and measure it using a suitable informal unit.

I can't describe here what a fluency activity would look like, because an activity that requires problem solving for one Year 2 class, may be a fluency activity for a class that has already met that concept.

To decide if a particular worksheet or activity is a fluency driven exercise, ask yourself: **Do these questions require my students to do anything other than recall and choose and use known mathematical facts and processes?**

If the answer is no, then it's probably just fluency.

Example 19: Personalised measuring tools

Making and using personalised (length) measuring tools

Measuring tools could be made using lots of different materials, some flexible, so they can measure curved surfaces, some not. Students can be encouraged to think about the benefits of each, for example, cardboard, timber, rope, clay, felt etc.

Examples of units that the tools could be based on include the:

- foot (width or length)
- cubit (distance from elbow to finger tip)
- hand (width or length)
- finger (width or length)
- whole body (height).

This activity can be adapted to create:

- **Opportunities to practice using informal units iteratively.** To do this, students should make a measuring tool from just one informal unit. The focus here would be on the student using the same unit repeatedly, without gaps or overlaps.
- **Opportunities to calibrate and use measuring devices.** To do this, students should make a measuring tool with multiple units marked on the tool. For example, a measuring tool with 10 hand-widths marked on it. To make such a tool the student would have the opportunity to use their hand length iteratively.

In using the tool the focus would be on measuring from the appropriate starting point and **reading from a scale** rather than counting each unit.

Working towards choosing an appropriate unit of measure

Make measuring tools of significantly different lengths. Having units that are very different in size provides teachers with the opportunity to ask students to choose the most suitable measure for a particular measuring task. For example, make tools where the unit is one finger width and tools where the unit is one shoe length or a stride length etc.

Extend to include fractions of units

Students can mark half, quarter and eighths on to their measures (as appropriate for their stage of development).

Connection to skip counting

For example, make a measuring tool that is 10 hands long, or 5 cubits long or 3 feet long etc. Then model and encourage skip counting by 10s, 5s and 3s. Making a measure that is ‘100 finger-widths’ could be used to provide opportunities to work with larger numbers.

SUPPORTING STUDENTS TO BECOME SUCCESSFUL IN USING SCALED INSTRUMENTS

Creating and using personalised measuring tools, with multiple units marked on the tool, provides the teacher with an opportunity to address the most common confusion in the use of scaled instruments; where to start measuring from.

Zero will need to be marked on the measure that is being created, so the teacher has the opportunity to talk about why the student might want to start their measure a little way into the cardboard/timber that they are using, so they have space to mark on the zero. When students use their instruments, if they measure from 1 instead of zero (as they should) the teacher is able to ask the student to check the measurement using their actual foot or cubit etc. Through doing this, the teacher will be able to bring the student to the point of realising that something is going wrong when they use their measure. **This is the perfect opportunity to establish, with felt meaning for the student, the rule about measuring from zero.**

Example 20: Small to large

Creating and ordering collections

We can ask students to:

Predict and find five (or two or three) objects, that you can fit into this shoebox (or matchbox or cereal box etc) and order them from smallest to largest.

We can add a level of difficulty by saying:

Find five objects, that you can fit into this shoe box (or matchbox or cereal box etc) and order them from smallest to largest in at least two different ways.

Example 21: Look at that truck!

Asking measurement based questions

A broader range of informal measures can be accessed through the use of photographs, such as the one in Figure 13 by Paula S from 'Dan Meyer's 101 questions' (http://www.101qs.com/2403-wide-load).

We can ask questions such as:

How many cars wide is this load?

How many men wide is this load?

(Refer to the marionette example on page 11.)

Figure 13

Example 22: Target measurement games

Measuring accurately using informal units of capacity

The teacher shows an empty container with a target line marked on it.

The student is challenged to fill the container up to the target line through following a set of instructions, such as:

To reach the target line you must measure out two yellow cups, five teaspoons and three blue bottles of water.

Each student/team can test their accuracy in measuring by following the recipe and then pouring their liquid into the container marked with the target amount.

GROWTH MINDSET—FOCUS ON IMPROVING YOUR ABILITY, RATHER THAN PROVING YOUR ABILITY

We can help students to develop a disposition towards having a growth mindset through using language that values improvement. For example, if a student is struggling to measure accurately, we can congratulate their initial attempts and ask:

How might you improve your accuracy? Do you think that you would be able to get closer to the target if you keep practising? (I do). How could you get some ideas about improving? Perhaps you'd like to watch/talk to someone who could show you how they improved.

Connections between 'using units of measurement' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use measurement as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Foundation	
Whilst working with units of measurement, connections can be made to:	**How the connection might be made:**
Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond.	Counting and recording using informal units.
Mathematics: Year 1	
Whilst working with units of measurement, connections can be made to:	**How the connection might be made:**
Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by twos, fives and tens starting from zero.	Making measuring tools, as described in **Example 19**.
Recognise, model, read, write and order numbers to at least 100. Locate these numbers on a number line.	Making measuring tools, as described in **Example 19**.
Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts.	Using their own measuring tools to work out the distance around a shape and adding the section lengths together.
Recognise and describe one-half as one of two equal parts of a whole.	Making measuring tools, as described in **Example 19**.
Describe and interpret different data sets in context.	Use examples that relate to units of measurements.

Mathematics: Year 2	
Whilst working with units of measurement, connections can be made to:	**How the connection might be made:**
Solve simple addition and subtraction problems using a range of efficient mental and written strategies.	Using their own measuring tools to work out the distance around a shape and adding the section lengths together.
Recognise, model, represent and order numbers to at least 1000.	Making measuring tools, as described in **Example 19**.
Recognise and interpret common uses of halves, quarters and eighths of shapes and collections.	Making measuring tools, as described in **Example 19**.
Count and order small collections of Australian coins and notes according to their value.	Ordering coins by value and then by other attributes. **Example 11** involves counting using coins.

‘Using units of measurement’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with measurement:

Using informal units for direct or indirect comparisons From **Foundation to Year 2** students focus on **informal units of measurement.**
Using standard metric units From **Year 3 to Year 8** students develop their **understanding of metric units of measure.** This begins with the use of familiar metric units and extends to include a greater range of metric units and the flexibility to convert between different units.
Establishing and applying formulae From **Year 5 to Year 10** students **establish and use formulae** of increasing complexity relating to perimeter, area and volume.
Estimating Australian Curriculum references to **estimation** in relation to measurement lie entirely in the **Numeracy Continuum.**

Year level	‘Using units of measurement’ content description from the AC: Mathematics
Foundation	Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.
Year 1	Measure and compare the lengths and capacities of pairs of objects using uniform informal units.
Year 2	Compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units.
Year 2	Compare masses of objects using balance scales.
Year 3	Measure, order and compare objects using familiar metric units of length, mass and capacity.
Year 3	Use scaled instruments to measure and compare lengths, masses, capacities and temperatures.
Year 4	Compare objects using familiar metric units of area and volume.
Year 5	Choose appropriate units of measurement for length, area, volume, capacity and mass.
Year 5	Calculate the perimeter and area of rectangles using familiar metric units.

Year 6	Connect decimal representations to the metric system.
Year 6	Convert between common metric units of length, mass and capacity.
Year 6	Solve problems involving the comparison of lengths and areas using appropriate units.
Year 6	Connect volume and capacity and their units of measurement.
Year 7	Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
Year 7	Calculate volumes of rectangular prisms.
Year 8	Choose appropriate units of measurement for area and volume and convert from one unit to another.
Year 8	Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.
Year 8	Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.
Year 8	Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
Year 9	Calculate the areas of composite shapes.
Year 9	Calculate the surface area and volume of cylinders and solve related problems.
Year 9	Solve problems involving the surface area and volume of right prisms.
Year 10	Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.
Year 10A	Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Numeracy Continuum: Using measurement	
End Year 2	Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects.
End Year 4	Estimate and measure with metric units: estimate, measure and compare the length, temperature, volume, capacity and mass of everyday objects using metric units and scaled instruments.
End Year 6	Estimate and measure with metric units: choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems.
End Year 8	Estimate and measure with metric units: convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems.
End Year 10	Estimate and measure with metric units: solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.0

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.