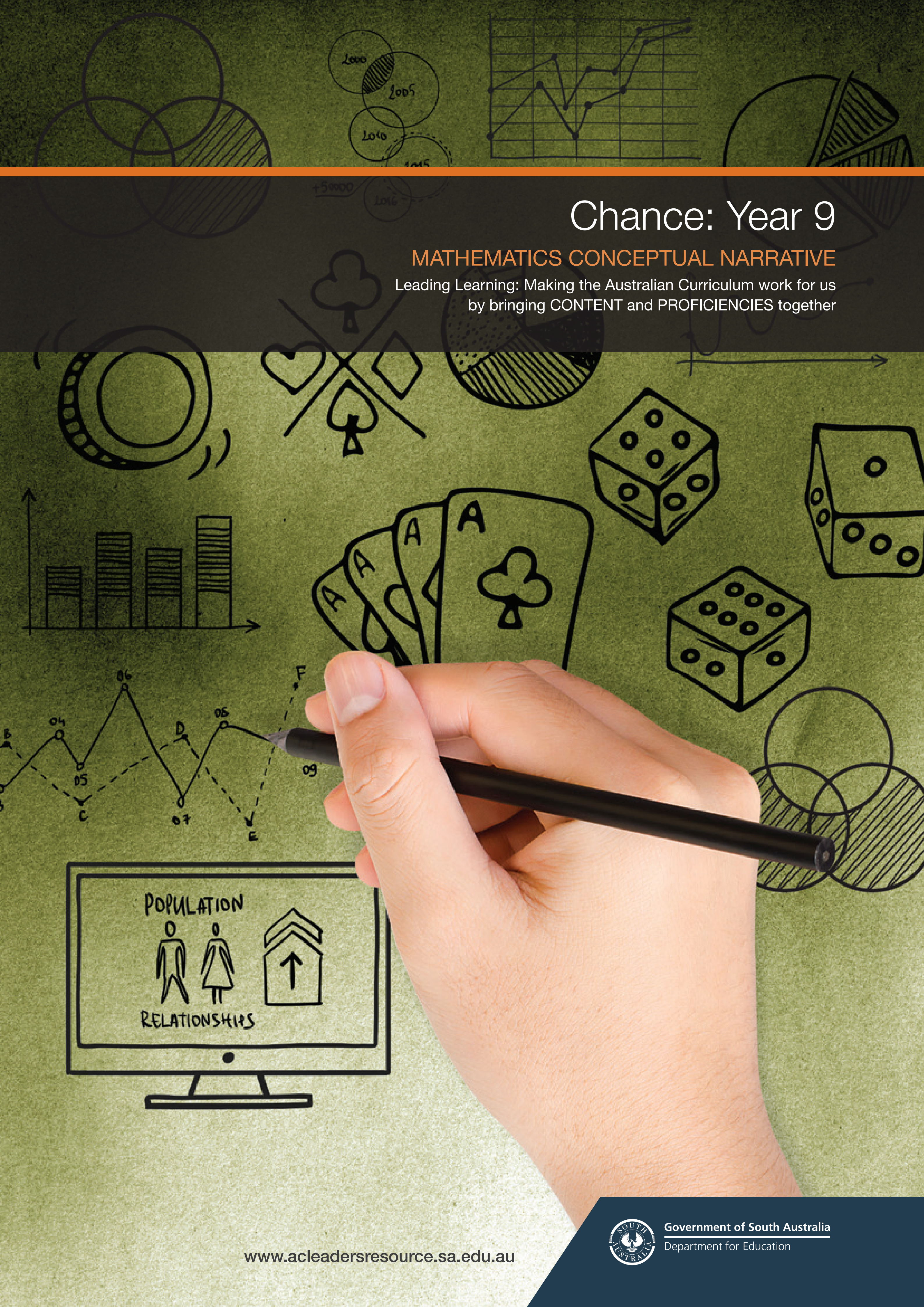
Chance: Year 9

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Chance' from Year 1 to Year 10A** (see page 21)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with Chance:

- Identify and order chance events
- Identify, describe and represent sample spaces
- Randomness and variation
- Observed frequencies and expected probabilities
- Language.

What the Australian Curriculum says about ‘Chance’

Content descriptions

Strand | Statistics and probability.

Sub-strand | Chance.

Year 9 ◆ ◆ | ACMSP225
Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.

Year 9 ◆ ◆ ◆ | ACMSP226
Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’.

Year 9 ◆ ◆ | ACMSP227
Students investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians.

Year level descriptions

Year 9 ◆ ◆ | Students explain the use of relative frequencies to estimate probabilities. Students list outcomes for experiments.

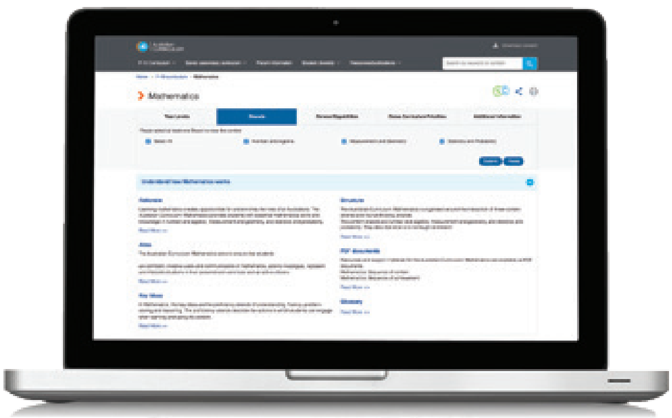
Achievement standards

Year 9 ◆ ◆ | Students calculate relative frequencies to estimate probabilities. Students list outcomes for two-step experiments and assign probabilities for those outcomes.

Numeracy continuum

Interpreting statistical information

End of Year 10 ◆ ◆ | Students explain the likelihood of multiple events occurring together by giving examples of situations when they might happen (Interpreting statistical information: Interpret chance events).



Source: ACARA, Australian Curriculum: Mathematics

Working with Chance

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 9 'Chance'

In Year 7, in line with students' developing ability to change between fractions, decimals and percentages, they would now be expected to change between these representations for any amount, rather than just for common quantities. Students in Year 7 are introduced to the term 'sample space'.

In Year 8 students use two-way tables to represent possibilities of experiments with two steps. Students are also introduced to the use of the terms 'at least', 'and' and 'or' in relation to chance events. They use two-way tables and Venn diagrams to calculate probabilities satisfying the 'at least', 'and' and 'or' criteria. Students are also introduced to the term 'complementary events' and use the sum of probabilities to solve problems.

In Year 9 students continue to work with two-step chance experiments, but now they consider the probabilities with and without replacement. To support this thinking, students learn to create tree diagrams and use such diagrams to assist in solving problems. Students estimate probabilities by calculating relative frequencies for collected or given data.

In Year 10/10A students describe the results of two- and three-step experiments, both with and without replacement. They investigate the concept of independence and conditional statements.

Statistics is about exploring variation and uncertainty, and in this way chance and data are intertwined. Probability is a measure of chance and students have had learning experiences that require them to order these events and quantify their likelihood which **links with their understanding of concepts within numbers**, ie fractions, decimals, percentages, ratio and proportion.

In addition to very real informal intuitions about the likelihood of events based on past experiences, students have some understanding of how to determine theoretical probabilities for simple events. **As events become more complex, their intuition can be misleading.** We should encourage students to use their intuition to predict likelihood before they conduct the experiments. When they encounter discrepancies between their 'gut feelings' and their results, students become interested in exploring further and are open to more sophisticated and theoretical ways to identify sample spaces and understand the relationships between compound events.

Students will still have strong intuition about the likelihood of events occurring, but now they will be dealing with large, complicated sample spaces for compound events that are not necessarily equally likely, and where it is not practical to list or represent every outcome. **It is still important for students to conduct these multi-step experiments themselves**, so they see how the probabilities will be affected if there is no replacement or no repetition. Tree diagrams are a good visual representation of the process and if **drawn as the experimentation takes place**, build a conceptual understanding of dependent events.

Once students understand the concepts, teachers can make links to the mathematical terminology and formal definitions; connect the mathematical names to the colloquial and everyday use of the language, such as 'independent', 'dependent', 'mutually exclusive', and 'disjoint'. Because language is so important, it is a good strategy for the teacher to **set up a highly visible class vocabulary list (eg word wall) for students to contribute to and refer to**. The word wall links the formal definitions, colloquial language, the students' own understandings and examples from authentic experienced contexts.

Engaging learners

Classroom techniques for teaching Chance

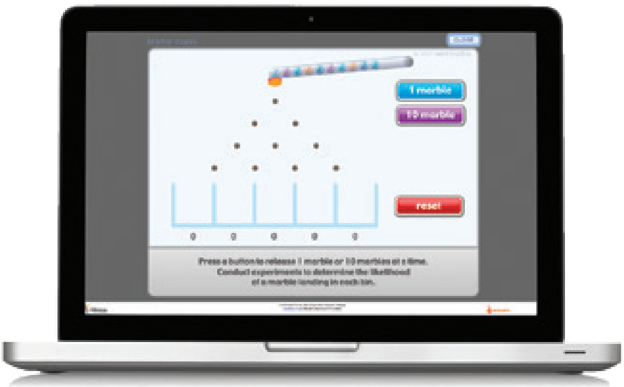
Chance provides opportunities to evoke curiosity and wonder in our students. Many genuine contexts allow students to draw on prior knowledge and intuition to engage them in their learning. Students personalise their learning when they make predictions about outcomes, design and conduct their own experiments and make meaningful inferences from what they have discovered.

Using relative frequency

Many websites such as Cambridge HOTmaths, Scootle, NRICH and the National Library of Virtual Manipulatives have a range of digital activities which students can use as random generators to experiment and collect relative frequencies to explore probabilities.

In most games, skill affects your chance of winning. However even in games where skill plays no part, each outcome is not always equally likely. In ***HOTmaths: Using relative frequency*** marbles fall from a tube and bounce through obstacles to land in 1 of 5 bins. By using the digital activity, students can calculate the relative frequencies for the bins the marbles land in. Students then gather data by conducting several experiments with different numbers of marbles and comment on the relative frequencies, as the number of trials in the experiment increases.

The digital activity can be found at:
http://tlf.dlr.det.nsw.edu.au/learningobjects/Content/L10574/object/



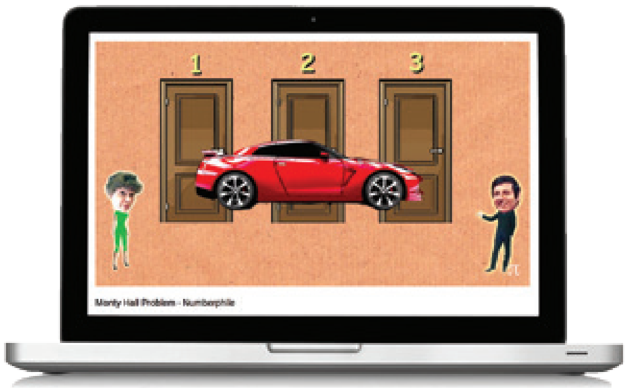
Source: *HOTmaths: Using relative frequency*, HOTmaths – The Le@rning Federation, 2009

Monty Hall problem

Famous problems such as the ***Monty Hall problem*** are counterintuitive and learners are quite often motivated by disbelief to explore the problem. Loosely based on the American television game show *Let's Make a Deal*, it is named after its original host, Monty Hall and focusses on the probability of opening the correct door to win the coveted prize.

Numberphile's examination of this problem features Lisa Goldberg, an adjunct professor in the Department of Statistics at the University of California, Berkeley USA.

The video can be found at:
https://www.youtube.com/watch?v=4Lb-6rxZxx0



Source: *Monty Hall Problem*, Numberphile, 2014

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–3)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the names 'mutually exclusive', 'complementary' and 'independent events'** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged **with questions that will result in them identifying the patterns that simplify finding the probability for different types of events**, so I don't need to design and instruct the details of the investigation for them.

At this stage of development, students can develop an understanding of chance, randomness and variation through conducting their own experiments. When teachers provide opportunities for students to predict, identify, describe and represent the outcomes of the experiments and compare them to theoretical expectations, they require their students to generalise. Telling students the laws of probability, removes this natural opportunity for students to make conjectures and verify connections that they notice.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to fully design a probability investigation, or set tasks that we want students to experience, rather than ask a question (or a series of questions) and support them to planning the stages of the investigation for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students use a specific set of skills, knowledge and procedures during the current unit of work, then it probably is quicker to tell them what to do. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the formulae is a false economy of time.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Odds and evens Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events. Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’.	**ACMSP225** ◆ ◆ **ACMSP226** ◆ ◆ ◆
Example 2: Yellow starbursts Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events. Students investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians.	**ACMSP225** ◆ ◆ **ACMSP227** ◆ ◆
Example 3: Who picks their card first? Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events. Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’.	**ACMSP225** ◆ ◆ **ACMSP226** ◆ ◆ ◆

Example 1: Odds and evens

ACMSP225 ◆ ◆
Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.

ACMSP226 ◆ ◆ ◆
Students calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/ calculate in different ways?

Reasoning proficiency:

In what ways can you communicate?

Instead of ***telling*** students about dependent events, we can challenge students to recognise events that are dependent and independent for themselves, by ***asking*** questions.

Part A

One of Marilyn Burn's: 10 Big Maths Ideas is to support learning with manipulatives (this is for the older students as well). This class activity can be used to introduce the concepts of dependent events, as well as ways of recording outcomes in two-step chance experiments.

Begin with a container that holds a set of numbered balls: 3, 4, 5, 6 and 7. Tell the students that you are trying to design a game using these balls but have not decided on the rules yet. You want it to be a fair game. You know you are going to draw out 2 balls, one after the other without replacement and you are going to add up the numbers and notice if the sum is odd or even. You are going to conduct a trial to see what sort of results you get, so you can decide if an even sum and an odd sum are equally likely. Ask the students:

- *What do you think? Fair? Unfair?*
- *If we conduct 10 trials what do you think will be the result?* (Run the trials.)
- *Does anyone want to change their mind now we have some more information?* (Note that drawing 2 balls simultaneously, is the same experiment as drawing them one after another without replacement and not recording the order they were drawn in.)

To gain a better understanding of the game, ask the students to represent all possible options in as many different ways as possible (do each way on a different piece of paper so it can be displayed on the board). If some groups need more support, you can ask the following questions:

- *What was the first result we got when we first drew out 2 balls? How could you record that? What was the second result? When we did our trials did we get all possible combinations? How do you know?*
- *You have made a list, but can you also use a table, graph, or grid of some sort?*

Ask groups to pin up their options and if possible to put similar methods together (lists, tables, grids, graphs, etc). Have some examples of systematic lists, tree diagrams, two-way tables and lattice diagrams. Explain that these are some of the ways that mathematicians have been recording data for years.

- *How are these the same or different to the ones we came up with?*

This is an opportunity to point out that they are capable of creating their own knowledge and that their methods made sense to other mathematicians as well.

- *What can we infer from these representations? What is the probability of getting the first result we got (eg 5 and 6). What about an odd sum? What about an even sum? Do you want to change your mind now we have more information? Is it a fair game?*
- *Would it be the same if we changed the numbers? What about the number of balls?*

The link to Marilyn Burn's ideas can be found at: http://www.scholastic.com/teachers/article/marilyn-burns-10-big-math-ideas

Use concrete materials in conjunction with digital activities

Starting with a digital activity and not manipulatives, can miss the opportunity to cognitively and emotionally engage the students with the challenge. Once the students have had an opportunity to play the game with concrete materials and are ready to apply their new knowledge, consider using a digital activity to speed up their data collection as they explore a related problem.

Part B

This activity is from the NRICH website and uses the knowledge the students have gained in Part A.

This activity has more than one set of numbered balls, which naturally creates an opportunity to compare and contrast results. Ask students:

- *Is it possible to change the numbers, but keep the probabilities of odd and even sums the same?*
- *Is it possible to change the number of balls, but keep the probabilities of odd and even sums the same?*
- *Can you design a fair game?*

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

The link to the problem on the NRICH website is: http://nrich.maths.org/4308

Example 2: Yellow starbursts

ACMSP225 ◆ ◆

Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.

ACMSP227 ◆ ◆

Students investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about complementary events, we can challenge students to recognise the relationships between the events for themselves, by ***asking*** questions.

This activity is a Dan Meyer ***Three-Act Maths Task***. It can be presented to students along with the question, *What's the first question that comes to mind?*

This activity includes a video that engages the students with this problem and shows that it is not always necessary to give students a lot of information or instruction, before they consider a problem. Ask students:

- *What questions do you have? What do you wonder?*
  This activity also explores assumptions about whether you would expect equal numbers of each colour produced and whether there would be equal numbers of the possible two-packs. As teachers, we should make explicit links to Statistics by asking students to notice how the data has been summarised and represented in different ways, when there are two or four categories. Make a point to discuss the fact that a choice was made to consider Yellow and Not Yellow, rather than considering all four colours as different options.
- *Why did they do that?*
  Even though they are not equally likely, the problem is simplified by having only two events which are complementary (that have probabilities that add to 1) and mutually exclusive (it is either one or the other, but can't be both).

There should also be a discussion about the fact that inferences are being made based on data from a sample of starbursts:

- *Would it be the same if you took another sample?*
- *How confident are you in the inferences you have made?*
- *Was the sample big enough?*

The **Yellow starbursts** activity can be accessed at: http://threeacts.mrmeyer.com/yellowstarbursts/

Example 3: Who picks their card first?

ACMSP225 ◆ ◆
Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.

ACMSP226 ◆ ◆ ◆
Students calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.

Questions from the BitL tool

Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/ calculate in different ways?

Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about dependent events, we can challenge students to recognise the relationships between the events for themselves, by ***asking*** questions.

This is a game played by two or more students, where each player nominates the suit (hearts, diamonds, clubs or spades) that they would like to draw from the pack. Each player draws a card which everyone sees and is not replaced. The first player to draw their nominated suit is the winner.

Before each draw, the player must determine the probability of drawing their suit on that particular draw (eg if there have been 4 cards drawn in the game and 2 of them were hearts, there would be 48 (=52–4) cards left and 11 (=13–2) of them would be hearts, so my chances of getting a heart on the next draw are $\frac{11}{48} \approx 23\%$).

Discuss the following with your students:

- *Does it matter who goes first? How can we make that fair?*
- *Am I better to choose the same suit as someone else or a different one?*

  For the second person there are various probabilities depending on what the first person drew and whether the players chose the same suit:
  - 1st and 2nd choose different suits (1st didn't get their suit and 2nd did) $\frac{3}{4} \times \frac{13}{51} = \frac{39}{204} \approx 19\%$
  - 1st and 2nd choose same suits (1st didn't get their suit and 2nd did) $\frac{3}{4} \times \frac{12}{51} = \frac{36}{204} \approx 18\%$
- *Is it a long or short game?*

  Drawing a suit is generally not a long game, as the first player has a 1 in 4 chance of drawing their suit. You may wish to collect class data on game length and use your knowledge of statistics to inform your decisions in this context.
- *How could we make it a more interesting game? Will it be longer or shorter?*

  Students may wish to choose two options, such as a suit and a face value (eg a heart **or** a jack: 14 cards). Here is an opportunity to demonstrate understanding of mathematical language relating to chance, such as the 'inclusive or'. They may choose a face value or range of values, a picture card, a colour (eg red **and** picture card: 6 cards). Allow students to make their own decisions about how the game could be changed. This is an opportunity to differentiate their learning, compare the length of the game for the different choices and relate this to theoretical probabilities.
- *Is it fair if players can choose different characteristics for their card? Is it fair if one chose picture cards and the other chose hearts? Are there any situations where this would be fair? In general, what has to be true?*

  Initially, the characteristics must have the same probability. The probability of a picture card is $\frac{12}{52}$, where hearts is $\frac{13}{52}$ and so the player choosing hearts would have a game advantage.

  (A similar activity can be found on page 10 of the Chance: Year 10/10A narrative.)

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 4–6)

Problems are described in the AC: Mathematics as ‘meaningful’ or ‘unfamiliar’. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as ‘fun’ nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, ‘I don’t know’, or ‘I’m not sure’. Students respond differently to this feeling; some shut down, others begin to ask, ‘But how could I work that out?’

In developing powerful learners we are aiming for all of our students to learn that ‘not knowing’ is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, ‘What could I do to work this out?’

Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they’ll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they’ll need
- a possible process to use.

Proficiency: Problem-solving examples	
Example 4: Horse race with different dice Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.	ACMSP225 ◆ ◆
Example 5: Path of termites Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.	ACMSP225 ◆ ◆
Example 6: Dice battles Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’.	ACMSP226 ◆ ◆ ◆

Example 4: Horse race with different dice

ACMSP225 ◆ ◆
Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This simple probability task can lead to the collection of sample data that will let students investigate notions of probability, whilst developing statistical investigation skills. Collection of data in a systematic and purposeful way will lead students to be able to make predictions about 'favourites', samples sizes, and explore experimental versus theoretical probability.

This activity involves simulating a twelve-horse race using 6, 4, 8 and 12-sided dice. Discuss with students:

- *What is a simulation? How and why are simulations used? Who uses them?*

Governments, businesses, scientists and particularly ecologists use simulations to model or imitate very important systems, where there is variation and high risk or cost for uninformed decisions. Students will be familiar with flight simulators and most of their computer games are simulations of real-life human actions.

It is engaging for students to act out the race. Mark a starting line and up to 10 different stages of the race (about a metre apart) on the ground, with masking tape or chalk. Using jockey caps or sashes numbered 1 to 12, select students to line up at the starting line. Roll two 6-sided dice. The sum of the dice will determine which 'horse' will advance to the next stage of the race. Continue to roll until one of the 'horses' crosses the finish line.

Prediction supports the development of conceptual understanding

When asked to make predictions, students are challenged to think more deeply about the process they are about to observe. Asking what the outcome 'might be' rather than what it 'will be' invites thinking, rather than 'seeking the correct answer'. Students are often motivated to experiment and take notice of the results to see how well the results match their predictions, but we can support a growth mindset through valuing their thinking, rather than the accuracy of their predictions.

Explain how the race will be run, but ask students to predict the outcome first by asking the following questions:

- *Which horse do you think will win? Is there a horse you know will not win?*

  Announce a scratching before the race begins. Horse number 1 is scratched as the possible sums (sample space) are the numbers 2 to 12.

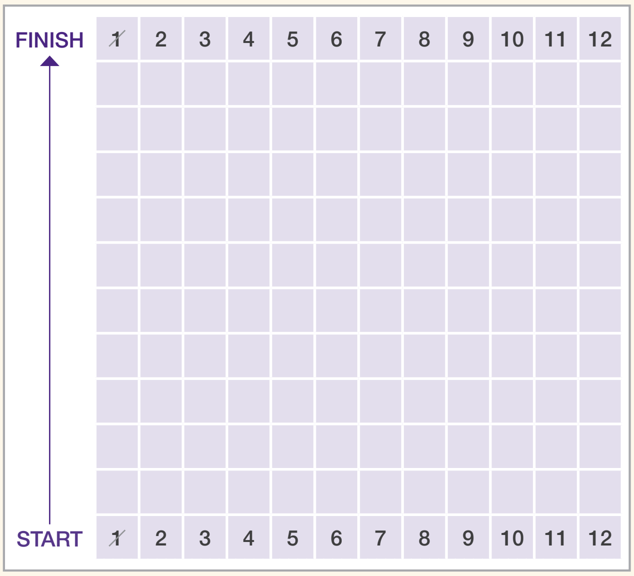


Figure 1

You can use a grid such as in Figure 1 to shade the squares as each horse advances. Sometimes we provide the students with a representation like this where we have made decisions about the most appropriate representation for the context. Rather than explain the features, it is better to question the students about what they notice, in order to cognitively engage them:

- *What are these numbers down the bottom? Why are they also at the top then?*
- *So, if I roll a 3 and a 5, what will happen? How can I record that? How will we know which horse has won?*

  This representation resembles a column graph, and the staged progress of the horses provides a visual image that prompts students to suspect that not all sums are equally likely.

- *What do you notice about where the horses are when the winner crosses the line?*

  It is important that you stop recording the race when the winner crosses the line. Students often wish to continue the race to see where they would have come, and some students may want to pose their own questions for an extended statistical investigation.
- *Are any of the 'horses' unhappy and think the race was not fair?*
- *Is this what you predicted? Would the result always be the same? How could we check our ideas?*

  It is important to allow students to simulate their own races, so they have a personal experience. It also provides more data from which students can form a conjecture. Ask the first pair that finish their race to design a table or graph, on which other students can record the winning horse from each of their races. This is an opportunity to consolidate student fluency in data representation, ensuring the table or graph is appropriately labelled, and accurate enough for the whole class to understand how to record their own results and retrieve the class information.
- *The data will most likely indicate that Horse 7 wins more frequently, but why?*
- *What do you notice? What do you wonder?*

  As with the structure of the Dan Meyer ***Three-Act Maths Tasks***, students can share the questions they have at this point. Record these and then share a question that you would like them to explore with you. Often, student questions will relate closely to the challenge you are setting. At the end of the activity, refer back to these questions to see if the students' questions can be answered by what you have learned, or whether they need to know something else. (The Dan Meyer *Three-Act Maths Tasks* can be accessed at: http://threeacts.mrmeyer.com).

Set the challenge for the class to make a conjecture about whether each horse has the same chance of winning, and if not, why? Does it matter which dice you use to decide which horse moves forward? Does it matter whether it is a long or short race? Ask the students to find out as much as they can about 'the race' in this type of simulation (always make a prediction before you test your ideas.)

If students are already familiar with the expected outcomes for the sum of two 6-sided dice, this will not be a problem-solving opportunity. For these students, you can ask them to explain why 'Horse 7' would be the 'favourite', before they predict and explore the simulations using a 4-sided, 8-sided and 12-sided die in a similar manner.

Interpret

What have you been asked to do? What information is helpful/not useful? (Establish that the student understands the data that has been collected and what it might suggest. If not, encourage them to run another race, but this time record more detail, ie what they rolled on each of the dice and which horse moved forward.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help if you listed all the possible things you could roll (outcomes)? Are there different ways that you could do that? What do you think would be easiest/most efficient/always work for you? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Teachers can become more supportive if students cannot make progress, by asking:

- *Can you describe all the ways for Horse 2 to move, Horse 3 to move … etc*
- *One student used this grid (Figure 2) to write their answers in. Can you complete it with your ways? (If they do not agree that 4,6 and 6,4 are different, see the discussion below). What does the symmetry tell us? Where have you seen a shape like this when running the race?* (The finishing positions of the races usually have this shape, and this makes a connection between the number of ways of rolling the sum and how the horses advance.)

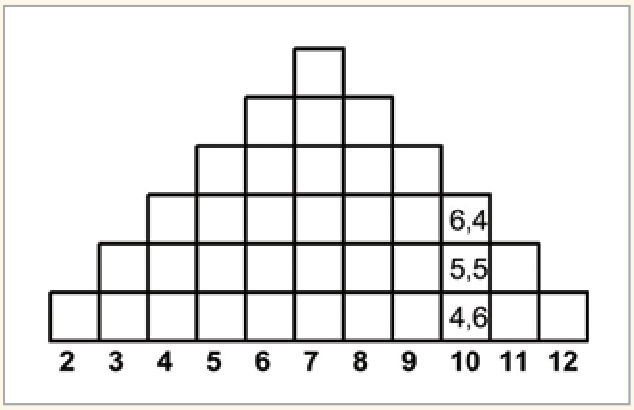


Figure 2

Solve and check

Is the 'favourite' the same if using the 4-sided, 8-sided or 12-sided die? How could you investigate that? Which race would take the longest to determine a winner? If you were 'Horse 2', would you prefer a long race or a short race? What other horses would feel the same way? Is there another way that you could have solved this problem?

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any values? How well does this simulate a real horse race? What is similar? What is different?

Students often do not realise that there are two ways to roll a 3 and a 5, but only one way to roll a 4 and a 4. This is an opportunity to interrogate this concept through discussion with students who find it challenging. Initially, make sure that students have two different coloured dice so that they can distinguish between 3R, 5B (3 on the red and 5 on the blue) and 5R, 3B (5 on the red and 3 on the blue).

Ask them to make an 8 in as many different ways as possible, by changing the dice. They can see the 4R, 4B is 4 on the red and 4 on the blue and that it does not change when swapped. The sample space can also be drawn using a lattice diagram (grid) to show there are two points at (3,5) and (5,3) but only one at (4,4).

Once grasped, the challenge is to use two dice of the same colour.

Ask them to make 8 using 3 and 5. Can they make it another way? (They must change the dice, so each shows a different result). Ask them to make 8 using 4 and 4. Can they make it another way? (They must change the dice, so each shows a different result). To consolidate their understanding, ask them to plan an activity so that they could show Year 8 students that when tossing two coins, there are two ways to get a head and a tail, but only one way to get two heads.

Example 5: Path of termites

ACMSP225 ◆ ◆
Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Students assign probabilities to outcomes and determine probabilities for events.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

This activity involves investigation and prediction. Explain to your class that a colony of termites are leaving a wood stack and are moving down a wire grid (see Figure 3).

Figure 3: http://pixabay.com/en/fence-chain-fencing-encaged-337867/
Source: shaunagm, Pixabay

Ask students:

- *What questions do you have? What do you wonder?*

Establish that the termites always move downwards and assume that when they get to a junction in the wire, they choose at random whether to move left or right. Students might suggest that they are likely to follow each other and hence not move randomly. This is a rich extension to the task. Ask students to research this fact and then share this with the class for further exploration, at a later date.

When mathematicians tackle complex problems, they solve a simpler problem first and this is usually by making some assumptions (ie the termites chose direction at random). Once the simpler problem is solved, they can then consider how to account for some of the most significant assumptions they have made (ie what if they follow the termite in front more often?).

Ask the students to investigate and answer the following questions:

If the termites came out of the hole, where would they all end up? What is the probability of them ending up at any given spot?

Don't always give the exact information needed to solve a problem

Often when we set students a challenge, we give them all the information they need and no extra unnecessary information. To solve a real problem, mathematicians have to make decisions about what information would be useful and what would not be.

Determine the following as a class:

- *What information do you need?* (How long is the wire grid? How many termites?)
- *Will that make a difference? How?* (The class can agree on a certain length of grid, number of termites, or decide that groups will consider different values and compare their findings. Alternatively, this can be a way of differentiating the task and working as a community of learners.)

Interpret

What have you been asked to calculate? What information is helpful/not useful? Have you decided on the important information? (Establish that the student is aware of how one termite might travel down the grid by simulating the path in some way, eg tossing a coin at each junction, spinner left/right, etc.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help if you did a simulation like we did with the horse race? Are there different ways that you could do that? What do you think would be easiest/most efficient/always work for you? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Teachers can become more supportive if students cannot make progress by asking:

- *Consider one termite. How could we decide the path it would take? Where did it end up? What about another? Do you have a new idea now? What do you think? How could you check?*

Solve and check

What do you think is happening, and why? What would happen if the wire was longer? Shorter? How many paths are there for a termite to get to this spot? What other point is similar to this one? Can you generalise? How did your results match the theory? Is there another way that you could have solved this problem?

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any values? What assumptions and limitations have you made? (It may be possible to collaborate with the technology department to allow students to build physical models such as marble runs. The number of different paths leading to an end point, determine the chance of a termite ending at that point).

Refer to Figure 4. There are 8 paths in total on this length of grid, so the probability of ending up at the 4 final points is $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$, $\frac{1}{8}$ respectively. Students should notice that they do not need to trace all the paths from the top but can add up the number of paths that lead to the points on the previous level that feed into the point. The connection to Pascal's Triangle can be an extension for students to research.

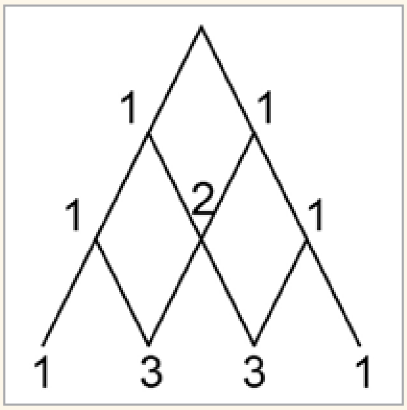


Figure 4

At the end of the investigation, ask students to use their knowledge to predict where a given number of termites will end up and run a class simulation. As teachers, we need to encourage discussion relating to the difference between the expected (theoretical probabilities) and actual outcomes (relative frequencies). Discuss with students:

- **Is that what we expected? Is this a highly unlikely outcome? Could this happen due to natural variation, or is there something else causing this result?**
- **What if we did it again? What if we used more 'termites'?**

If students wish to explore the possibility that a termite might follow the one in front, change the probability, eg 80% of time the termite is following the one in front and run some simulations. This introduces the concept of conditional probability where events are dependent. It will also require students to design a random process that does not have equally likely outcomes (eg make a spinner with 5 sections and shade 4, roll a 10-sided die, generate a random number (1–10) using technology; if the result is 8 or less, follow the termite in front). This is an open-ended task and gives students an opportunity to design, conduct and report on their own.

Example 6: Dice battles

ACMSP226 ◆ ◆ ◆
Students calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.

Questions from the BitL tool
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by ***asking*** questions.

Students at this stage of development are familiar with expected outcomes when throwing conventional dice, but it becomes a problem-solving task when outcomes are no longer equally likely.

Consider three different dice, with the numbers on the 6 faces as follows:

Die 1: 3 3 3 3 3 6
Die 2: 2 2 2 5 5 5
Die 3: 1 4 4 4 4 4

In a dice battle between two players, each player picks a die. The two dice are then rolled together and whoever gets the highest value wins. The ultimate winner is the player with the most wins in 10 rolls. Discuss the following with your class:

- Is this a fair game?
- Which die should you choose and why?

These dice are like *Rock, Paper, Scissors* as Die 1 beats Die 2 beats Die 3 beats Die 1 (non-transitive). Students will notice this very quickly but not necessarily be able to understand why it happens. Encourage students to represent the sample space. At Year 9 level, this can be done using a table (Lattice diagram as shown in Figure 5 below), or tree diagram of all possible outcomes so you can see which of the 36 options are won by which die.

Die 2	5	w2	w2	w2	w2	w2	w1
	5	w2	w2	w2	w2	w2	w1
	5	w2	w2	w2	w2	w2	w1
	2	w1	w1	w1	w1	w1	w1
	2	w1	w1	w1	w1	w1	w1
	2	w1	w1	w1	w1	w1	w1
		3	3	3	3	3	6
		Die 1					

Figure 5

This is an opportunity for some students to consider a 'weighted' tree diagram when they find that a tree diagram showing all 36 simple events is large and difficult to draw. As a teacher, you can present an incomplete, or not completely correct sample of work (see Figure 6) from a fictitious student to stimulate discussion:

'A student decided that a tree diagram was too big and decided to draw it like this to work out the probability of Die 1 or 2 winning.'

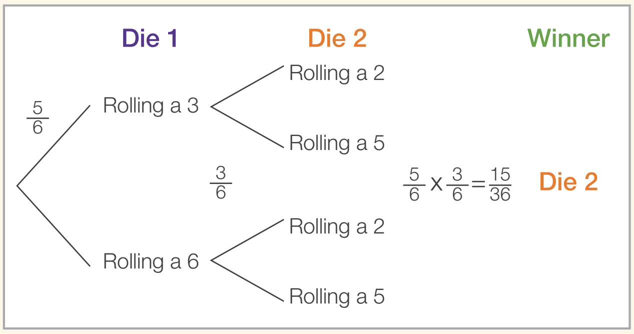


Figure 6

Ask students:

- *Does it look right to you? Would it always work? Would it work for Die 1 and Die 3? Could you use this method to see who would win out of Die 2 and 3?*
- *How is this the same, or different, to the Lattice diagram?*
- *Why does it work?* ($\frac{5}{6}$th of the time you expect to get a 3 and then $\frac{3}{6}$ of that ($\frac{3}{6}$ of $\frac{5}{6} = \frac{3}{6} \times \frac{5}{6}$) you expect to get a 5 so the probability of rolling a 3 on Die 1 and a 5 on Die 2 would be $\frac{15}{36}$.)

When students attempt to work out the probability of Die 1 winning, it leads them to the sum of mutually exclusive events which conceptually can be linked back to different areas in the Lattice diagram.

If students are familiar with this battle, or it does not present a challenge, ask them to explore what might happen if each player rolls two of their dice, with the highest sum being the winner. Interestingly, it is still non-transitive (*Rock, Paper, Scissors*) but the order of dominance changes.

(A similar activity can be found on page 18 of the *Chance: Year 8* narrative.)

Interpret

What have you been asked to do? What information is helpful/not useful? (Establish that the student understands how the game is played and even encourage them to guess which one they think might be better.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help if you played a few games? Are there different ways that you could do that? How many games will be enough? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Teachers can become more supportive if students cannot make progress by asking:

- *How could you determine which of Die 1 and Die 2 is most likely to win? Would one game be enough? Is 50 enough? What will you do if you find that Die 1/ Die 2 is best? Do you have a new idea now?*

Solve and check

What did you learn from the games? Does this make sense? How could you investigate that? What is unusual about what you discovered? What's that similar to? Could you explain why this happens? How might you represent the ways that Die 1 could win against Die 2? How likely would that be? Is there another way that you could have solved this problem?

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for other strange dice? What if they were 4-sided or 8-sided dice – could you still use your method? What if you rolled two of each?

Connections between ‘Chance’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use Chance as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 9	
Whilst working with Chance, connections can be made to:	**How the connection might be made:**
Students solve problems involving direct proportion. Students explore the relationship between graphs and equations corresponding to simple rate problems. ACMNA208	Refer to: **All examples.**
Students identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources. ACMSP228	Refer to: **All examples.**
Students construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bimodal’. ACMSP282	Refer to: **Example 4: Horse race with different dice** **Example 5: Path of termites**
Students compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread. ACMSP283	Refer to: **Example 4: Horse race with different dice** **Example 5: Path of termites**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Chance’ from Year 1 to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Chance:

Identify and order chance events ◆
In Year 1 to Year 2 students identify and order chance events.

Identify, describe and represent sample spaces ◆
In Year 1 to Year 5 students describe and represent sample space. In Year 6 to Year 8 students mostly describe more complex sample spaces and assign probabilities. In Years 9 to Year 10A students mostly represent complex sample spaces and events.

Randomness and variation ◆
In Year 3 students conduct chance experiments and recognise variation in results. In Year 9 students calculate from given or collected data. In Year 10A students investigate reports of studies in digital media and elsewhere for information on their planning and implementation.

Observed frequencies and expected probabilities ◆
In Year 3 to Year 5 students consider expected probabilities of simple events. In Year 6 students quantify probabilities. In Year 9 and Year 10 students determine probabilities of compound events.

Language ◆
Throughout Year 1 to Year 10A students use the language of chance in increasingly sophisticated ways. In Year 8 students explore the particular language relating to the exclusive or inclusive references to events. In Year 10 students make and appraise inferential statements relating to chance.

Year level	‘Chance’ content descriptions from the AC: Mathematics
Year 1 ◆ ◆	Students identify outcomes of familiar events involving chance and describe them using everyday language such as ‘will happen’, ‘won’t happen’ or ‘might happen’. ACMSP024
Year 2 ◆ ◆ ◆	Students identify practical activities and everyday events that involve chance. Describe outcomes as ‘likely’ or ‘unlikely’ and identify some events as ‘certain’ or ‘impossible’. ACMSP047
Year 3 ◆ ◆ ◆	Students conduct chance experiments, identify and describe possible outcomes and recognise variation in results. ACMSP067
Year 4 ◆ ◆	Students describe possible everyday events and order their chances of occurring. ACMSP092
Year 5 ◆ ◆ ◆	Students list outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions. ACMSP116
Year 6 ◆	Students describe probabilities using fractions, decimals and percentages. ACMSP144
Year 7 ◆ ◆	Students construct sample spaces for single-step experiments with equally likely outcomes. ACMSP167
Year 8 ◆ ◆	Students identify complementary events and use the sum of probabilities to solve problems. ACMSP204
Year 8 ◆	Students describe events using language of ‘at least’, ‘exclusive or’ (A or B but not both), ‘inclusive or’ (A or B or both) and ‘and’. ACMSP205
Year 8 ◆ ◆	Students represent events in two-way tables and Venn diagrams and solve related problems. ACMSP292
Year 9 ◆ ◆	Students list all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events. ACMSP225
Year 9 ◆ ◆ ◆	Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’. ACMSP226

Year 9 ◆ ◆	Students investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians. ACMSP227
Year 10 ◆ ◆ ◆	Students describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence. ACMSP246
Year 10 ◆	Students use the language of ‘if ....then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language. ACMSP247
Year 10A ◆	Students investigate reports of studies in digital media and elsewhere for information on their planning and implementation. ACMSP277

Numeracy continuum: Interpret chance events	
End Foundation	Recognise that some events might or might not happen.
End Year 2	Identify and describe familiar events that involve chance.
End Year 4	Describe possible outcomes from chance experiments using informal chance language and recognising variations in results.
End Year 6	Describe chance events and compare observed outcomes with predictions using numerical representations such as a 75% chance of rain or 50/50 chance of snow.
End Year 8	Describe and explain why the actual results of chance events are not always the same as expected results.
End Year 10	Explain the likelihood of multiple events occurring together by giving examples of situations when they might happen.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

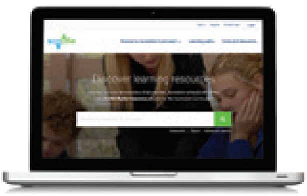
http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Scootle

https://www.scootle.edu.au/ec/p/home



This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.

reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

Numeracy in the News

http://www.mercurynie.com.au/mathguys/mercury.htm

Numeracy in the News is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.

TIMES modules

http://schools.amsi.org.au/times-modules/

TIMES modules are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.

Top drawer teachers – resources for teachers of mathematics (statistics)

http://topdrawer.aamt.edu.au/Statistics

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.

Double Helix Extra

https://blog.doublehelix.csiro.au/

This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.

CensusAtSchool NZ

http://new.censusatschool.org.nz/tools/random-sampler/

CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics. It aims to:

- *'foster a positive attitude to statistics through using data that is both relevant and real*
- *improve understanding of a data gathering process, its purposes and benefits to society*
- *provide access to large and meaningful multivariate data sets*
- *encourage effective IT teaching and learning*
- *enhance the process of statistical enquiry across the curriculum.'*

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.