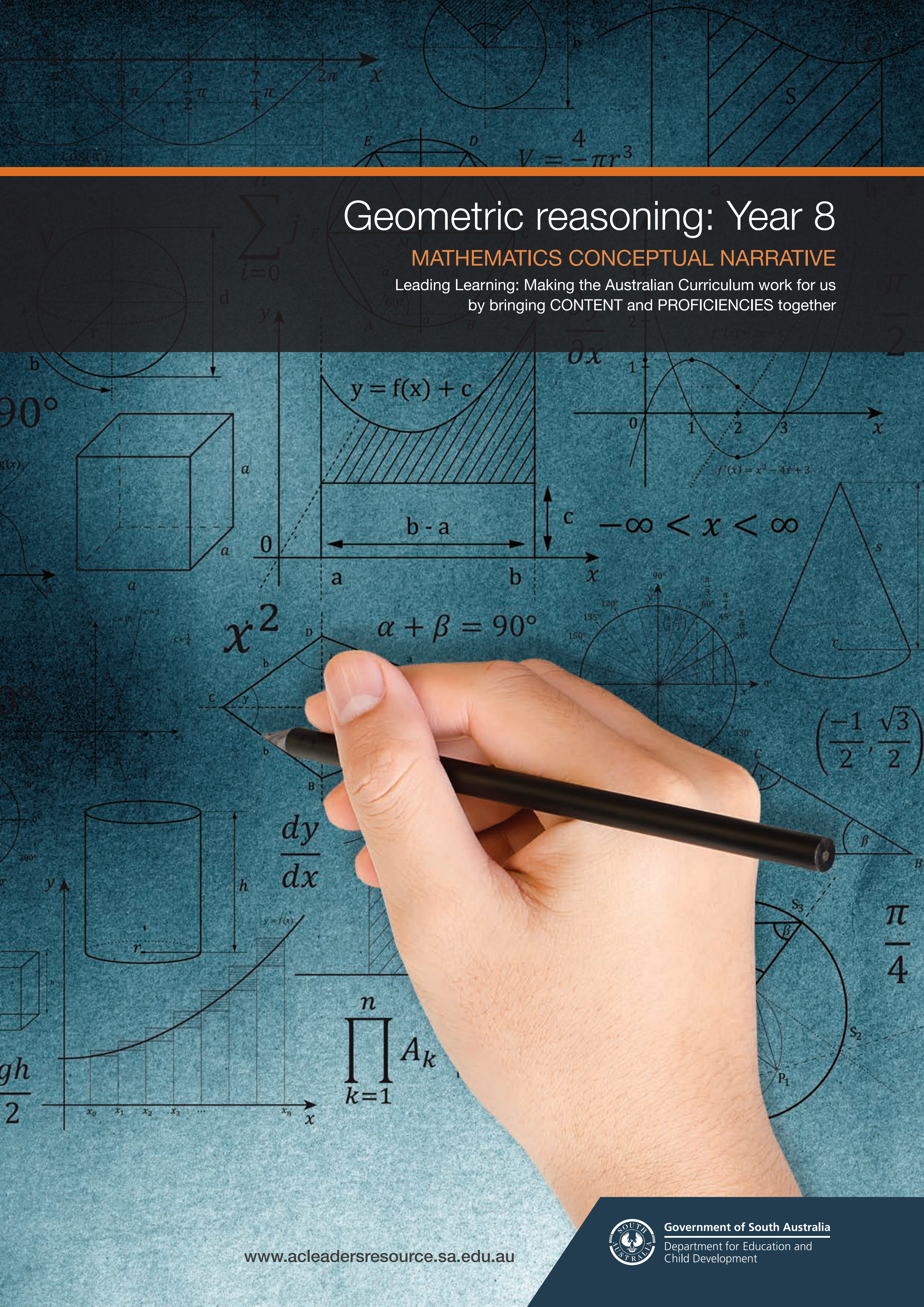


Geometric reasoning: Year 8

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us by bringing CONTENT and PROFICIENCIES together

www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks': http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Geometric reasoning' from Foundation to Year 10A** (see page 25)— we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with geometric reasoning:

- Estimate/measure/compare/identify/classify using geometrical reasoning
- Investigate and generalise using geometrical reasoning
- Solve and prove using geometrical reasoning.

What the Australian Curriculum says about 'geometric reasoning'

Content descriptions

Strand | Measurement and geometry.

Sub-strand | Geometric reasoning.

Year 8 ◆ | ACMMG200
Students define congruence of plane shapes using transformations.

Year 8 ◆ | ACMMG201
Students develop the conditions for congruence of triangles.

Year 8 ◆ ◆ | ACMMG202
Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Year level descriptions

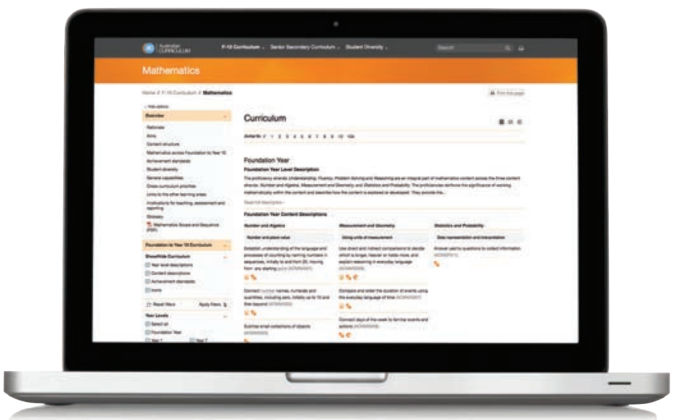
Year 8 ◆ | Students use congruence to deduce properties of triangles.

Achievement standards

Year 8 ◆ ◆ | Students identify conditions for the congruence of triangles and deduce the properties of quadrilaterals.

Numeracy continuum

End of Year 8 ◆ ◆ | Students visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects (Using spatial reasoning: Visualise 2D shapes and 3D objects).



Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Working with 'geometric reasoning'

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 8 geometric reasoning

In Year 7 students investigate and establish the conditions for two lines to be parallel and as a result of this the terms 'corresponding', 'alternate' and 'co-interior' angles are introduced. Students demonstrate angle sums in triangles and quadrilaterals. They also use observations of side length and angles in triangles to be able to sort and classify triangles and to describe features of quadrilaterals.

In Year 8 students use their observations of the properties of quadrilaterals to sort, classify and solve problems involving quadrilaterals. The term 'congruence' is introduced in Year 8 and students establish the (minimum) conditions necessary for congruence of triangles.

In Year 9 students continue to make observations about angles and side lengths of triangles and they use observations together with an understanding of enlargement and ratio to explain similarity. Students establish the conditions for similarity in triangles and they compare these observations to the conditions required for congruence of triangles.

- **Notice that it is *only* in the numeracy continuum that we see an acknowledgement of the need for students to *visualise* shapes.** Visualisation is an important skill for young mathematicians to be developing especially when they are formulating geometric arguments. The numeracy continuum also states the need to *describe and apply* their understanding of the features and properties of 2D shapes and 3D objects. This could be easily overlooked when only reading the content strands and achievement standards.
- The formal mathematical written protocols for a congruency proof are quite abstract. **At this stage of development the conceptual understanding is more important than setting out** and should be considered as 'exposure to' rather than 'mastery of' the written protocols. The AC Mathematics states that Year 8 students need to ***develop*** the conditions for congruent triangles and ***use*** congruent triangles to establish properties of quadrilaterals. While it is not explicit that students need to be able to communicate their reasoning using the abstract mathematical language and symbols used universally to record a formal proof, teachers could see this as an opportunity to immerse learners in the mathematical language.
- **At this stage of development, students often have difficulty distinguishing between:**
  - **the information you can use as truth**
  - **things that look like they are true**
  - **what you are trying to prove.**

  As teachers we should support students to develop a habit of clarifying these three things before they start to formulate an argument or proof for the general case by asking the following questions:
  - *What do you know is true?* (Only what you are given, or what you can deduce.) *How could you show that on the diagram?*
  - *What do you think might be true but you don't know for sure?*
  - *What do you want to prove?*

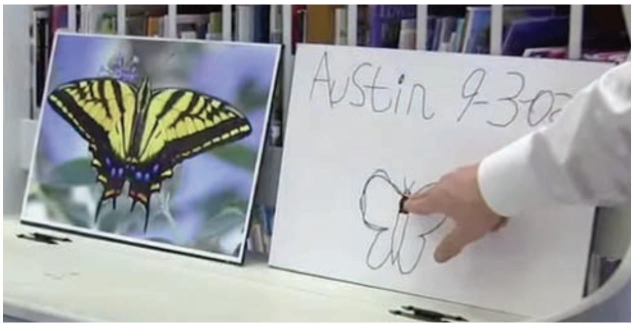
Engaging learners

Classroom techniques for teaching geometric reasoning

Developing geometric reasoning is very challenging for students, requiring risk taking, persistence and the expectation that you will make mistakes and learn from them.

Austin's butterfly

This video tells the story of a first-grade student who received constructive feedback from his peers in his efforts to draw a butterfly. It is a great story to share with your class when they are about to tackle a significant learning challenge.

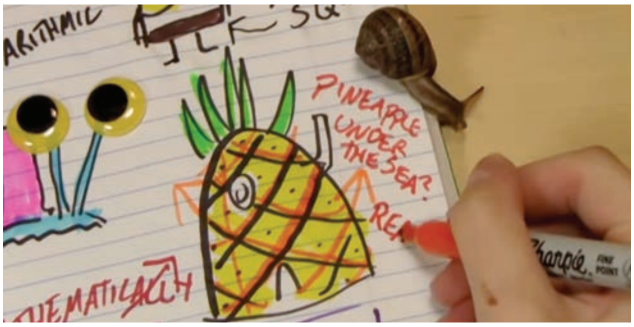


Source: *Critique and Descriptive Feedback: The Story of Austin's Butterfly*, Ron Berger at Presumpscot Elementary, Portland, ME https://www.google.com.au/webhp?sourceid=chrome-instant&ion=1&espv=2&ie=UTF-8#q=austin%27s%20butterfly%20youtube

Vi Hart's art

Introduce your students to the stimulating work of Vi Hart, an artist, musician and mathematician.

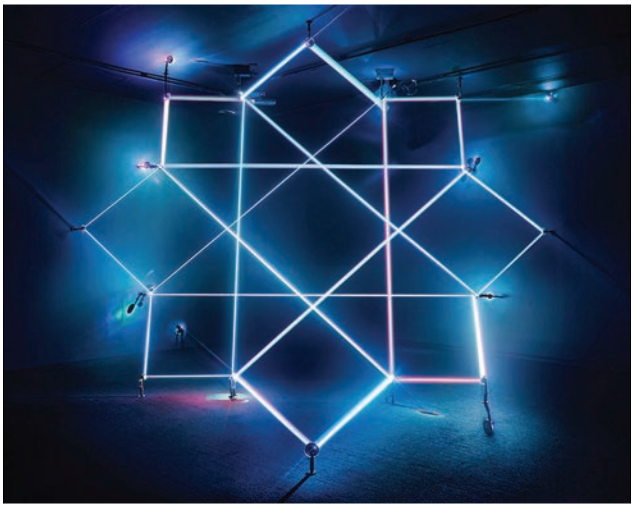
Vi's doodling in mathematics fascinates students with her rapid speech, skilful sketching and unusual explorations into questions such as 'Is SpongeBob SquarePants' home really a pineapple?'



Source: Vi Hart (2012) *Open Letter to Nickelodeon, Re: SpongeBob's Pineapple under the Sea*, accessed at https://www.youtube.com/watch?v=gBxeju8dMho

James Nizam

James is a Canadian artist who has created intriguing stacking sculptures but also some interesting geometric light sculptures.



Source: Tracy Tonnu, *Visual News*, 2016 – Image James Nizam: https://www.visualnews.com/2016/05/01/light-sculptures/

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–3D)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

At this stage of development, students can develop an understanding of **what congruence is through visual recognition when generating and matching congruent shapes**. When teachers provide opportunities for students to **identify and describe the properties of congruent shapes, they require their students to generalise from the classifications they have made**. Telling students **properties** removes this natural opportunity for students to make conjectures and verify connections that they notice.

Teachers can support students to **identify properties of congruence** by asking questions as described in the **Understanding** proficiency. **What patterns/connections/relationships can you see?** The intent of this question is to promote learning design that intentionally plans for students to develop a disposition towards looking for patterns, connections and relationships.

For example, no amount of reasoning will lead my students to **create the names for the congruency tests** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged with questions that will result in them identifying **the minimum information to identify a particular triangle**, so I don't need to instruct that information.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true, or not, really depends on what we have established as our goal. If our goal is to have students recall and apply a particular rule, or theorem during the current unit of work, then it probably is quicker to tell them the rule and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the rule is a false economy of time. When we challenge our students to *establish* a theorem, we are modelling that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Clones and mutants Students define congruence of plane shapes using transformations	ACMMG200 ◆
Example 2A: Identification check – Is it the same triangle? Students develop the conditions for congruence of triangles	ACMMG201 ◆
Example 2B: Congruency checks (SAS) Students develop the conditions for congruence of triangles	ACMMG201 ◆
Example 2C: Congruency checks (AAcorrS) Students develop the conditions for congruence of triangles	ACMMG201 ◆
Example 2D: Congruency checks (RHS) Students develop the conditions for congruence of triangles	ACMMG201 ◆
Example 3A: Proof using congruent triangles Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning	ACMMG202 ◆ ◆
Example 3B: Consider a kite Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning	ACMMG202 ◆ ◆
Example 3C: Consider a parallelogram Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning	ACMMG202 ◆ ◆
Example 3D: Consider an isosceles trapezium Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning	ACMMG202 ◆ ◆

Example 1: Clones and mutants

ACMMG200 ◆

Students define congruence of plane shapes using transformations.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

Instead of ***telling*** students about congruence, we can challenge students to recognise the relationships between congruent shapes for themselves, by ***asking*** questions.

Ask students to design their own simple plane shape and cut it out of card. You may wish to place restrictions on the shape such as it must fit inside a 20 cm square, have straight sides, or a given number of sides.

Ask them to make at least 4 different 'clones' of their shape by rotating, reflecting, translating, or a combination of these transformations, tracing the image and cutting it out.

- *Why do you think they might be referred to as 'clones'? In what sense might these shapes be considered to be 'genetically identical individuals'?*

Students at this stage are familiar with the transformations of rotation (turn), reflection (flip) or translation (slide). Teachers can challenge their students to adapt the known to the unknown, by asking questions such as:

- *What do you notice when you rotate, reflect, translate it? What do you think changes/remains the same about the shape?*

If the gene that determines size of shapes in our classroom mutates due to radiation, create a 'mutant' of your shape.

- *How will you change the size without changing other features of the shape? What other features do you need to keep the same?*

Students will quite often be very creative in the methods they use. If their shape is complex and they are finding it challenging, support them through questioning to halve, or double the lengths of the sides but keep the orientation the same by making sure the mutant sides are parallel to the original shape.

Place all the students' shapes in a box and get each one in turn, to draw out 5 and sort them into groups on the board, explaining and demonstrating why they think they belong together as 'clones' (congruent, or whether they are 'mutants' (in this case, similar).

'Clones' is not a mathematical term. Mathematicians refer to these as **congruent** shapes.

- *What does congruent mean? How could we find out?*

These shapes that you have grouped together as clones, mathematicians describe as **congruent** shapes. Ask students:

- *How would you describe congruent shapes to a Year 7 student? How could they check if shapes were 'congruent'?*

Check a maths dictionary for a definition of congruent. Ask students:

- *How is this definition the same, or different to yours?*

Example 2A: Identification check – Is this the same triangle?

ACMMG201 ◆

Students develop the conditions for congruence of triangles.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the four different ways of establishing whether triangles are congruent, we can challenge students to recognise the minimum conditions for themselves, by ***asking*** questions.

Links to authentic contexts

Using examples/analogies from real life situations engages students and can contextualise the purpose of a mathematical process. Determining the minimum requirements for two triangles to be congruent can be related to the '100 points' identification check used by the Australian Government.

Determining the minimum requirements for establishing congruent triangles can be related to the '100 points' identification check that the Australian Government uses to reduce fraud in financial transactions through people stealing someone else's identity. People can select from a range of different forms of identification such as a passport, birth certificate, or bank account information, but they must have a total of 100 points. Students can research this as an introduction to the task and as a connection to Civics and Citizenship.

Triangles have 3 angles and 3 sides. Do we have to check all 6 measures to be sure that two triangles are the same? What are the minimum requirements to prove that they are identical? (The equivalent of 100 point check.)

Provide opportunities for **authentic creation of knowledge**

This task can be simplified by doing the activity with only one triangle at a time, beginning with Triangle 3 only. However, it is also important that students learn to deal with uncertainty and experience that not all questions will be neatly resolved in one lesson. Conceptual understanding is also built from knowing 'what it isn't' as well as 'what it is'. So for this task, it would be appropriate to introduce all 4 triangles at the beginning and explore and resolve each situation in turn over a longer period of time.

Display 4 triangles (drawn to scale), as below, on the board:

Triangle 1: 4 cm, 3 cm and 35°
Triangle 2: 35°, 45° and 7 cm
Triangle 3: 5 cm, 8 cm and 11 cm
Triangle 4: 90°, 7 cm and 5 cm

Give 3 pieces of information for each triangle and ask:

- *Is this enough information to identify these triangles or is it possible to draw other triangles with this information that are different?* (A fraud! This is the same as saying, that not every triangle drawn with 4 cm, 3 cm and 35° is identical.)
- *Can you draw a triangle that has the same information as any one of these triangles but is not identical to it?*

Students can choose any of the triangles, or may share the tasks in their groups or between groups. If students are unsure how to construct the triangles, encourage them to find a student who can help them in the task. This is an opportunity for students to practise geometric construction using a protractor and a compass.

If students are not fluent in these skills and this is likely to interfere with their exploration, you can facilitate the investigation using prepared manipulatives of strips of paper of the given lengths and angles of the given measures which they can combine and manipulate to form triangles. If students believe they have created a different triangle they can cut it out, and use it to convince the class that it is different by comparing it to the triangle on the board.

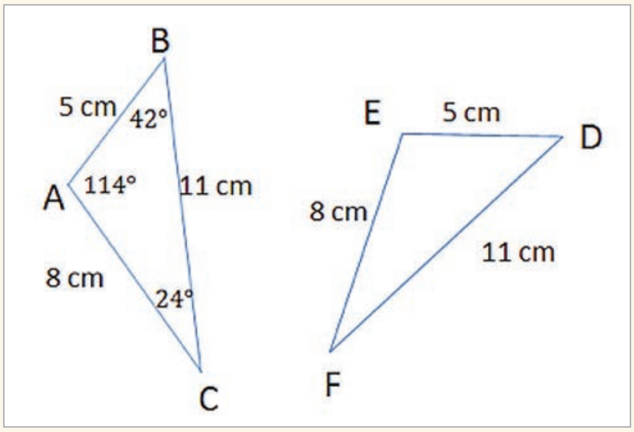
Share the learning:

- *What have we learnt?*
- *Was the type of information we were given the same for each triangle? Does that make a difference?*
- *Was there any information for which we could only draw one possible triangle?*

Only the lengths of the 3 sides is sufficient information to deduce that two triangles would be identical (congruent).

The connection that needs to be made here is if we know that the lengths of the 3 sides are the same for both triangles then we know they are identical (congruent) then we can deduce that the 3 other measurements of the triangles we didn't know before (in this case the angles) must be the same without even checking. It is like getting 'three for free'.

- *If I was to mark the measures of the angles on my triangle, what do you think you might be able to say about yours? Why? How could you check?*



- *What do you notice? What can you deduce from this information? Convince me.*

As teachers, we can take this opportunity to model for students how what they are thinking can be recorded in a short but precise way using mathematical language that all mathematicians will understand.

Construction of knowledge: building a bridge between their reasoning and formal mathematical language

When students have applied their conceptual understanding to solve a problem:

- ask them to explain their thinking verbally
- scaffold how this could be recorded using mathematical symbols in a precise and accurate way.

This is an example of how to scaffold the connection between the students' reasoning and a more mathematical way of recording their thinking. While this is an important connection to be made, it must not be considered more important than conceptual understanding and being able to explain the concept in a way that makes sense to them. Encourage and reward the use of mathematical symbols but require all students to justify their thinking even if they need to write this in a worded response. This is time consuming and, in most cases, is sufficient motivation for them to adopt at least some of the mathematical language which simplifies their recording.

- *What did you say first?* (The three sides in both triangles are the same.)
- *How could we record that? Any ideas? How about AB=ED? Is that enough? What did you say next?* (If the sides are the same, the triangles have to be exactly the same, ie congruent.)

AB=DE= 5
BC=EF= 8
CA=FD=11

- *What do you think this line of mathematical symbols means?*

$\Delta\, ABC \cong \Delta\, DEF$ SSS

What does this symbol in the middle look like? Are the triangles really equal? (≅ means 'is congruent to'.) *Why could you say this?* (Because if all three sides are the same, they must be the same triangle.)

- (Mathematicians would write SSS.) *Does that make sense to you? Why? When you had convinced me that the triangles were the same, you then went on to deduce something that wasn't given.* (If the triangles are the same, I know that the angles have to be the same.)
- *But how do you know which ones are which?* (You match them up. The one that is opposite the 11 cm has to be 114°. The one that is opposite the 5 cm has to be 24° and the one that is opposite the 8 cm has to be 42°.)
- *What do you think the mathematical reasoning shown below means? Could you write it in words?*

Hence:

$\angle B = \angle D = 42°$ corr $\angle$s in $\cong \Delta$s
$\angle C = \angle F = 24°$ corr $\angle$s in $\cong \Delta$s
$\angle A = \angle E = 114°$ corr $\angle$s in $\cong \Delta$s

ie Corresponding angles in congruent triangles

Developing a community of learners: We all speak the same language

To convince students about the universal language of mathematics, get some students to take the solution you have written to other maths teachers in the school or senior students and record them as they read it out in words. Students are quite often amazed by the similarity of the interpretations.

Example 2B: Congruency checks (SAS)

ACMMG201 ◆

Students develop the conditions for congruence of triangles.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the four different ways of establishing whether triangles are congruent, we can challenge students to recognise the minimum conditions for themselves, by ***asking*** questions.

Ask students, 'What about the other three triangles?'

Share the learning:

- *What have we learnt?* (SSS congruency test.)
- *Was the type of information we were given the same for each triangle?* ('What about the other three triangles?')
- *Was there any information for which we could only draw one possible triangle?*

For **Triangle 1**, it is possible to construct different triangles with these three pieces of information – **4 cm, 3 cm and 35°**. Ask students:

- *Is there an extra condition you could make when you use these three pieces of information so that there is only one possible triangle that could be drawn?* (If the angle had to be in between the two given sides, the first drawn triangle would be the only possibility.)

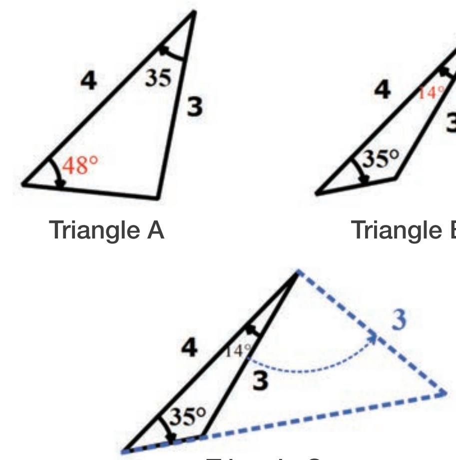
For students that need extra support:

- *What about if you restrict where the angles were in relation to the sides?*
- (Besides SSS, mathematicians use SAS and AAcorrS and RHS.) *What might these things mean?*

From Triangle 1, students might realise that **the angle has to be between the two sides** to be the same as the triangle on the board and this can be connected to SAS terminology. (SAS)

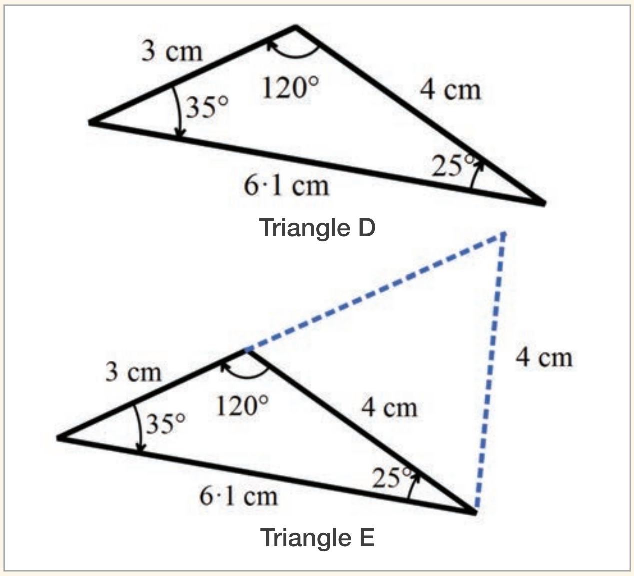
Encourage them to convince themselves and others that there is only one triangle that can be drawn with these sets of conditions.

We need to draw out from this discussion that the triangles that students have drawn that meet these extra conditions are congruent and the others aren't.



Triangle A is the only triangle that can be drawn if the 35° angle must be placed between the sides with length 4 cm and 3 cm.

If the 35° angle does not need to be between the sides of length 3 cm and 4 cm, then two possible triangles can be drawn. If the side of length 3 cm in Triangle B was moved to the position as indicated by the blue dotted line, it would create a third and different triangle, Triangle C, which can also be described as having sides of 4 cm, 3 cm and an angle of 35°.



Triangle D is another possible triangle that can be drawn. Ask students:

- *How might you change this triangle in a similar way, to create a fifth and different triangle which has also sides of 4 cm, 3 cm and an angle of 35°?* (The side of 4 cm could be re-positioned to create Triangle E. The 120° angle would become 60°.)

The students have used only three of the six characteristics (3 sides and 3 angles) of the triangles to prove they are identical. The next stage is for students to realise that now they have established that the two triangles are congruent they can deduce that the other three, previously unknown measurements must be the same as well, giving us 3 new bits of information about the triangle. It is like getting 'three for free'. Ask students:

- *If I was to mark the measures of the other angle and the unmarked side on my triangle, what do you think you might be able to say about yours? Why? How could you check?*
- *How might you write an explanation for another Year 8 student to explain your reasoning, either in words, or using mathematical language as we did with the SSS?*

As teachers we can take this opportunity to model for students how what they are thinking can be recorded in a short but precise way using mathematical language that all mathematicians will understand.

Construction of knowledge: building a bridge between their reasoning and formal mathematical language

When students have applied their conceptual understanding to solve a problem:

- ask them to explain their thinking verbally
- scaffold how this could be recorded using mathematical symbols in a precise and accurate way.

This is an example of how to scaffold the connection between the students' reasoning and a more mathematical way of recording their thinking. While this is an important connection to be made, it must not be considered more important than conceptual understanding and being able to explain the concept in a way that makes sense to them. Encourage and reward the use of mathematical symbols but require all students to justify their thinking even if they need to write this in a worded response. This is time consuming and, in most cases, is sufficient motivation for them to adopt at least some of the mathematical language which simplifies their recording.

Example 2C: Congruency checks (AAcorrS)

ACMMG201 ◆

Students develop the conditions for congruence of triangles.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the four different ways of establishing whether triangles are congruent, we can challenge students to recognise the minimum conditions for themselves, by ***asking*** questions.

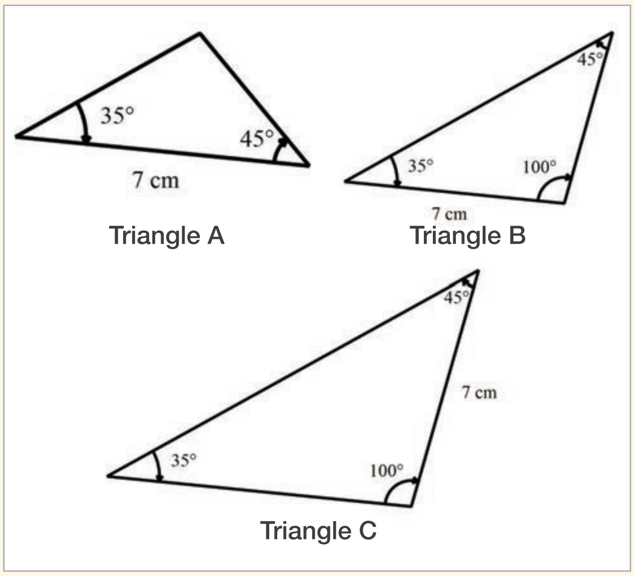
Ask students, 'What about the other two triangles?'

Share the learning:

- *What have we learnt?* (SSS and SAS congruency tests)
- *Was the type of information we were given the same for each triangle?* ('What about the other two triangles?')
- *Was there any information for which we could only draw one possible triangle?*

Triangle 2: 35°, 45° and 7 cm

- *What do you notice about the different triangles we were able to draw using the information about Triangle 2?*



Triangle A is the only triangle that can be drawn if the 7 cm is between the 35° and 45° angles.

If the 7 cm side is not in the corresponding position, that is, between the 35° and 45° angles, then two further possible triangles can be drawn. Ask students:

- *What do you notice about Triangle B and Triangle C?* (They are similar triangles because all their angles are the same but they are not the same size.)

From Triangle 2, the two angles being the same for both triangles means that the third angle will have to be too, so that they both add up to 180°. Having all three angles the same will make the triangles similar. To be exactly the same, **sides that match, or correspond, have to be equal** so the side between the 35° and 45° angles must be 7 cm to be congruent to the one on the board (AAcorrS – Angle/Angle/Corresponding Side).

The students have used only three of the six characteristics (3 sides and 3 angles) of the triangles to prove they are identical (two angles and a side were given to be the same). The next stage is for students to realise that now they have established that the two triangles are **congruent**, they can deduce that the other three characteristics (an angle and two sides) must be equal. These previously unknown measurements must be the same as well, giving us 3 new bits of information about the triangle. It is like getting 'three for free'. Ask students:

- *If I was to mark the measures of two other sides on my triangle, what do you think you might be able to say about yours? Why? How could you check?*
- *How might you write an explanation for another Year 8 student to explain your reasoning, either in words, or using mathematical language as we did with the SSS and SAS?*

Example 2D: Congruency checks (RHS)

ACMMG201 ◆

Students develop the conditions for congruence of triangles.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

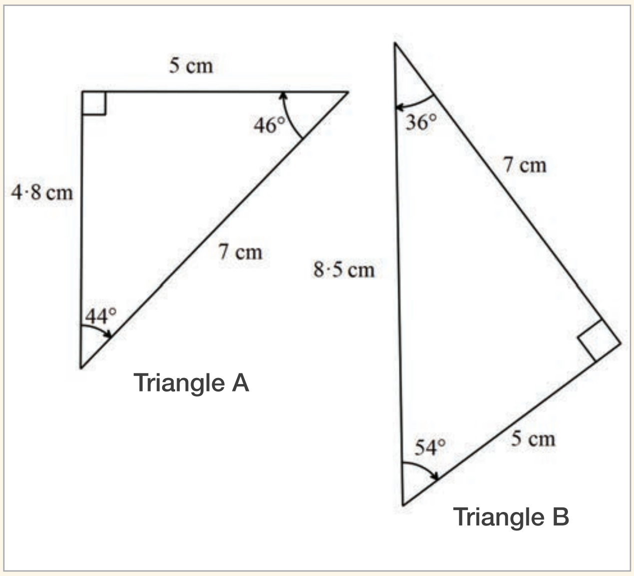
Instead of ***telling*** students about the four different ways of establishing whether triangles are congruent, we can challenge students to recognise the minimum conditions for themselves, by ***asking*** questions.

Ask students, 'What about the final triangle?'

Triangle 4: 90°, 7 cm and 5 cm

Share the learning:

- *What have we learnt?* (SSS, SAS and AAcorrS congruency tests)
- *Triangle 4 information is 2 sides and an angle. How was the type of information we were given the same as for Triangle 1 and how is it different?*
- *Was there any information for which we could only draw one possible triangle?*



Triangle A is the only right angled triangle that can be drawn if the 7 cm is the hypotenuse and one of the sides is 5 cm.

If the longer of the two given sides is not the hypotenuse as in Triangle B, then the right angled triangles will not be the same (congruent).

From Triangle 4, this is in fact the same type of information as Triangle 1, knowing the measurement of 2 sides and an angle. However, because the triangle is right angled the angle does not have to be in between the two sides. As long as the longest side is the hypotenuse, it will be the same as the triangle on the board (RHS – Right angle/Hypotenuse/Side).

Provide students with the opportunity to make deductions that follow from this congruency test and encourage them to explain their reasoning in words and/or mathematical language.

It is also important to consider whether there may be other tests which have not been considered by looking at these 4 triangles. Ask students:

- *Have you covered all possibilities for three bits of information involving sides and angles?*

The possible combinations would be 3 sides (SSS), 2 sides and an angle (SAS, RHS) and 2 angles and a side (AAcorrS), but what about 3 angles? Three angles will only establish similarity which can be related to the 'mutant' shapes in Example 1: Clones and mutants.

Example 3A: Proof using congruent triangles

ACMMG202 ◆ ◆

Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

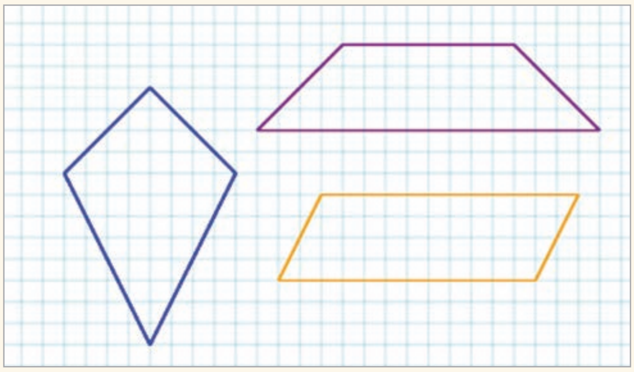
Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the properties of quadrilaterals, we can challenge students to recognise these for themselves, by ***asking*** questions.

Students have established that they are able to identify congruent triangles from 3 given facts. They have also been made aware of the inferences that can be drawn once they have established that they are congruent (identical) without measuring or being given the information. While we as teachers tell them that this is going to be very useful, it is much more powerful to support students in using this new knowledge to create new knowledge. It is also an opportunity for students to **prove** conjectures which they have made based on data they have collected to **identify** or **verify** properties of plane shapes in measurement and geometry.



- *Can you recognise and name these quadrilaterals? Can you list as many properties as possible that you know about each shape?*
- *How do you know these properties are true? Convince me. Prove it!*
- *What is the difference between proving and verifying?*

Proving means you have convinced other mathematicians that it will be true all the time and they are so convinced they do not need to check any other cases.

Verifying is when you are checking some cases to see if they work. If they don't work then you have disproved the theory or conjecture and have shown it does not work all the time because it didn't work once. If it does work, you have not proved it as you have not checked every possible case which is not always possible anyway.

When discussing these ideas with students it is important to refer back to learning experiences that they have had previously. '*Remember when we collected data about the connection between the circumference of a circle and its diameter. We verified (checked) that C = π x d but we did not prove it.*' (DECD, 2017, *Using units of measurement: Year 8 – Mathematics Conceptual Narrative*, Example 6)

Example 3B: Consider a kite

ACMMG202 ◆ ◆

Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

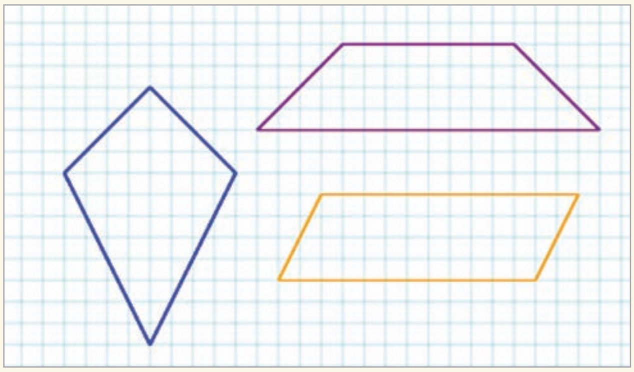
Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the properties of quadrilaterals, we can challenge students to recognise these for themselves, by ***asking*** questions.



- *Which quadrilateral is a kite? What is your definition of a kite? How does that match with our maths dictionary?*
- *Given that it needs to have two pairs of adjacent sides equal, is this shape a kite? Convince me.* (As this has been drawn on a grid, students can deduce that adjacent sides are equal. They can calculate the length using Pythagoras's Theorem but they can also argue that $a^2 + b^2$ has the same value for both triangles.)
- *What additional properties did you list for this kite? Why did you think this would be the case? Can you convince me? Prove it.* (Sometimes students say because 'it is obvious', 'it looks like it', or because 'I measured it'.)
- *How can we be sure that it is always the case for every possible kite we could draw?*
- *How many kites would we have to draw to be sure it would be true?*

As teachers, we must continue to challenge students' beliefs that verification is sufficient to prove conjectures. A formal proof must convince us that it would work for all kites so it must be a general discussion and cannot use specific measurements.

At this stage of development, students often have difficulty distinguishing between:

- the information you can use as truth
- things that look like they are true
- what you are trying to prove.

As teachers, we should support students to develop a habit of clarifying these three things before they start to formulate an argument or proof for the general case.

- *What do you know is true?* (Only what you are given or what you can deduce. If I know it is a kite, I know the adjacent sides are equal even if I don't know the actual length.) *How could you show that on the diagram?*
- *What do you think might be true but you don't know for sure?* (I am pretty sure one pair of angles is the same. They look it but they aren't marked/labelled the same and I have not been told how big they are.)
- *What do you want to prove?* (I would like to prove that those angles are equal.)

We could use what we have learnt about congruent triangles to help us think about this. Ask students:

- *What is it about this problem that might make me think to use congruent triangles?* (The kite can be divided into triangles. I have some information about the 6 measurements of the triangle but not all. Using the idea about congruent triangles, I might be able to use the 3 particular facts that the two triangles have in common, to deduce that the remaining 3 must be the same even if I did not know them before.)

Ask students to draw their own kites on graph paper, using the grids to make sure adjacent sides are equal in length. Then ask:

- *Can you identify any pairs of congruent triangles in a kite? How do you know they are congruent?*

Even if students can quickly identify SSS, ask them to physically demonstrate that the three sides of the two triangles are the same by tracing, or cutting them out and matching them. Students commonly have difficulty in identifying shared or common sides as being the third equal side.

Putting the cut outs of the congruent triangles together, physically matching the sides to form the diagonal of the kite supports students in visualising this. Ask students:

- *If you know the triangles are congruent, what can you deduce about the triangles and hence about the kite? Is this a property you knew before, or a new one?*

Students can now use the physical models they made earlier to match up the equal angles.

Students may wish to check their findings by measuring the angles of their own kite but remind them that they are verifying their findings not proving that it is true for all kites. Ask students:

- *Can you write an argument to convince other Year 8 students who understand congruent triangles that this new property will be true for all kites?*
- *Can you identify a different pair of congruent triangles? How can you use this pair to deduce more about the properties of a kite?*

Mathematical protocols and representations: Linked to conceptual understanding

Whenever students develop the understanding themselves, there will come a stage when they need to be made aware of the way other mathematicians record their thinking. As teachers we must facilitate a transition period where a student's current understanding is connected to mathematical protocols in a blended fashion. If this is a sudden and irreversible process, it creates a disjunction where the benefit of students creating their own knowledge will be lost. To facilitate the transitioning, requires all students to write their reasoning in words when they solve a problem. Model the universally used mathematical protocols for the proof and make explicit links to the student's written argument. At this level it is 'exposure to' rather than 'mastery of' this type of communication.

Choose other shapes and investigate how you could use congruent triangles to prove or establish properties of the shape.

Allowing students to choose which shape they investigate differentiates their learning and provides an opportunity where they can share and compare their learning with others. Students can also be challenged to try the third shape, or another quadrilateral, or polygon, of their choosing to investigate. As teachers, we tend to think students should complete the same problems but it is more important that they engage meaningfully with the intended learning.

Example 3C: Consider a parallelogram

ACMMG202

Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

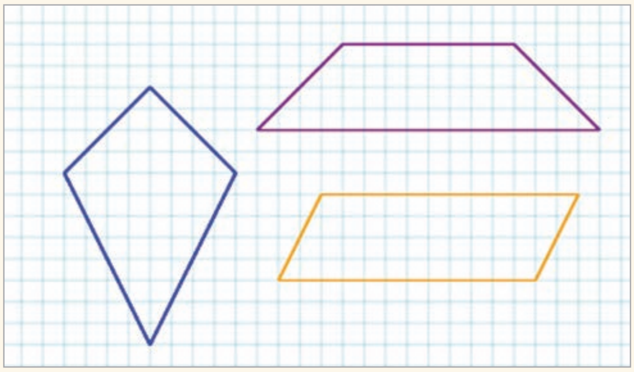
Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the properties of quadrilaterals, we can challenge students to recognise these for themselves, by ***asking*** questions.



- *Which quadrilateral is a parallelogram? What is your definition of a parallelogram? How does that match with our maths dictionary?*
- *Given that it needs to have two pairs of opposite sides parallel, is this shape a parallelogram? Convince me.* (As this has been drawn on a grid, students can deduce that the opposite sides are parallel because they are always the same distance apart and have the same slope.)
- *What additional properties did you list for this parallelogram? Why did you think this would be the case? Can you convince me? Prove it.* (Sometimes students say because 'it is obvious', 'it looks like it', or because 'I measured it'.)
- *How can we be sure that it is always the case for every possible parallelogram we could draw?*
- *How many parallelograms would we have to draw to be sure it would be true?* (Again, we challenge students' beliefs that if it works from all the ones we check it must be true. Verification is not proof.)

- *What do you know is true?* (Only what you are given or what you can deduce. If I know it is a parallelogram, I know the opposite sides are parallel.) *How could you show that on the diagram?*
- *What do you think might be true but you don't know for sure?* (I am pretty sure the opposite sides are the same length and opposite angles are equal. They look it but they aren't marked, or labelled the same and I have not been told how big they are.)
- *What do you want to prove?* (I would like to prove that the opposite sides are the same/opposite angles are equal.)

In this situation we do not know any information about the angles/the sides. The only thing we know is that the lines are parallel (given). Ask students:

- *What do we know about parallel lines?* (What can we deduce?)

Year 7 students have explored the angles on parallel lines and this is an opportunity to revisit corresponding, alternate and co-interior angles. Ask students:

- *What is it about this problem that might make me think to use congruent triangles?*
- *Can you identify any pairs of congruent triangles in a parallelogram? How do you know they are congruent?*
- *Can we use our knowledge about parallel lines to deduce something about the triangles?*

At this level of development not all students can reason in a general case. Being able to cut up a parallelogram that does not have its measurements labelled, and use it to verify that alternate angles will be equal helps students to accept that this would always be possible.

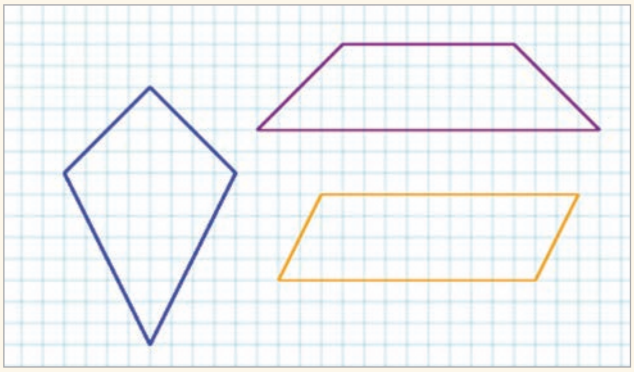
Quite often students do not notice that the line which is common is a side in both triangles and so would be equal sides. Cutting the parallelogram along the diagonal and separating them so the two triangles can be seen, supports students in making this connection and provides a visual image for students to recall in later situations where common sides exist.

Example 3D: Consider an isosceles trapezium

ACMMG202 ◆ ◆
Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

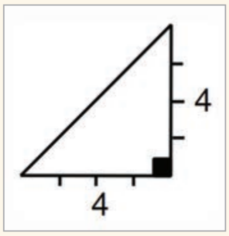
Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about the properties of quadrilaterals, we can challenge students to recognise these for themselves, by ***asking*** questions.



- *Which quadrilateral is an isosceles trapezium? What is your definition of an isosceles trapezium? How does that match with our maths dictionary?*
- *An isosceles trapezium needs to have:*
  - *one pair of opposite sides parallel and,*
  - *the sides that are not parallel, equal in length.*

  So, is this shape an isosceles trapezium?

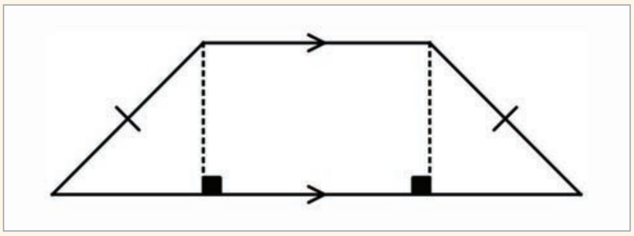
  

  (As this has been drawn on a grid, students can deduce that the opposite sides are parallel because they are always the same distance apart. The non-parallel sides are both equal in length because they are the hypotenuse of the same right angled triangle.)
- *What additional properties did you list for this isosceles trapezium? Why did you think this would be the case? Can you convince me? Prove it.* (Sometimes students say because ‘it is obvious’, ‘it looks like it’, or because ‘I measured it’.)
- *How can we be sure that it is always the case for every possible isosceles trapezium we could draw?*
- *How many isosceles trapezia would we have to draw to be sure it would be true?* (Again, we challenge students’ beliefs that if it works from all the ones we check it must be true. Verification is not proof.)

- *What do you know is true?* (Only what you are given or what you can deduce. If I know it is a trapezium, I know that one pair opposite sides are parallel. If it is an isosceles trapezium, I know that the non-parallel sides are the same length.) *How could you show that on the diagram?*
- *What do you think might be true but you don’t know for sure?* (I am pretty sure that in an isosceles trapezium, the top adjacent angles are equal and the bottom adjacent ones are too. They look it but they aren’t marked, or labelled the same and I have not been told how big they are.)
- *What do you want to prove?* (I would like to prove that two pair of the adjacent angles are equal.)

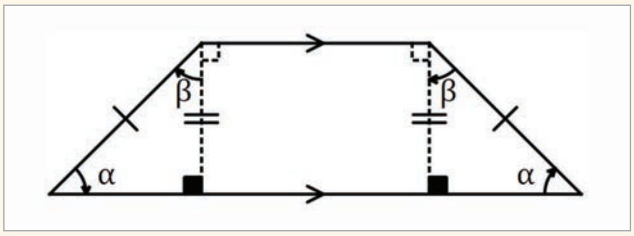
In this situation, we do not know any information about the angles. The only thing we know about an isosceles trapezium is that the lines are parallel (given) and that one pair of sides is the same length. Ask students:

- *What do we know about parallel lines?* (What can we deduce?)



- *If I label the trapezium as shown, what information can you read from the markings?*
- *If you know the lines are parallel, what can you deduce about the distance between them?*
- *What is it about this problem that might make me think to use congruent triangles?*

- *Can you identify any pairs of congruent triangles in an isosceles trapezium? How do you know they are congruent?* (The triangles are congruent because of RHS. It can be deduced that the other two angles in the triangles are the same because they are corresponding angles in congruent triangles.)



(Hence the base angles are equal and the top pair are both $\beta + 90°$.)

At this level of development not all students can reason in a general case. Being able to cut up a trapezium that is not labelled with its measurements, and use it to verify that the vertical heights between the parallel lines will be equal, and superimpose the two right angled triangles, will help students to accept that this would always be possible. Ask students:

- *Would this work for all trapezia?*

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 4–6)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling, some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 4: Irregular tetrahedron Students visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects (Visualise 2D shapes and 3D objects) Students define congruence of plane shapes using transformations	NC LEVEL 5 ◆ ◆ ACMMG200 ◆
Example 5: Hex Students define congruence of plane shapes using transformations	ACMMG200 ◆
Example 6: An engaging animation Students calculate the surface area and volume of cylinders	ACMMG217 ◆

Example 4: Irregular tetrahedron

NC LEVEL 5 ◆ ◆

End Year 8: Students visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects (Visualise 2D shapes and 3D objects).

ACMMG200 ◆

Students define congruence of plane shapes using transformations.

Questions from the BitL tool

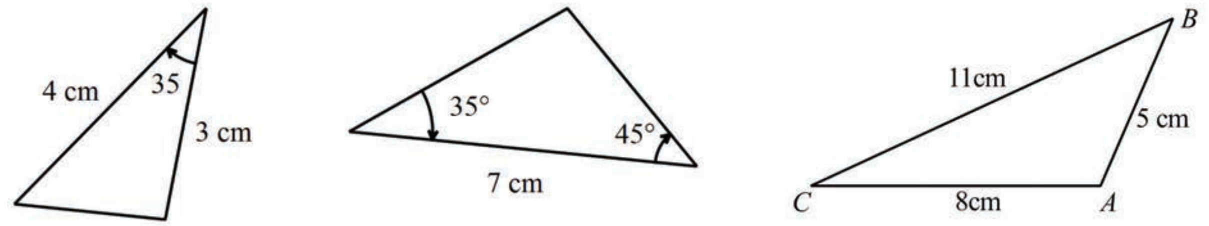
Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students, the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.



Triangle 1 Triangle 2 Triangle 3

This activity requires students to design a net for an irregular tetrahedron for which the faces have the same perimeter. We should ensure students have the opportunity to work with shapes other than those which are regular and symmetrical. When students design the net themselves rather than using one we provide, they are required to think more deeply about the necessary dimensions of the 2D shape from which it is constructed. They must understand the need for the adjacent sides of the triangular faces to be the same length. These experiences support conceptual understanding of surface areas of solids which, while mentioned in the numeracy continuum and year level description, is not explicit in the content descriptions.

Provide three different scalene triangles, for groups to use as the bases of their tetrahedron (you may wish to use Triangles 1, 2 and 3 from the *From tell to ask* examples 2B, 2C and 2D, as above). This gives students the chance to compare and notice that although they all have the same base the tetrahedrons are different. Having three different bases suggests that they are not special bases and helps students generalise as well as compare their results.

Activity

Construct an irregular tetrahedron using one of the scalene triangles as your base so that the perimeter of all the faces is the same. Ask students:

- *What do you notice about the faces?*
- *Can you prove that this would always be the case?*

Interpret

What are you being asked to find out or show? What information will be useful? What do you know about the faces? How can you make sure they have the same perimeter? (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Will you draw some diagrams? How can you be sure the net will work? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

How could you use your tetrahedron so that you can see patterns or connections between the faces? Are there measurements you could make? How could you check your ideas? Have you verified your conjecture? How could you prove it?

Reflect

Did other people do it differently? Is there something that you would do differently next time? (Students can show the faces of their tetrahedron are congruent by superposing them on each other. They can also verify it by measuring the sides and using SSS. A formal proof requires them to make a general argument which will be based on the fact that the perimeters are the same.)

Example 5: Hex

ACMMG200 ◆
Students define congruence of plane shapes using transformations.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students, the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

In this activity from the NRICH website, students are asked to reform a regular hexagon into 3 congruent hexagons using a range of congruent triangles.

The link to this problem on the NRICH site is: http://nrich.maths.org/795

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Example 6: An engaging animation

ACMMG217 ◆
Students calculate the surface area and volume of cylinders.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students, the information they'll need and the steps they should take, we can ***ask*** them to identify the information they'll need and the steps they could take.

This animation activity from the NRICH website—*Notes on a triangle*—could be used as a stimulus for a physical re-enactment using congruent triangles.

The link to this problem on the NRICH site is: http://nrich.maths.org/5920

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Connections between ‘geometric reasoning’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use geometric reasoning as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Years 7–8	
Whilst working with geometric reasoning, connections can be made to:	**How the connection might be made:**
Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries. ACMMG181 (Year 7)	Refer to: **Example 1: Clones and mutants**.
Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal. ACMMG163 (Year 7)	Refer to: **Example 3A: Proof using congruent triangles** **Example 3B: Consider a kite** **Example 3C: Consider a parallelogram** **Example 3D: Consider an isosceles trapezium**.
Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning. ACMMG164 (Year 7)	Refer to: **Example 3D: Consider an isosceles trapezium**.
Classify triangles according to their side and angle properties and describe quadrilaterals. ACMMG165 (Year 7)	Refer to: **Example 3A: Proof using congruent triangles** **Example 3B: Consider a kite** **Example 3C: Consider a parallelogram** **Example 3D: Consider an isosceles trapezium**.

Making connections

We know that when students meet a concept frequently, and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individuals or small groups of students.

‘Geometric reasoning’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with geometric reasoning.

Estimate/measure/compare/identify/classify using geometrical reasoning ◆
In Foundation to Year 2 through the ‘shape’ content descriptions, students sort/recognise/name/describe/draw/classify. In Year 3 to Year 5 students estimate/measure/compare/identify/classify using geometrical reasoning.

Investigate and generalise using geometrical reasoning ◆ ◆
In Year 6 to Year 8 students mostly investigate and solve.

Solve and prove using geometrical reasoning ◆
In Year 9 and Year 10 students mostly solve and prove by applying logical reasoning.

Year level	‘Shape’ content descriptions from the AC: Mathematics
Foundation ◆	Students sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment. ACMMG009
Year 1 ◆	Students recognise and classify familiar two-dimensional shapes and three-dimensional objects using obvious features. ACMMG022
Year 2 ◆	Students describe and draw two-dimensional shapes, with and without digital technologies. ACMMG042
Year 2 ◆	Describe the features of three-dimensional objects. ACMMG043
Year level	**‘Geometric reasoning’ content descriptions from the AC: Mathematics**
Year 3 ◆	Students identify angles as measures of turn and compare angle sizes in everyday situations. ACMMG064
Year 4 ◆	Students compare angles and classify them as equal to, greater than or less than a right angle. ACMMG089
Year 5 ◆	Students estimate, measure and compare angles using degrees. Construct angles using a protractor. ACMMG112
Year 6 ◆ ◆	Students investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Students use results to find unknown angles. ACMMG141
Year 7 ◆	Students identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal. ACMMG163
Year 7 ◆ ◆	Students investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning. ACMMG164
Year 7 ◆ ◆	Students demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral. ACMMG166
Year 7 ◆	Students classify triangles according to their side and angle properties and describe quadrilaterals. ACMMG165
Year 8 ◆	Students define congruence of plane shapes using transformations. ACMMG200
Year 8 ◆	Students develop the conditions for congruence of triangles. ACMMG201
Year 8 ◆ ◆	Students establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning. ACMMG202
Year 9 ◆ ◆	Students use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar. ACMMG220

Year level	‘Geometric reasoning’ content descriptions from the AC: Mathematics *continued*
Year 9 ◆	Students solve problems using ratio and scale factors in similar figures. ACMMG221
Year 10 ◆	Students formulate proofs involving congruent triangles and angle properties. ACMMG243
Year 10 ◆	Students apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes. ACMMG244
Year 10A ◆	Students prove and apply angle and chord properties of circles. ACMMG272

Numeracy continuum: Using spatial reasoning	
End Foundation	Visualise 2D shapes and 3D objects: sort and name simple 2D shapes and 3D objects.
End Year 2	Visualise 2D shapes and 3D objects: identify, sort and describe common 2D shapes and 3D objects.
End Year 4	Visualise 2D shapes and 3D objects: visualise, sort, identify and describe symmetry, shapes and angles in the environment.
End Year 6	Visualise 2D shapes and 3D objects: visualise, sort, describe and compare the features of objects such as prisms and pyramids in the environment.
End Year 8	Visualise 2D shapes and 3D objects: visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects.
End Year 10	Visualise 2D shapes and 3D objects: visualise, describe and analyse the way shapes and objects are combined and positioned in the environment for different purposes.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Each time we refer to one of the NRICH resources we have provided you with a link to that activity.

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Dan Meyer’s blog: 101 questions

http://www.101qs.com

Dan’s blog contains images and short films that can be presented to students along with the question: *What’s the first question that comes to mind?*

A spreadsheet of ***Dan Meyer’s Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

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