Linear and non-linear relationships: Year 8

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together

www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Linear and non-linear relationships' from Year 7 to Year 10A** (see page 22)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with linear and non-linear relationships:

- Plotting and sketching skills and strategies
- Connection between numerical, algebraic and graphical representations and methods
- Strategies for solving equations
- Using digital technologies.

What the Australian Curriculum says about ‘Linear and non-linear relationships’

Content descriptions

Strand | Number and algebra.

Sub-strand | Linear and non-linear relationships.

Year 8 ◆ ◆ | ACMNA193
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

Year 8 ◆ ◆ | ACMNA194
Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.

Year level descriptions

Year 8 ◆ ◆ | Students connect rules for linear relations and their graphs.

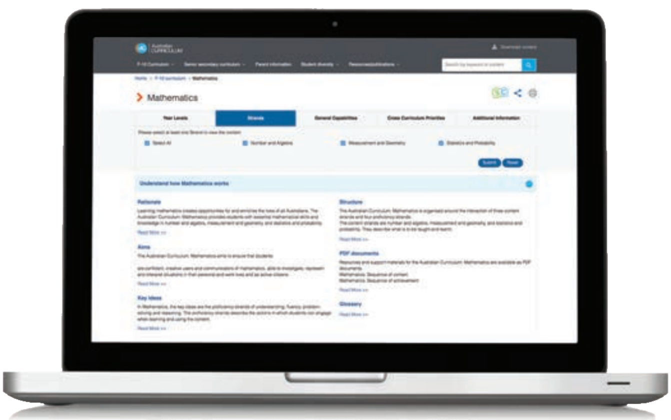
Achievement standards

Year 8 ◆ ◆ | Students solve linear equations and graph linear relationships on the Cartesian plane.

Numeracy continuum

Recognise and use patterns and relationships

End of Year 8 ◆ | Students identify trends using number rules and relationships (Recognise and use patterns and relationships: Linear and non-linear relationships).



Source: ACARA, Australian Curriculum: Mathematics

Working with Linear and non-linear relationships

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 8 ‘Linear and non-linear relationships’

In Year 7 students plot given coordinates and make observations about where points lie on a straight line or otherwise. Students make the step from solving problems that have been expressed as number sentences, to solving simple linear (algebraic) equations.

In Year 8 students continue to plot coordinates on a Cartesian plane, but now they are expected to generate coordinates from linear equations. This will be done with and without digital technologies. Developing this skill enables students to explore the use of graphical techniques as well as algebraic methods for solving simple linear equations.

In Year 9 students continue to solve linear equations graphically, but they develop efficiency through understanding that a linear graph can be sketched from plotting just two points. Students investigate gradients of linear graphs and develop techniques for calculating the distance between two points and the midpoint of lines on a Cartesian plane.

In Year 10 students use their understanding of gradients to solve problems involving parallel and perpendicular lines. They continue to solve linear equations that now may involve simple algebraic fractions. Students transfer their skills from solving linear equations to solving linear inequalities. They apply their ability to plot linear equations (with and without digital technologies) to solve simultaneous equations graphically and explore the use of algebraic technique. Quadratic equations, equations of circles, and exponents are introduced and students explore the connection between the algebraic and graphical representation. At this stage, students are only expected to sketch and solve equations of quadratics.

In Year 10A students sketch parabolas, circles and exponential functions, and solve simple exponential equations. They investigate the features of graphs and develop an understanding of the connection between the graphical and algebraic representation of polynomials so that they are able to sketch a range of curves and describe the features of the curves from looking at their equation. Factorisation of quadratics extends to non-monic quadratic equations.

- Developing linear relationships from practical and real-life situations, supports students in gaining a conceptual understanding that the straight line is representing a relation between two varying quantities. Once this understanding has been established, concepts of coordinates, equations, slope and Y-intercept have some significance to the learner; particularly if they have encountered linear relationships in a range of different contexts.
- As well as providing situations where linear relationships occur, it is valuable to identify non-linear relationships as well, so that learners realise that not all relationships will be linear. In the James Nizam sculpture of the stacking goblets (see Example 8: Stacking sculpture), the number of goblets in each level is a linear relationship (Level 1 has 1 goblet, Level 2 has 2 goblets, etc) but the number of goblets in the entire sculpture as it grows is not (after one level the structure has 1 goblet, after two levels it has 3, after three levels it has 6 …).

Engaging learners

Classroom techniques for teaching Linear and non-linear relationships

Coordinate grid games

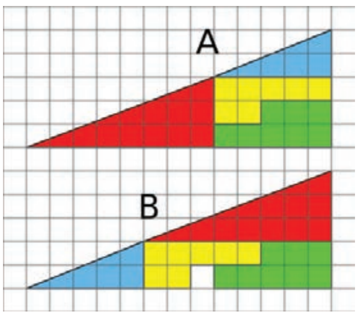
Coordinate grid games can be an engaging way to build skills in coordinate geometry.

Coordinate grid games can be found at: https://www.mathnook.com/math/skill/coordinategridgames.php

Source: *Math Nook [Math Games and More]*

Missing square puzzle

Puzzles, such as the 'Missing square', intrigue students. The solution relates to gradients and straight lines.



When these 4 shapes are rearranged to form what appears to be an identical triangle, an extra square appears. These can be made into a physical puzzle that students can manipulate onto a grid. When they do this they may discover that the hypotenuse is not a straight line on either 'triangle'. For Triangle A, it bows slightly inward and Triangle B, slightly outward, accounting for the extra square.

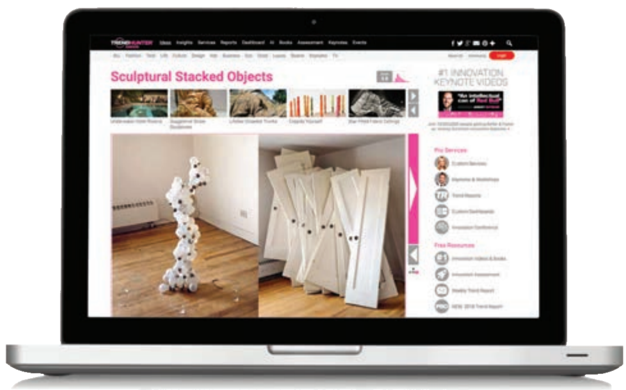
Further information can be found at: https://en.wikipedia.org/wiki/Missing_square_puzzle

Uniform and non-uniform stacking

Stacking is a ready-made context for 'linear and non-linear relationships'.

James Nizam created an intriguing series of stacking sculptures that can be used to prompt questions about the patterns to be found in uniform and non-uniform stacking.

The ***Memorandoms*** photo series can be accessed at: https://www.trendhunter.com/trends/james-nizam-photo-series



Source: *Trendhunter*

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–6)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **write the equation of a line in algebraic form** themselves. They need to receive this information in some way. However, it *is* possible my students can be challenged with questions that will result in them identifying the **relationship between the coordinates**, so I don't need to instruct that information.

At this stage of development, students can **develop an understanding of the patterns and relationships that can exist on a coordinate system**. When teachers provide opportunities for students to **recognise, create and describe relationships between coordinates of points**, they require their students to generalise. Telling students rules and relationships removes this natural opportunity for students to make conjectures and verify and apply connections that they notice. Using questions such as the ones described here, supports teachers to replace 'telling' the students information, with getting students to notice for themselves.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular rule or theorem during the current unit of work, then it probably is quicker to tell them the rule and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the rule is a false economy of time.

When we challenge our students to **establish a theorem**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Bridge building Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.	ACMNA193 ◆ ◆
Example 2: Hexagonal train Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA193 ◆ ◆ ACMNA194 ◆ ◆
Example 3: Cup stack Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA193 ◆ ◆ ACMNA194 ◆ ◆
Example 4: Homework rates Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA193 ◆ ◆ ACMNA194 ◆ ◆
Example 5: Colour-coded points Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA193 ◆ ◆ ACMNA194 ◆ ◆
Example 6: Colour-coded lines Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA193 ◆ ◆ ACMNA194 ◆ ◆

Example 1: Bridge building

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the linear relationship between two variables, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

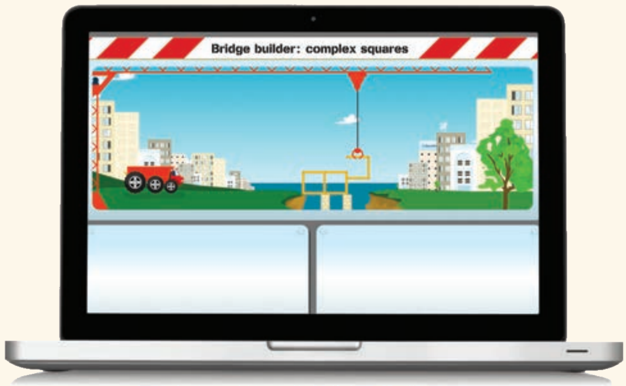
This digital activity from the Scootle website introduces students to linear patterns in a simulated context and is recorded in table and graph form.

Discuss with students:

- *What did you learn from this digital object? Can you generalise?*
- *Convince me. Can you do it another way? In words or a story? Using an equation?*

Use concrete materials in conjunction with digital objects

Using a digital object only and not manipulatives or student-constructed representations, can miss the opportunity to cognitively and emotionally engage the students with the challenge. Once the students have had an opportunity to explore with the digital object, provide them with the opportunity to reflect on and apply their new knowledge.



The Bridge building activity can be accessed at: http://www.scootle.edu.au/ec/viewing/L1925/index.html

Example 2: Hexagonal train

ACMNA193

Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

ACMNA194

Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

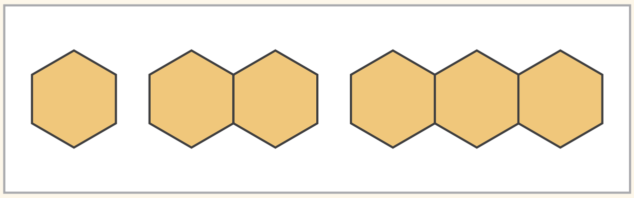
Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of *telling* students about the linear relationship between two variables, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

Begin a discussion by explaining that a series of trains of different lengths, can be made out of hexagonal shapes.



Ask students:

- *What quantities do you notice varying from one train to the next?* (The number of hexagons, the area of the train, the perimeter, the number of edges, the number of vertical edges, the number of corners, the number of joins … etc.)

The two variables I want to know more about are the number of hexagons and the perimeter of the shape (train).

- *What do you think the perimeter of the train made from 10 hexagons might be?*
- *How might you find out?*
- *Convince me. Can you do it another way? Could you use a table? A graph?*

There are many levels of entry for this task. Students can:

- Build the train and count. (It is a common misconception that the perimeter of the second train is 12, which you can encourage students to self-correct by asking them to show you how they determined that. Ask students that have the correct answer as well.)
- Draw a table using values from the first 3 and then continue the pattern for each train by adding on 4 each time, up to the 10th term.

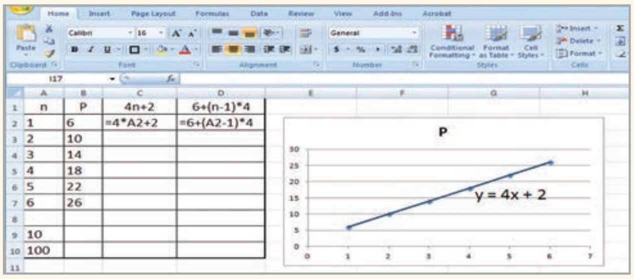
Term	P	
1	6	
2	10	+4
3	14	+4

- Begin the table then realise that it would be 6, with 9 lots of 4 added on. (It is a common misconception that it would be 10 lots of 4. Ask them what they would have thought the 3rd train would have been with that thinking (6 + 3x4), yet it is actually 14, which is 6 + 2x4.)

  t = 1 P = 6 + 0x4
  t = 2 P = 6 + 1x4
  t = 3 P = 6 + 2x4
  …
  t = 10 P = 6 + 9x4

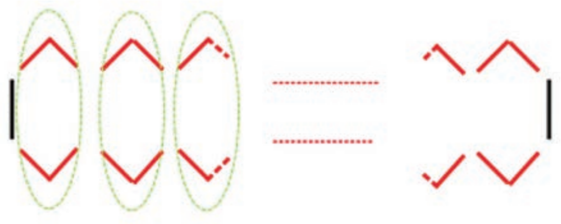
- Rather than do an iterative pattern, students might identify a relationship between the two variables. (The perimeter is 4 times the term, plus 2 for each train (4t+2). Check that this works for the trains you have (4x1 + 2 = 6, 4x2 + 2 = 10, etc.))

Using an Excel spreadsheet, students can use two or more of the expressions as formulae to verify that they have the same values for a large number of train lengths.



They can also draw a chart and fit a linear model to the data, selecting ‘show equation’ to compare this to the rule they found. Ask students:

- *How is this equation the same/different to the one you identified? How might you check?* (While this pattern works for the values in the table, it may not work for all trains. We need to think about **why** this works.)



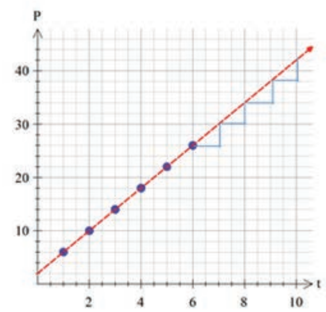
There is one side at the start and one at the end, making 2. Then there is a set of 4 for each hexagon in the train (4t+2).

Figure 1

There are multiple ways to describe the pattern. This is explored further in the Patterns and algebra: Year 8 narrative examples.)

- *Can you generalise? Can you write a description for a Year 7 student to explain how you would determine the perimeter of a train of 100 hexagons, or a hexagon train of any length?*
- *Is it possible to have a train with a perimeter of 2016? Convince me.*

Some Year 8 students recorded their thinking on this problem.



A number of hexagons each have 6 sides but you lose 2 sides when you join them together. There are always one less join than hexagons.

Begin with the 6 sides, pull one of the sides out and add 4 sides, (2 at the top and 2 at the bottom) to create another hexagon in the train.

We plotted our answers from the table for the perimeter, against the number of hexagons. They were all in a straight line. As the number of hexagons increased by 1, the perimeter increased by 4, so we kept the steps going up to 10 to see that the perimeter would be 42.

Figure 2

Discuss the following with your class:

- *In what ways have the students used the graphs to demonstrate their thinking?*
- *Does this make sense to you? What feedback would you give?* (They have identified the pattern and extended it accurately. It would be important to discuss whether they think this pattern would continue, by referring back to the building of the train.)
- *What are some good features of the graph? What could be improved?* (The graph is neat, the axes labelled and the scale appropriate, so that 4 steps up is exactly 2 squares. Showing the steps on the graph helps the reader to understand the explanation.)
- *Why have the students used a dotted line?* (They wanted to show the linear pattern, but they realised if they used a solid line people might think that it was possible to have fractions of hexagons in the train. In maths, if you draw a dotted line the boundary is not included in the region.)
- *What is the answer when t=0? Does this make sense?* (From the graph, P=2. The explanation in Figure 1 suggests that you have the 2 vertical edges, but have not yet made a hexagon. In this case you would not have a perimeter either, so the rule doesn't work unless ***t*** is a positive integer. It is common in practical situations that you have to restrict the domain of values for when the rule applies.)
- *How might these students find the perimeter of a train with 100 hexagons?* (There are many different methods they might use. They could extend the graph, but it would be very large unless they changed the scale. Describing the pattern using an algebraic expression means you could work it out for any value of ***t***.)

(This activity also appears in the *Patterns and algebra: Year 8* narrative.)

Example 3: Cup stack

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

ACMNA194 ◆ ◆
Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool

Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?

Reasoning proficiency:
In what ways can your thinking be generalised?

Instead of ***telling*** students about the linear relationship between two variables, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

Begin a discussion by explaining that most cups are designed to stack. Ask students:

- *How many cups would you need to make a stack as tall as you are?*
- *What information do you need? What different methods can you think of for answering this question?*
- *How might you adapt your thinking to find how many cups would be needed to stack them to a height as tall as me?*
- *Can you generalise?*

Choose another object that stacks (eg chairs) and find out how many of them would be required to make a stack of your height:

- *Can you generalise?*

Example 4: Homework rates

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

ACMNA194 ◆ ◆
Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the equations of a linear relationship, we can challenge students to recognise the relationships between the coordinates of the points for themselves, by ***asking*** questions.

Lisa Simpson is planning to set up a 'Homework Help' business. When you hire Lisa to assist you with your homework, there is an initial charge of $2 and then an ongoing charge that is equivalent to $5 per hour. Ask students:

- *Do you have any questions?*
- *What might it cost you to get help with your homework? Is this reasonable?*
- *How might Lisa show 'customers' her price schedule? Is there another way?*
- *How might you generalise her charges? How might you write it in words/express it using algebraic notation?*
- *What are some different ways that Lisa might increase her income? Which way is better? Convince Lisa.* (Encourage students to graph several options, so that they can see that increasing the initial charge affects the vertical axis intercept, and that the hourly rate affects the slope. They can also see that as there is a positive slope; the more students she tutors, the more she will earn.)
- *What does this remind you of? Do any other businesses use a similar method of charging for their services? How is it the same or different?*

Students may be familiar with tradespeople who have a callout fee and a charge for every 15 minutes, or a cab charge which has a flag fall then a per km/time charge which is continuous. Challenge students to find different examples. Choose one and represent it in as many ways as possible, for example: in words, in a table, in a graph, and as an algebraic expression.

Ask groups to share their examples and in each case draw connections between the initial value and: the slope as seen in the table, on the graph, in the equation, and how it relates to their context.

If none of the examples have a negative slope, challenge students to consider if this is possible. A simple example is phone credit which reduces with the length of a call.

Example 5: Colour-coded points

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

ACMNA194 ◆ ◆
Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the equations of a linear relationship, we can challenge students to recognise the relationships between the coordinates of the points for themselves, by ***asking*** questions.

Whilst this activity involves the use of a whiteboard, it could be done instead in the yard, or with a grid marked on the pavement or oval. An image can then be viewed or captured from the second floor (if applicable). Alternatively, you could mark the classroom floor and ask students to stand in the place that represents the points, or place a large mat with a coordinate grid on the floor and use counters, etc.

The materials you will need for this activity are:

- A set of coordinate axes drawn or projected onto a whiteboard (magnetic). If working outside, mark a grid on the pavement or grass. Portable grids can be made from lengths of shade cloth. These will have multiple uses.
- A set of cards with the coordinates of points, eg (-3, -2) that obey the rule y = x+1 (at least one card per student pair).
- A round red-coloured magnet.

Instruct the students to form pairs and discuss where the magnet should be placed on the magnetic board for their card. Once the magnets have been placed, ask:

- *What do you notice?* (The magnets should be in a straight line. Students will often make errors with points that lie on the axes or they will reverse the x and y coordinates. The order of the coordinates (x first and y second) can be compared to a situation, such as a country deciding which side of the road we drive on. The most important thing about the decision, is that we all understand and comply with the protocol. This is also an opportunity to create a growth mindset in your classroom by valuing mistakes.)

Making the most of mistakes

Create a language and culture shift in your classroom. Design activities to actively teach students that sometimes their first thoughts may not be consistent with mathematical protocols.

They have a choice to make when this happens. They can think:

- I was wrong. I'm not good at this.
  OR
- I've changed my mind. I've learnt something new.

We can promote a growth mindset, by asking the class:

'Who changed their mind/discovered something new to them? Congratulations! You have just changed your brain. Good mathematicians change their mind when they get new information.'

Make a list of all the points and ask the students to look for a pattern in the coordinates:

- *What do you notice? Can you suggest another point that might be on the line? How can we check?* (Allow students to add different points to the axes. Usually students use positive integers only.)
- *Would it work for different types of numbers?*
- *Would the point (-90, -91) be on the line? Why/why not? What would the following points on the line be if the first number was 1002? What if the second number was 534?*
- *How might you describe the pattern in words?* (It is important that students write the relationship in words before they attempt to write an algebraic equation.)

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

Discuss the following:

'I think that the second number is the first number, plus 1.'

- *How might you write that relationship using mathematical symbols?* ($y = x + 1$)

'I wrote it differently. I said that you take one from the second number to get the first number, $y - 1 = x$'.

- *Does that make sense? Does it work? How would that look in mathematical symbols? Is there another way?* (These discussions are important in supporting students to understand the mathematical symbols but also to recognise equivalent representations. Emphasise that 'the rule that the points obey is the "name" of the line' and ask students to write a brief sentence for a Year 7 student to explain what it means. For example, *'It tells you that every point on the line obeys the rule and if it doesn't obey the rule, the point is not on the line.'*)

This activity can be continued and differentiated by preparing more sets of cards by using:

- Sets of coordinates for $y = x+c$, using different colours for different values of c
- Use sets of coordinates for $y = kx$, using different colours for different values of k
- Ask students to make up their own sets of points for other students, using their own rules; first describing in words and then in mathematical symbols.

Example 6: Colour-coded lines

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

ACMNA194 ◆ ◆
Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students how to use a table of values to plot a line, we can challenge students to go from the known to the unknown for themselves, by ***asking*** questions.

This activity is similar to **Example 5: Colour-coded points**, but students calculate the coordinates of the points themselves.

The materials you will need for this activity are:

- A set of coordinate axes drawn or projected onto a whiteboard (magnetic)
- Red, blue and green magnets
- Cards: $x =$ (*a number*)
- Cards: $y =$ (*a number*) [if applicable]
- Written linear equations on the board. Select the equations according to what you wish the students to notice, for example:
  Y-Intercept: Red: $y = 2x$, Blue: $y = 2x+1$, Green: $y = 2x-2$
  Slope: Red: $y = x$, Blue: $y = 2x$, Green: $y = \frac{1}{2} x$
  Negative slopes: Red: $y = - 2x$, Blue: $y = 2x$, Green: $y = 2x-1$

Students work in groups and discuss their thinking. Ensure each student has their own magnet and card with $x =$ (*a number*). The magnet will determine the equation the student is to solve. Whether the number is an integer, positive or negative can be a way of differentiating the task. Introducing the $y =$ (*a number*) card is also more challenging.

An example of this activity is as follows:

A student with a red magnet uses their card $x = 3$ and the red equation for the Y-Intercept, $y = 2x$ to generate the coordinate pair (3,6). The student then places the red magnet on the set of axes on the white board to plot the point.

This activity can also be completed using the grid created for Example 5.

Ask students:

- *What do you notice? What does this make you think?*
- *Does that make sense? Does it work?*
- *What questions do you have? How could we explore this further?*

Technology such as graphic calculators support students as they explore their thinking and test their conjectures.

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 7–10)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem-solving examples	
Example 7: Beelines Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.	ACMNA193 ◆ ◆
Example 8: Stacking cups and sculptures Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.	ACMNA193 ◆ ◆
Example 9: Cracking the concrete Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA194 ◆ ◆
Example 10: Stacking dice Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution.	ACMNA194 ◆ ◆

Example 7: Beelines

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students, we can challenge them to identify the information for themselves, by ***asking*** questions.

Whilst this activity from the NRICH website can be explored without technology, you could also use the GeoGebra applet. This task links with the concept of constant gradient and slope.

Interpret

What have you been asked to find out? What might you need to work out to answer that question? What information is helpful? What information is not useful? What extra information do you need? What information can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? Would it help if you thought about a similar problem for a simpler situation first? (Ask students to speak to someone who they think is being a good problem-solver today and ask them to show what they are trying.)

Solve and check

What would be the maximum number of squares? What would be the minimum?

Questions to be used only after students have grappled with the problem for a few minutes:

Does that seem right to you? Do other people think that too? Could you give a range of possible values? Are there some situations which could be considered similar to each other? What is the simplest type of situation? What could you do to check your thinking?

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

The link to the problem on the NRICH website is: http://nrich.maths.org/737

Example 8: Stacking cups and sculptures

ACMNA193 ◆ ◆
Students plot linear relationships on the Cartesian plane with and without the use of digital technologies.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students, we can challenge them to identify the information for themselves, by ***asking*** questions.

This activity is a Dan Meyer ***Three-Act Maths Task***. It can be presented to students along with the question, *What's the first question that comes to mind?*

There are many ways to solve this problem, including using linear relationships. Allow students to use their own method first, then compare and contrast the approaches. If the students do not use linear relations, ask them whether that thinking could be used to solve the problem; then compare this method to their own.

The **Stacking cups** activity can be accessed at: http://www.101qs.com/1897-stacking-cups--act-1

Artistic extension

James Nizam is an artist who has created a number of stacking sculptures. The images prompt students to ask their own questions, that may lead to linear and non-linear patterns when counting.

The ***Memorandoms*** photo series can be accessed at: https://www.trendhunter.com/trends/james-nizam-photo-series

Source: © *James Nizam 2010, via Trendhunter*

Example 9: Cracking the concrete

ACMNA194 ◆ ◆
Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students, we can challenge them to identify the information for themselves, by ***asking*** questions.

An electrical contractor has to install a power point at position P, from the electrical supply at point O, using underground cabling across a pavement that is 4 pavers wide (see Figure 3). She will damage several concrete pavers in the process and these will need to be replaced. This cost must be included in her quote.

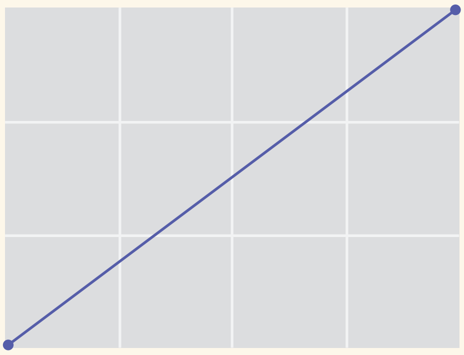


Figure 3

She finds this situation occurs a lot in her work and would like to have a way of determining how many pavers will be affected without counting them, particularly over long sections of pavement. Is that possible?

Interpret

What have you been asked to find out? What might you need to work out to answer that question? What information is helpful? What information is not useful? What extra information do you need? What information can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? Would it help if you thought about a similar problem for a simpler situation first? (Ask students to speak to someone who they think is being a good problem-solver today and ask them to show what they are trying.)

Solve and check

Are there some situations which could be considered similar to each other? What is the simplest type of situation? Does that seem right to you? Do other people think that too? What could you do to check your thinking? (For rectangles whose dimensions are relative prime, eg 4 by 3, the amount of broken pavers can be calculated as (4 + 3-1) = 6. In general, for relative prime dimensions **4** and **a**, the number of broken pavers is: **(4 + a-1) = a + 3**. This is because the line will crack **4** pavers from left to right. It will also crack **a - 1** pavers extra from bottom to top. For non-relative prime numbers, eg 6 by 8, this is calculated as 2 sets of 3 by 4, which will have damage ((4 + 3 – 1) x 2 = 12 pavers.)

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way? (How would your thinking change if the width of the path was different to 4? How would your thinking change if the paving pattern was like the one shown below?)

(This activity also appears in the Patterns and algebra: Year 8 narrative.)

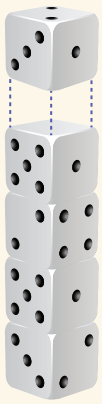
Example 10: Stacking dice

ACMNA194 ◆ ◆
Students solve linear equations using algebraic and graphical techniques. Verify solutions by substitution.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students, we can challenge them to identify the information for themselves, by ***asking*** questions.

Consider a stack of any number of dice with a ‘2’ as the top face. Is there a way of knowing the sum of all the dice faces that you can not see?



Interpret

What are you trying to find out? What do you need to show to answer that question? What information is helpful? What information is not useful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is stealing the opportunity for them to realise for themselves later on in the task. Not having a set number of dice is a stumbling block for some students. Ask what is making the problem difficult and what information would they like, then encourage them to decide the number of dice for themselves to get started. Setting the top number to always be 2, means there is only one variable determining the sum, which is ‘n’, the number of dice.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help to start by thinking about a smaller version of this pattern? (Ask students to speak to someone who they think is being a good problem-solver today and ask them to show what they are trying. Students can identify from the faces they see which are missing and will total those, but this does not lead to generalisation. Putting totals for various different size stacks with a 2 on the top face into a table and/or graph, will help them recognise the number pattern emerging.)

Solve and check

Questions to be used only after students have grappled with the problem for a few minutes:

How might you answer the question for a small number of dice, say 4? Is there a more efficient way to find the sum? How would the sum change if you had 5 dice? What does this make you think? Does that seem right to you? Do other people think that too? (Observing the number patterns can reveal that adding another dice to the stack increases the sum by 7. It can be reasoned that this is because it also adds two unseen faces which are always opposite each other. A property of conventional dice is that the opposite faces sum to 7.)

Reflect

Pair up with someone who did it differently. How do your methods compare? What do you like about each other’s strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way? (Students may have written or verbal explanations, or a range of different algebraic expressions to determine the sum of the unseen faces, such as 7(n-1) + 5 = 7n-2).

Check that these solutions are the same in as many ways as possible, such as using the distributive law, diagrammatic model, Excel spreadsheet, etc. Ask students:

- *Is it possible for the sum of unseen faces on a dice stack to be 223? Why? Why not?*
- *How many possible values are there less than 100? What about between 200 and 300?*
- *What if the number on top was something other than 2?*

A more general version of this problem in the *Patterns and algebra: Year 8* narrative allows the number on top and the number in the stack to vary, leading to a range of answers such as:

- A stack of 4 dice with any number on top: 7 x 3 + (7-t) = 28-t
- A stack of **n** dice with the number **t** on top: 7(n-1) + 7-t = 7n-7+7-t = 7n-t

(This activity also appears in the *Patterns and algebra: Year 8* narrative.)

Connections between 'Linear and non-linear relationships' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use linear and non-linear relationships as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 8	
Whilst working with Linear and non-linear relationships, connections can be made to:	**How the connection might be made:**
Students simplify algebraic expressions involving the four operations. ACMNA192	Refer to: **Example 2: Hexagonal train** **Example 4: Homework rates**
Students extend and apply the distributive law to the expansion of algebraic expressions. ACMNA190	Refer to: **Example 2: Hexagonal train** **Example 9: Cracking the concrete**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Linear and non-linear relationships’ from Year 7 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Linear and non-linear relationships:

Plotting and sketching skills and strategies ◆
In Year 7 to Year 9 students focus mostly on plotting and sketching skills and strategies.

Connection between numerical, algebraic and graphical representations and methods ◆
In Year 8 to Year 10A students focus mostly on using and comparing multiple representations and methods.

Strategies for solving equations ◆
In Years 10/10A students focus mostly on strategies for solving equations.

Using digital technologies ◆
In Year 8 to Year 10A students use digital technologies to identify and represent functions and relations and also solve related problems.

Year level	‘Linear and non-linear relationships’ content descriptions from the AC: Mathematics
Year 7 ◆	Students solve simple linear equations. ACMNA179
Year 7 ◆	Students investigate, interpret and analyse graphs from authentic data. ACMNA180
Year 8 ◆ ◆	Students plot linear relationships on the Cartesian plane with and without the use of digital technologies. ACMNA193
Year 8 ◆ ◆	Students solve linear equations using algebraic and graphical techniques. Students verify solutions by substitution. ACMNA194
Year 9 ◆ ◆	Students find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software. ACMNA214
Year 9 ◆ ◆	Students find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software. ACMNA294
Year 9 ◆ ◆	Students sketch linear graphs using the coordinates of two points and solve linear equations. ACMNA215
Year 9 ◆ ◆ ◆	Students graph simple non-linear relations with and without the use of digital technologies and solve simple related equations. ACMNA296
Year 10 ◆	Students solve problems involving linear equations, including those derived from formulas. ACMNA235
Year 10 ◆ ◆	Students solve linear inequalities and graph their solutions on a number line. ACMNA236
Year 10 ◆ ◆ ◆	Students solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology. ACMNA237
Year 10 ◆	Students solve problems involving parallel and perpendicular lines. ACMNA238
Year 10 ◆ ◆	Students explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate. ACMNA239
Year 10 ◆	Students solve linear equations involving simple algebraic fractions. ACMNA240
Year 10 ◆ ◆	Students solve simple quadratic equations using a range of strategies. ACMNA241
Year 10A ◆	Students solve simple exponential equations. ACMNA270
Year 10A ◆	Students describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations. ACMNA267

Year 10A ◆ ◆	Students apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation. ACMNA268
Year 10A ◆	Students factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts. ACMNA269

Numeracy continuum: Recognising and using patterns and relationships	
End Year 6	Identify and describe pattern rules and relationships that help to identify trends.
End Year 8	Students identify trends using number rules and relationships.
End Year 10	Students explain how the practical application of patterns can be used to identify trends.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Visual patterns

http://www.visualpatterns.org/

This website has multiple visual patterns that students can consider and describe in a way that makes sense to them. Students will have multiple interpretations of each image, promoting multiple ways of visualising, solving problems and stimulating dialogue. They can be used as lesson starters.

Scootle

https://www.scootle.edu.au/ec/p/home

This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.

reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

Numeracy in the News

http://www.mercurynie.com.au/mathguys/mercury.htm

Numeracy in the News is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.

TIMES modules

http://schools.amsi.org.au/times-modules/

TIMES modules are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.

Top drawer teachers – resources for teachers of mathematics (statistics)

http://topdrawer.aamt.edu.au/Statistics

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.

Double Helix Extra

https://blog.doublehelix.csiro.au/

This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.

CensusAtSchool NZ

http://new.censusatschool.org.nz/tools/random-sampler/

CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics. It aims to:

- *'foster a positive attitude to statistics through using data that is both relevant and real*
- *improve understanding of a data gathering process, its purposes and benefits to society*
- *provide access to large and meaningful multivariate data sets*
- *encourage effective IT teaching and learning*
- *enhance the process of statistical enquiry across the curriculum.'*

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by the Learning Improvement division