

Data representation and interpretation: Year 8

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together

Contents

What the Australian Curriculum says about ‘Data representation and interpretation’	3
Content descriptions, year level descriptions, achievement standards and numeracy continuum	
Working with Data representation and interpretation	4
Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum	
Engaging learners	6
Classroom techniques for teaching Data representation and interpretation	
From tell to ask	7
Transforming tasks by modelling the construction of knowledge (Examples 1–6)	
Proficiency: Problem-solving	17
Proficiency emphasis and what questions to ask to activate it in your students (Examples 7–10)	
Connections between ‘Data representation and interpretation’ and other maths content	25
A summary of connections made in this resource	
‘Data representation and interpretation’ from Foundation to Year 10A	26
Resources	28

Resource key



The ‘AC’ icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.



The ‘From tell to ask’ icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about ‘Transforming Tasks’:
http://www.aclleadersresource.sa.edu.au/index.php?page=into_the_classroom



Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.



The ‘Bringing it to Life (BitL)’ tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the ‘Bringing it to Life’ tool:
http://www.aclleadersresource.sa.edu.au/index.php?page=bringing_it_to_life



Throughout this narrative—and summarised in ‘**Data representation and interpretation’ from Foundation to Year 10A** (see page 26)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with Data representation and interpretation:

- ◆ Posing a question
- ◆ Collecting and organising data
- ◆ Summarise and represent data
- ◆ Interpretation and inference.

What the Australian Curriculum says about ‘Data representation and interpretation’

Content descriptions

Strand | Statistics and probability.

Sub-strand | Data representation and interpretation.

Year 8 ◆ | ACMSP284

Students investigate techniques for collecting data, including census, sampling and observation.

Year 8 ◆ | ACMSP206

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

Year 8 ◆ | ACMSP293

Students explore the variation of means and proportions of random samples drawn from the same population.

Year 8 ◆ | ACMSP207

Students investigate the effect of individual data values, including outliers, on the mean and median.

Year level descriptions

Year 8 ◆ ◆ | Students explain the purpose of statistical measures.

Year 8 ◆ | Students find estimates of means and proportions of populations.

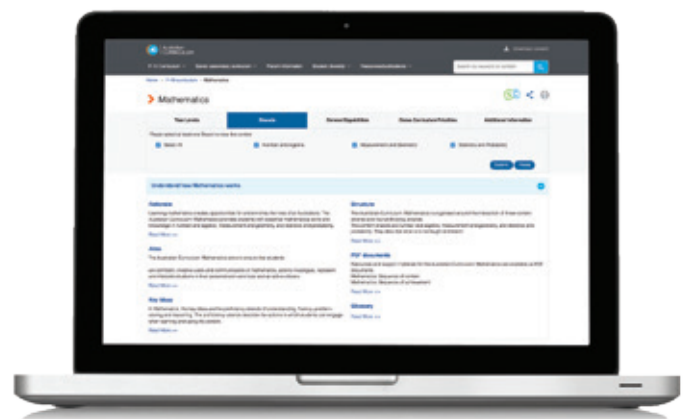
Achievement standards

Year 8 ◆ ◆ ◆ | Students explain issues related to the collection of data and the effect of outliers on means and medians in that data.

Numeracy continuum

Interpreting statistical information

End of Year 8 ◆ | Students compare, interpret and assess the effectiveness of different data displays of the same information (Interpreting statistical information: Interpret data displays).



Source: ACARA, Australian Curriculum: Mathematics

Working with Data representation and interpretation

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 8 ‘Data representation and interpretation’

Through Foundation to Year 10A, students identify questions, collect, represent and interpret data using increasingly sophisticated methods.

In Year 7 students construct graphs including stem-and-leaf plots. Students investigate issues about the collection of data and they calculate mean, median, mode and range for interpreting data.

In Year 8 students explore the practicalities of different techniques for data collection such as census, sampling and observation, and the variation in random samples from the same population.

In Year 9 students identify an issue where they need to collect both categorical and numerical data directly or from secondary sources. They graph back-to-back stem-and-leaf plots and histograms, and use mean, median and range to compare them. They also calculate these values from histograms, stem-and-leaf diagrams or dot plots.

In Year 10/10A students use terminology, including the terms ‘quartile’ and ‘interquartile range’, they then use these measures to enable them to create box plots. Students compare data that has been represented in the different forms that they have become familiar with in the last couple of years, for example, comparing the shapes of a box plot to its corresponding histogram. Students develop approaches to investigate relationships between two numerical variables over time. Students in Year 10 apply their understanding of statistics to evaluate statistical reports found in the media. In 10A standard deviation is used as a measure of spread and students use mean and standard deviation to compare data sets. Students also investigate techniques for establishing the ‘line of best fit’ when using digital technologies to investigate bivariate data sets.

- **Statistical investigations** help us gain a better understanding of an observed variation. It involves a series of important processes:
 - posing a question
 - collecting and organising data
 - summarising and representing data
 - interpreting the results
 - drawing inferences based on the evidence.

We give students a lot of opportunity to practise **summarising** (finding mean, medians, mode and range) and **representing** data (drawing a graph) but not always in a meaningful context. Sometimes we ask them to collect their own data (eg the number of cars that go past the school), but we have to ask, ‘Who would be interested in this information?’ It might be a meaningful exercise if the school were approaching the council to have a school crossing placed out the front. Often as teachers, we see that it’s important to tell students when a task ‘**might**’ be meaningful, but it is far more powerful to do a task that ‘**is**’ important to them.

- **The Year 8 AC: Mathematics content descriptions do not cover the full range of processes that constitute a statistical investigation. They are mostly about the variations in sampling.** To become fluent, students can practise their skills in other strands by conducting statistical investigations in the context of their learning. To be effective, we must make the statistical processes and decision-making explicit in the task. Identify for students that they are applying their statistical knowledge when they are collecting,

tabling and graphing data. Ask them to explain their choices and decisions, and encourage students to give examples of where they have used statistics in other learning areas. Be mindful too, that the Numeracy Continuum for Year 8 students requires them to *compare, interpret and assess the effectiveness of different data displays of the same information.*

- **Statistical investigations can be used to support inquiry-based learning in maths and other subjects.** Students can collect data to make generalisations to develop their own rules in algebra and patterning, to derive formulae in measurement (*Using units of measurement: Year 8*, ‘Example 7: How far will this ... roll?’) or discover properties in shape.

In Chance, students use statistics to conduct experiments and compare their outcomes to theoretical probabilities. Students can use statistics in physical education, science, history and English to provide evidence for arguments in their persuasive writing.
- **The statistical process is used to answer questions.** Teachers and students need to realise that findings can’t be proven using statistics, but rather conjectures can be made with varying levels of confidence. The findings are based on evidence and an argument is presented by the student/statistician. Reasoning is important at every stage of a statistical report:
 - *Convince me that this is a random sample; that it is representative and that it is big enough.*
 - *What will this evidence tell me about the question that has been asked?*

– *What assumptions and limitations are there to your investigation? How confident are you in your findings?*

- **Using representative data requires getting a fair sample.** In Year 8, students begin to ask more sophisticated and socially significant questions about bigger data sets and so use representative data, or samples, from both primary and secondary sources. It is now essential to know more about sampling and how it affects the information collected. There are challenges in obtaining representative data. How, when and where data is collected, are important decisions when choosing a fair sample.

For convenience, students often survey their own class when collecting data, without considering whether their class, as a group, represent:

- a sample (eg How much pocket money do **students** receive?), or
- a population (eg How much pocket money do **students in my class** receive?) for the question they had posed.

If the question is about all students in Year 8 in SA (the population), does their class represent a fair, representative sample that would provide enough information to answer the question?

Teachers need to provide opportunities for students to discuss whether such a sample is:

Fair

Does every student in the population (Year 8 students in SA) have the opportunity of being selected, or represented? For some questions, there may be a difference between country and city students and so, in that case, our class will not be a fair representation of the population.

Representative

For such a situation, there should be country and city representation, but there are more Year 8 students in the city than in the country in SA. So, there should be more Year 8 students from the city in the sample.

Large enough so you can make generalisations about the whole population with some confidence

If there are 12,000 Year 8 students in SA, then 110 (which is approximately the square root of 12,000, as a rule of thumb), would be considered large enough. But if the population has a lot of variation, the sample has to be big enough to show the same variation, as exists in the population.

If the question is 'Which AFL team do you follow?' there are 18 teams, so a sample size of 30 (class size) is not big enough to start showing any patterns. However, if the question is 'Which team do you prefer out of the Crows and Power?', there are only three options: Crows, Power or neither, and a trend is more likely to be seen in a sample of 30. (In 'Problem-solving', 'Example 7: Dot picture', there is a lot of variation in the distribution of the dots in the picture, so the students that only sample a small percentage of the area—eg 5%—will not predict the number of dots with any accuracy or confidence.)

- **How does biased sampling and variation impact statistical investigations?**

It is also important for teachers to provide students with opportunities to see the impact of biased data. (When doing 'Problem-solving', 'Example 6: Mark and recapture', replacing the marked buttons on the top of the jar and not shaking the jar up before re-sampling will give more of the marked buttons in the sample than there are in the population, and this will impact on the accuracy of the inferences made.)

The examples of bias due to sampling that students would be most familiar with, exist in advertising and the media. (Resources: Teachers should provide students with the opportunity to ask probing questions, such as those asked about the articles in 'Example 4: Numeracy in the news' from *The Mercury* newspaper, or better still, examples from local newspapers such as *The Advertiser*. This will encourage students to become more critical of whether the sampling process has been appropriate and how much confidence they should have in the claims being made.)

Even if the sample is fair, there is still the question of whether the data gives any useful information about the question you posed.

If using secondary data for your class investigation, always discuss how it might have been collected, why this type of information helps us in the investigation, and if there was other data that might have been useful or more appropriate.

Of course, the best way for students to appreciate these challenges is to make decisions about collecting data for their own questions.

In 'From tell to ask', 'Example 4: Mystery bag', students observe fair and random sampling from the same population, yielding a wide range of different samples. They see that the natural variation leads to a lot of different inferences (often misleading) about the population. Seeing and discussing the natural variation that occurs in sampling, will help them when they describe and interpret the information they get from samples.

Engaging learners

Classroom techniques for teaching Data representation and interpretation

Statistics engage students when they are posing questions that interest them and are relevant to their lives.

It is a valuable tool to help them make decisions and reasonable choices. They generate their own new knowledge and understanding based on evidence from the data they have collected.

Statistical investigation

Statistical investigation is an important real-life human endeavour and students should appreciate and experience this. Where there is variation, we seek to measure, represent and describe the variations. This helps us to predict changes, or to use this knowledge to control things like global warming, extinction of animal species, or the spread of disease.

Hans Rosling's 200 Countries, 200 Years, 4 Minutes – The Joy of Stats (BBC Four), is a dynamic demonstration with commentary that explores the development of nations over the years since 1800.

The video can be found at:

<http://www.youtube.com/watch?v=jbkSRLYSojo>



Source: *Hans Rosling's 200 Countries, 200 Years, 4 Minutes – The Joy of Stats* – BBC Four, BBC, 2010

Statistical investigations can be used to improve performance in sport, develop medical treatments, improve profits and plan for the future.

The New York Times examines modern Olympic results, in the context of 116 years of the Olympic Games, for a range of sports. **One Race, Every Medalist Ever** considers how far ahead of the field, Usain Bolt really is in the men's 100-metre sprint.

The video and statistical information can be found at: http://www.nytimes.com/interactive/2012/08/05/sports/olympics/the-100-meter-dash-one-race-every-medalist-ever.html?_r=0



Source: *One Race, Every Medalist Ever*, by Kevin Quealy and Graham Roberts, *The New York Times*, 2010

Teachers can create simulations that allow students to work mathematically in the same way as an ecologist, biologist, etc would in the field (eg 'Example 6: Mark and recapture' on page 15 of this narrative).

Statistics can also be used to deceive or persuade us through advertising or scamming. We need to be critical users and consumers of statistics, so we can improve our decision making (eg 'Example 8: What sort of detective are you? Can you pick a fake?' on page 21 of this narrative).

Using statistics in these contexts with topical and current issues is engaging for students.

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–11)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a ‘stand and deliver’ model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the name ‘mean’ or ‘histrogram’** for themselves. They need to receive this information in some way. However, it is possible for students to be challenged to **identify their own question about something of interest and design their own investigation to answer it**, so we don’t need to design and instruct the details of the investigation for them.

When we are feeling ‘time poor’ it’s tempting to believe that it will be quicker to fully design a statistical investigation, or set tasks, that we want students to experience rather than ask a question (or series of questions) and support them to planning the stages of the investigation for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students use a specific set the skills, knowledge and procedures during the current unit of work, then it probably is quicker to tell them what to do. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the formulae is a false economy of time.

On the other hand, we could start with a problem and support students in the design of an investigation, to explore a question that interests them. They will still practise skills and procedures, but in an authentic context while they are engaged in a problem that has some meaning for them, with the opportunity to think and work mathematically. **Telling** students how to conduct a statistical investigation removes a natural opportunity for students to create their own knowledge. When students plan and conduct their own investigation, they are in a much better position to analyse results and report on findings. This is the part of the process that students find most difficult.

Curriculum and pedagogy links

The following icons are used in each example:



The ‘**AC**’ icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.



The ‘**Bringing it to Life (BitL)**’ tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life



The ‘**From tell to ask**’ icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving ‘From tell to ask’ is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the ‘Transforming Tasks’ module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom



Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
<p>Example 1: Is our class a fair sample? Students investigate techniques for collecting data, including census, sampling and observation.</p>	ACMSP284 ◆
<p>Example 2: A mean (or median) game Students investigate the effect of individual data values, including outliers, on the mean and median.</p>	ACMSP207 ◆
<p>Example 3: Contrasting the median and mean Students investigate techniques for collecting data, including census, sampling and observation. Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes. Students investigate the effect of individual data values, including outliers, on the mean and median. Students explore the variation of means and proportions of random samples drawn from the same population.</p>	ACMSP284 ◆ ACMSP206 ◆ ACMSP207 ◆ ACMSP293 ◆
<p>Example 4: Numeracy in the news Students investigate techniques for collecting data, including census, sampling and observation. Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.</p>	ACMSP284 ◆ ACMSP206 ◆
<p>Example 5: Mystery bag Students investigate techniques for collecting data, including census, sampling and observation. Students explore the variation of means and proportions of random samples drawn from the same population. Students investigate the effect of individual data values, including outliers, on the mean and median.</p>	ACMSP284 ◆ ACMSP293 ◆ ACMSP207 ◆
<p>Example 6: Mark and recapture Students investigate techniques for collecting data, including census, sampling and observation. Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.</p>	ACMSP284 ◆ ACMSP206 ◆

Example 1: Is our class a fair sample?



ACMSP284 ◆

Students investigate techniques for collecting data, including census, sampling and observation.



Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you answer backwards questions?

Reasoning proficiency:

In what ways can you communicate?



Instead of **telling** students what would be a fair sample for a statistical investigation, we can challenge students to recognise how a fair sample depends on the question to be investigated, by **asking** questions.

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

Begin this activity by presenting the following questions to your students:

- **What statistical question might be asked, where our class would represent a fair sample? Why would we want to answer this question?** For example:
 - How do Year 8 students at our school travel to school? (Planning for bike racks.)
 - How many Year 8 students in our school order their lunch from the canteen? (Campaigning for a special queue for Year 8 students to collect their lunch.)
 - Which radio station do most Year 8 students in the metropolitan area listen too? (Advertising a Year 8 music festival.)

Teachers should draw students' attention to the nature of the questions asked and why you might be interested in this information.

- **How are these statistical questions, where our class would be a fair sample, the same? How are they different?**

It is apparent that these are not global issues, but mostly relate to their school and age group.

A good activity for students, is to undertake or even create, their own card match. Place a range of possible questions to be investigated on blue cards, and a range of possible sample groups that might be surveyed on red cards. Ask students to match the questions to the appropriate sample groups. Do not have the same number of each colour, and make it possible for there to be more than one appropriate match.

Blue cards:

- How do Year 8 students at our school travel to school?
- How many Year 8 students in our school order their lunch from the canteen?
- Which radio station do most Year 8 students in the metropolitan area listen too?
- Should students in the Middle Years wear school uniform?
- Should Year 8 students have a representative in the student leadership?

Red cards:

- 30 Year 8 students are randomly selected from 15 private and public schools throughout the state.
- All Year 8 students in our school are surveyed.
- 10 students from each Year 8 class in our school are surveyed.
- Half of the Year 8 students in our school are randomly selected, along with 20 students from both Years 9 and 10.
- 20 schools in the metropolitan area are randomly selected, and 30 Year 8 students are surveyed from each.

Provide some spare cards so students can make up their own questions that might match the given samples, and samples that might match the given questions.

Connecting a fluency activity to a context

Often, a fluency activity requires students to perform one stage of the statistical process in isolation from a context or investigation. For example, single tasks which instruct students to calculate the mean, draw a stem-and-leaf plot for the given data and read information from the graph. As teachers, we assume that students can connect these problems to meaningful situations, but it is important to ask questions to make the context more explicit:

Who would be interested in this?

Where might this data have been collected and from whom?

How do we know what this graph is about?

Why would you be interested in the mean, rather than the median or the mode?

Example 2: A mean (or median) game



ACMSP207 ♦

Students investigate the effect of individual data values, including outliers, on the mean and median.



Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Reasoning proficiency:

In what ways can you communicate?



Instead of **telling** students the difference between a mean and median score, we can challenge students to recognise the difference between the two central measures, by **asking** questions.

Most students at this level can accurately calculate a mean. They are often less familiar with how to calculate a median, what information it provides and when it is better to use a median (rather than a mean), as a measure of the centre of the data set. As teachers, we can support students to gain fluency in calculating, as well as a better understanding of these measures by introducing them to a game.

In this activity students play the game in pairs. Draw some playing cards from a pack and consider their numerical value (Ace:1, Jack:11, Queen:12, King:13). The game is played in rounds. In each round, students take it in turns to draw a given number of cards and decide by just looking at the cards, whether they want the mean or median for their score, then calculate it.

They draw 2 cards each in the first round and then 3 in the next, until in the last round they are drawing 10. The student with the highest score wins.

Students often forget to order the data when working with a string of numbers to calculate the median. In this game it is easy to arrange the numbers into ascending order by physically sorting the cards. After playing a few games, students will have developed strategies to win the game, based on the differences between means and medians that they have discovered for themselves. They realise that the mean is affected by extreme values, so if there is one card higher than the others, they are better to choose a mean and if there is one lower card, a median.

Have a class discussion to share strategies and then offer all students the opportunity to practise with their new knowledge. Students are practising calculating means and medians, as well as mental computation. The game can be varied so that the winner has the lowest score, or include the range of the data, as a third option.

Discuss the following with your students:

- **What have you noticed? What is the same about each round and what is different as the number of cards change?** (Students should notice that when 2 cards are dealt, that the mean and median are the same. This would also occur when the data is symmetrical – eg 3, 5, J, K mean and median = 8.)
- **Are there any unusual hands of cards?** (Students may consider having all cards the same as odd. This is an opportunity to talk about the mode and when it would be a sensible central measure to use in a practical situation – eg when a shoe manufacturer was determining which size(s) would be most profitable to manufacture.)

You could also collect examples from the media where means and/or medians are used. Ask students to make judgements based on what they have learnt from the game, whether the best measure has been used and why. For example, in real estate the median house prices of suburbs are frequently referred to rather than the mean value, because of the extreme variables in property data (eg beach-front homes as compared to those on a busy road).

(This activity is also on page 10 of the [Data representation and interpretation: Year 9](#) narrative.)

Example 3: Contrasting the median and mean



ACMSP284 ♦ Students investigate techniques for collecting data, including census, sampling and observation.

ACMSP206 ♦ Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP207 ♦ Students investigate the effect of individual data values, including outliers, on the mean and median.

ACMSP293 ♦ Students explore the variation of means and proportions of random samples drawn from the same population.



Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Reasoning proficiency:

In what ways can you communicate?



Instead of **telling** students the difference between a mean and median score, we can challenge students to recognise the difference between the two central measures, by **asking** questions.

This activity by the Australian Association of Mathematics Teachers is from their ‘Top drawer teachers – resources for teachers of mathematics’ website. It is designed to help students distinguish between means and medians in symmetric and skewed data sets.

Students can apply their understanding about means and medians from ‘Example 2: A mean (or median) game’ in a statistical investigation about the variation in two contrasting data sets. The students collect the length of fictitious students’ arm spans and the minted dates on 10 cent coins. The different contexts provide two data sets: one where symmetric data are expected and the other where the data are expected to be skewed.

The **Contrasting the median and mean** activity can be accessed at: <https://topdrawer.aamt.edu.au/Statistics/Activities/Contrasting-the-median-and-mean>



Example 4: Numeracy in the news



ACMSP284 ◆

Students investigate techniques for collecting data, including census, sampling and observation.

ACMSP206 ◆

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.



Questions from the BitL tool

Understanding proficiency:

Can you answer backwards questions?

Reasoning proficiency:

In what ways can you communicate?



Instead of **telling** students about the misuse of statistics, we can challenge students to identify the misrepresentation by **asking** questions.

'Numeracy in the News' is a website which holds multiple articles from the Tasmanian newspaper, *The Mercury*. The intention of the website is to raise the consciousness of students as critical readers of media reports. In this activity, we focus on terms of data collection and sampling in particular.

This resource, along with teacher discussion notes, is a great model for showing how you can adapt student questions to suit the articles from our local papers, such as *The Advertiser*.

By accessing the 'Data collection and sampling' page (via the icon as shown adjacent) you will find a range of questions you can use to ask your students about sampling, which can be applied to any newspaper article.

'Numeracy in the News' can be accessed at:
<http://www.mercury.com.au/mathguys/mercury.htm>



Source: 'Numeracy in the News', *The Mercury*, News Limited. News articles © 1997–2001, Copyright reserved, News Limited. Discussions © 1997–2001, Copyright reserved, Commonwealth of Australia and University of Tasmania.

Example 5: Mystery bag



ACMSP284 ◆

Students investigate techniques for collecting data, including census, sampling and observation.

ACMSP293 ◆

Students explore the variation of means and proportions of random samples drawn from the same population.

ACMSP207 ◆

Students investigate the effect of individual data values, including outliers, on the mean and median.



Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you answer a backwards question?

Reasoning proficiency:

In what ways can you communicate?



Instead of **telling** students about the natural variation in sampling, we can challenge students to experience this and its implications by **asking** questions.

Prediction and experimentation supports the development of conceptual understanding

To help students understand the way samples vary, we should allow them to sample from populations that can be counted at a later time, even if it is tedious to do so. This way, the true proportions can be checked and compared to the sampling results and the inferences that had been drawn. Using questions to prompt students to share what they notice, what connections they can make and what they can infer from their results, supports teachers to replace 'telling' the students information, with getting students to notice for themselves.

Mystery bag is a class activity that explores sampling and principles of probability, as well as providing practice in ratios. Students make individual and collaborative decisions about an unseen population, based on small samples.

This activity allows every student to take their own sample from the bag and make their own prediction about its contents. Look for every opportunity to allow students to guess, estimate, predict and make choices. This engages passive learners, giving them more ownership of their learning in a low-risk situation.

Materials

Create a 'mystery bag' by filling an empty bag with 60 coloured items as follows:

- 20 red
- 15 blue
- 10 green
- 10 yellow
- 4 white
- 1 black.

Activity

Present the mystery bag to the class and tell them there are 60 items in the bag.

Next, pass the bag around the class, instructing each student to take out 5 items. Without letting anyone else see what they have, ask each student to record the colour/s of their items, before replacing all the items back in the bag and passing it to the next student. (To make this task easier, you could consider using a mystery bag at each student table).

Once the students have completed this task, you can ask:

- *What is the connection between the sample you have and the contents of the bag (the population)?*
- *Can you predict the contents of the bag?*
- *Which colour has the greatest and the least number of items in the bag?*

Each student is to make a prediction about the contents of the mystery bag based on their selection and record it and their reasoning.

At this stage, the students will have different **levels of confidence** about their predictions. Based on their sample (eg 3 red, 1 blue and 1 white) ask them to make a statement that:

- they **know is true** (there are red, blue and white items in the bag)
- they are **very confident about** (there are more red items than blue or white)
- one that they are **somewhat confident about** (there are more than 10 red items in the bag)
- one that they are **not confident about** (there are 36 red items (3x12), 12 blue items (1x12) and 12 white items (1x12).

Students will quite often state the last point as one that they are confident about, because it has been calculated numerically which, for them, gives it more credibility. It is, however, unlikely that a sample of 5 exactly predicts a population of 60. As teachers, we should encourage students to use their understanding of ratios to interpret numerically what they have observed from their samples, but also help them to realise the limitations of this sampling when predicting the population.

Students then pair up and share their information to make new predictions based on the extra information.

Share some of the results from students' samples with the class:

- **Which one of the samples is the 'odd one out'?** *Why?* (A sample that contains a colour no one else has, or a sample with all the items the same colour.)
- **What if we combine the results of the samples? Are the predictions still the same as our first predictions?** (Students are now aware that there were colours in other samples that they did not have. More information, means they now know something about the population that they did not know before and should be given the option to change their prediction.)
- **Were our first predictions wrong?** (Students often feel the process is 'unfair' as their sample did not contain one of the colours. It is important for students to realise that when making inferences based on data, that their first predictions were not 'wrong'. They were completely valid for the information they had at the time.)

This illustrates that even if the samples are random, natural variation means that predictions based on samples may not be accurate.

This activity links strongly with chance and students could be asked to determine quantitatively, how likely these results were. The probability of picking the black on its own is $\frac{1}{60}$. The probability of getting 5 red items is $(\frac{20}{60} \times \frac{19}{59} \times \frac{18}{58} \times \frac{17}{57} \times \frac{16}{56})$. While students at this level are not generally familiar with the concept of independent events, students can be told about how theoretical probabilities can be worked out. Students can be asked if this *makes sense*. Sample spaces can be demonstrated with a tree diagram and evaluated to give some sense of the unlikely nature of the event.

Maths class is a place for talk

Reasoning is critical to an effective statistical investigation. Look for opportunities to ask your students to explain their thinking to you, to themselves, or to their peers.

Discuss the following with your class:

- **If Student A predicted 36 red, 12 black and 12 white; what might their sample have been?**
- **Would their prediction always be the same ratio as the sample?** (If we had taken a sample of 7 which is not a factor of 60, the ratio would not yield a prediction of whole numbers, which is required for this experiment.)
- **What other factors might they consider?** (Recognising that samples vary, particularly in big populations, you might round off to the nearest 5 or 10 when making predictions; recognising that the process does not support high levels of accuracy.)
- **What can you conclude if your sample was 5 red?** (If you do not have any other information about the contents, you would assume that the population contains 60 red items.
An outlier, or unusual result such as 5 red, is not always excluded from the data. Outliers are removed if they are errors, such as: a mistake in counting (brown mistaken as red); or recording (marking a blue as a red); or the subject doesn't belong to the set (the sample was taken from a different bag). If the result is not due to an error, the decision on keeping or discarding depends on the question being asked. In this case, the sample supports the prediction that there are more red items than other colours in the bag.)

Teachers can support students to better understand important considerations when sampling, by getting them to think about sampling in different ways:

- **Is there another way to sample?**
- **Would combining the results from 12 students (each with a sample of 5) provide sufficient information to predict the exact contents of the mystery bag?** (There were 60 (12x5) items in the bag, but the samples were replaced before the next draw, so it is not certain that all 60 items were drawn in 12 samples. If the samples were not replaced, then the entire population would be known.)
- **What sample size is sufficient to be absolutely certain of the contents?** (This emphasises that only by considering the whole population (a census) can you be certain of the contents.)

To extend this activity further, fill a mystery bag with a large number of coloured items that is unknown, and get groups to plan a statistical investigation that relates to determining the contents of the bag.

(A similar activity can be found on page 18 of the *Chance: Year 10/10A* narrative.)

Example 6: Mark and recapture



ACMSP284 ◆

Students investigate techniques for collecting data, including census, sampling and observation.

ACMSP206 ◆

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.



Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you answer backwards questions?

Reasoning proficiency:

In what ways can you communicate?



Instead of **telling** students about the natural variation in sampling, we can challenge students to experience this and its implications by **asking** questions.

'Mark and recapture' is a method used by ecologists to estimate the population size of animals.

The steps are as follows:

- 1 Some animals from a population are captured, marked, and then released.
- 2 At a later stage, a select number of the animals from the population are captured at random.
- 3 They count how many of the animals in the sample are marked.
- 4 The proportion of the marked animals in the sample, is considered to be the same as the proportion of marked individuals in the whole population. Based on this information, they can estimate the total population size.

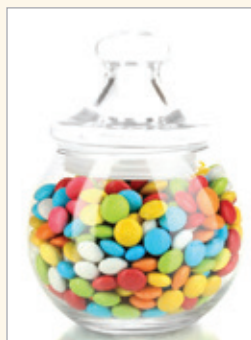
While students can't use 'mark and recapture' to estimate the size of real animal populations (unless you have an enclosed ant colony in your classroom), teachers can create a simulation which allows students to work mathematically, in the same way as an ecologist would in the field.

How many buttons are there are in the jar?

When tackling a problem such as this, it is not necessary to determine the 'best' way to investigate. Considering different ways to sample, calculate, or represent allows students to think more deeply about the problem and provides opportunities for them to use their reasoning skills to explain and justify their choices.

Present a jar containing buttons to your class and ask, 'How many buttons are there in this jar?':

- **Can you estimate?** (Students will probably suggest to simply count them.)
- **What is another way to work that out?**



Consider how banks determine how many coins are in a packet. It is by weight. Banks use balancing scales to check that the customer's packet of coins, balances with a packet with the correct number of coins. Ask students:

- **Can you explain why that works? Why do they do it that way, rather than counting? Could we use this idea to work out how many buttons there are?**
- **What is the same, and what is different, about counting coins and counting buttons? What if we used buttons instead of coins, would that still work?** (Buttons may be lighter than coins and it may be hard to determine if they balance exactly. Are the buttons all the same size?)
- **Is there another way to use a weighing strategy?** (You could find the weight of one button, then weigh the total population. You will need scales that can measure small weights accurately.)
- **How accurate can you be? Does it make a difference? Will they all weigh the same?** (Encourage students to devise ways to test their ideas, eg weighing several different buttons, getting several students to weigh the same button, weighing 10 buttons to determine the average weight, etc.)

Can we use 'mark and recapture?' Take an appropriate number of buttons from the jar, mark and then return them:

- **How many buttons should I take? How many would be too few? How many would be too many?** (If you choose too small a number, they are not likely to be in the sample you recapture; if you choose too many it will take a long time and you might as well have counted the whole jar.)

After you have returned the marked buttons to the jar, ensure that the jar is shaken thoroughly. If you put the buttons on the top of the jar and then commence to select your sample, students will often point out their concerns that it will not be a random sample. If they don't, you can take the sample and then ask:

- **What if we shake up the jar, will we get the same results? Let's try it.**

- *How many shall we recapture?*
- *Can we sample in a different way?*

Students may want to have larger or smaller samples, sample once, or sample multiple times. Allow students to approach this in different ways, clearly stating the reasons for their choice. It gives them control over their learning and also provides good data for comparison when the final results are shared.

Consider if you had marked 50 and returned them to the population. When you recaptured a sample of 100, it is noted that 10 of them were marked.

The proportion of marked buttons in the sample, P_s , must equal the proportion marked in the population, P_p .

$P_s = \frac{10}{100}$ must equal $P_p = \frac{50}{\text{Population}}$. This can be solved using equations, equivalent fractions or trial and error, developing fluency in Number and Algebra.

$$\frac{10}{100} = \frac{50}{\text{Population}}$$

$$\frac{1}{10} = \frac{50}{\text{Population}}$$

$$\text{Population} = 10 \times 50 = 500$$

To extend this activity, you can ask 'How many of each colour are in the jar?':

- *How might you find out how many of each colour are in the jar? Can you use your sampling to determine how many of each colour there are in this particular jar?* (Students can apply the skills they gained in the 'Example 5: Mystery bag' activity on page 13.)
- *If a manufacturer always produced buttons in the same proportion as they are in this jar, how big should the container be if there were 100 red buttons?*

Levels of confidence

The reliability of results in the Statistics and probability strand depends on range of factors. Students can be made more aware of this by asking them to identify their level of confidence about their findings or predictions.

Ask them to make a statement that they **know is true**, one that they are **very confident about**, one that they are **somewhat confident about** and one that they are **not confident about**.

This may lead them to decide that they need to collect more, or different, information in order to improve the significance of their findings.

Even when a final answer is reached, a teacher can provide the opportunity for students to consider how confident they are in the result and consider what assumptions and limitations exist in their strategy. This develops skills in reasoning.

Students can also consolidate and demonstrate their understanding by considering a backwards or related question. This might be one the teacher poses, or one the students themselves are curious about.

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 7–10)

Problems are described in the AC: Mathematics as ‘meaningful’ or ‘unfamiliar’. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as ‘fun’ nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, ‘I don’t know’, or ‘I’m not sure’. Students respond differently to this feeling; some shut down, others begin to ask, ‘But how could I work that out?’

In developing powerful learners we are aiming for all of our students to learn that ‘not knowing’ is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, ‘What could I do to work this out?’

Engaging in problem-solving supports the move *from tell to ask*

Instead of **telling** students:

- the problem to solve
- the information they’ll need
- the steps they should take.

We can **ask students to identify**:

- the problem to solve
- the information they’ll need
- a possible process to use.

Proficiency: Problem-solving examples

Example 7: Dot picture

Students investigate techniques for collecting data, including census, sampling and observation. Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP284 ◆

ACMSP206 ◆

Example 8: What sort of detective are you? Can you pick a fake?

Students investigate techniques for collecting data, including census, sampling and observation. Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP284 ◆

ACMSP206 ◆

Students explore the variation of means and proportions of random samples drawn from the same population.

ACMSP293 ◆

Example 9: Three in a row – a winning strategy

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP206 ◆

Students explore the variation of means and proportions of random samples drawn from the same population.

ACMSP293 ◆

Example 10: Random or not – analyse numbers of jubes (1:1:1)

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP206 ◆

Students explore the variation of means and proportions of random samples drawn from the same population.

ACMSP293 ◆

Example 7: Dot picture



ACMSP284 ◆

Students investigate techniques for collecting data, including census, sampling and observation.

ACMSP206 ◆

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.



Questions from the BitL tool

Problem-solving proficiency:

**Interpret; Model and plan;
Solve and check; Reflect.**

Reasoning proficiency:

What can you infer?



Instead of **telling** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by **asking** questions.

Use sampling to determine how many dots there are in this painting and what colours they are.



To present this activity to your class, you could use an Aboriginal dot painting (as used here), a large picture of an ant colony, an aerial photo of a concert crowd, or ideally an image relevant to your class.

Interpret

What have you been asked to calculate? What information is helpful/no use? Why is sampling suggested? (There needs to be too many dots for the students to consider counting, so that the task is more purposeful.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Can you estimate? What sampling methods have you used before? Are there any you could use here? What do you think would be easiest/most efficient way to sample a section of the painting? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

While students have taken samples out of the 'Mystery bag' and used 'mark and recapture' for the buttons, here the dots cannot be physically manipulated, and so they need to devise a more appropriate sampling technique.

Teachers can become more supportive if students cannot make progress by asking:

- *How could you randomly select a smaller piece of the painting? How many samples will you need to count?* (Students may use a range of area-related techniques, eg: dividing the painting into a grid like a street directory, using coordinates to label each square and then using random numbers to determine which square(s) they will count; or consolidating their understanding of concepts in coordinate geometry and location. They may want to try making a shape from wire and randomly dropping it on the painting a number of times. Be careful not to direct students to any particular method before they have had an opportunity to devise their own. They often come up with very original and effective methods that you have not considered.)

Solve and check

How can you use the samples to predict how many dots there are? How accurate do you think your method might be? How could you investigate that? Could you sample in a different way? Is there another way that you could have solved this problem? (If there is a lot of variation in the painting as there is with this one, one sample is not enough, and they may choose to find a mean or median for the number of dots. If the sampled area represents one hundredth of the painting area, then the results can be multiplied by 100. Some students may want to actually count the number in the original painting. Generally, to see if they were 'right', but it also provides good information to refer to in their reflection.)

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any dot painting? (Encourage students to find their own dot paintings and explain why the method may or may not be appropriate.)

Assumptions and limitations

Teachers must require students to state the limitations and assumptions in their findings. This does not come naturally to students who feel they are criticising their own mathematical thinking, and this might affect your assessment of their work. It is the responsibility of the teacher, to emphasise that this is an essential part of using statistics ethically in the search for new, purposeful knowledge.

Example 8: What sort of detective are you? Can you pick a fake?



ACMSP284 ◆

Students investigate techniques for collecting data, including census, sampling and observation.

ACMSP206 ◆

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP293 ◆

Students explore the variation of means and proportions of random samples drawn from the same population.



Questions from the BitL tool

Problem-solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?



Instead of *telling* students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by *asking* questions.

In this activity, you can challenge students to investigate a suspected counterfeit ring using their own observations, knowledge of random events and problem-solving skills.

This task challenges students' expectations for a process that is very familiar to them, tossing a coin. In 100 tosses, students expect there to be approximately 50 of each outcome: heads and tails. While they do not expect these two outcomes to alternate, they don't expect there will be a long run of one outcome or the other. Surprisingly in 100 tosses, it is almost certain that there will be a run of at least 6 heads or 6 tails.

Distribute a numbered recording sheet to each student in the class for anonymous identification purposes. As a homework task, ask students to complete a record of the outcomes of 100 tosses of a coin. Before they begin recording, instruct them to toss the coin once. If the result is a head, the students must genuinely toss the coin 100 times. If, however, the result is a tail, they are to record the outcomes they might expect if they were tossing the coin. (Impress upon them that it is important that you have accurate data if it is to be a valid investigation.)

If you examine the recording sheets, you can identify the authentic data sets as they will have a significant number of 'clusters', or runs of heads or tails. Students who are 'faking' the data will not have long runs of heads or tails. Use this knowledge to sort the sheets into two piles. Ask the students to identify if you have placed their sheet in the correct pile.

It is important to stress that you are not magical, or performing any trick. You have only used your mathematical skills to make decisions. As detectives, they are challenged to determine a method that can be used by others to identify fake data.

Note: there is a digital activity that students can use as part of their investigation called 'Coin tossing' from the National Library of Virtual Manipulatives: <http://nlvm.usu.edu>. The activity is also on Scootle: <http://www.scootle.edu.au/ec/viewing/L3515/index.html>.

(This activity also appears in the *Chance: Year 8* narrative where the empirical evidence can be used to challenge students' beliefs about randomness and expected outcomes for independent events.)

Interpret

What have you been asked to find? What information is helpful/not useful? (Students know if their own data is authentic or not. Observing the sheets in each pile and trying to identify what is the same about the sheets in the authentic pile, and how that is different to the sheets in the 'faked' pile, would give significant clues as to the method you used when sorting. The task is most challenging if they receive the unsorted pile of the data sets, as you did.)

Model and plan

Do you have an idea? How might you start? Would it help if you organised the data? Are there different ways that you could do that? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying. Students do not often value the information they get from visually checking the data for patterns, and that is a key skill in approaching this problem.)

Can you see any differences, or similarities, between the authentic and faked data results?

Solve and check

If the data looks different, how could you investigate that? Is there something about the data that you could count and record? How could you represent this so that others could see the pattern? (Students could record the number of runs of different lengths, ie: a run of 1 (head or tail); or a run of 2 (head/head or tail/tail), etc for authentic and 'faked' data to compare. Notice that it does not matter if it is a run of heads or a run of tails, it is only the length that is of interest. These runs can be recorded and graphed. Choosing an appropriate table and graph type are evidence of fluency. Once students have developed a conjecture (theory) for how to tell the difference, they can develop a written statement about their process to help others identify a 'fake'. This is an opportunity for students to demonstrate their reasoning skills.)

How could you check your findings? (To extend this activity, you could provide a new collection of recording sheets; some that have been randomly generated and others that a group of teachers have faked, and challenge the students to catch the 'criminals'.)

Supporting writing in mathematics

For students who need more support in their writing, develop a wordlist they could choose from, or provide some sentence starters. You could also consider recording the student explaining it to someone else, and then using that to sequence their written explanation.

Reflect

Could you improve your method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Would you always pick the fake correctly? Will your method work for any data sets? (Now that the students have more knowledge, if you conducted the experiment again, the results would not be the same.)

Tackling misconceptions: Knowing what it isn't

It can work well if the teacher compiles a number of examples that reflect their students' misconceptions. The examples don't have to be presented as their misconceptions, just as examples of thinking that are commonly seen when people are learning to work with data. Once the students have identified the issues, they can reflect on the effect that it has had on their own thinking. For example:

One group said they threw 10 heads in row. Another group said that was impossible with a fair coin, so they must have cheated. What do you think? (It is highly unlikely. Possible, but not probable.)

Example 9: Three in a row – a winning strategy



ACMSP206 ♦

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP293 ♦

Students explore the variation of means and proportions of random samples drawn from the same population.



Questions from the BitL tool

Problem-solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?



Instead of **telling**

students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by **asking** questions.

'Three in a row' is a game played between two people. The object of the game is to choose the sequence that will appear from tossing a coin 3 times, eg heads, tails and tails (HTT). Each player discloses which sequence they think will win, prior to playing the game. In this task, students have the advantage of knowing the sequence their opponent has selected (HTT), and can use this knowledge to identify a tactic that will allow them to choose a winning sequence.

Interpret

What have you been asked to find? What information is helpful/not useful? (Students might consider the results from tossing a coin 100 times, and think about what would they need to choose to beat their opponent. If they check the data, they will soon realise it must be HHT or THT, as HT are the first 2 outcomes in your opponent's winning sequence.)

Model and plan

Do you have an idea? How might you start? What might you look for in the data? Are there different ways that you could do that? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.) Then ask students: *What about other opponent combinations? What might your sequence need to be so that it appeared before theirs?*

Solve and check

Does that give you any ideas? How could you investigate that? Is the strategy the same for all your opponent's combinations? How could you represent this, so that others could see the pattern? How could you check your findings?

Reflect

Could you improve your method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Would you always win the game? Will your method work for any combination? What about '4 in a row'?

The aim of a statistical investigation is to gain more information about complex situations where we observe random variation, so we need to be able to generalise about our findings. Students at this level are often challenged by the fact that most often there is no definitive or 'right' answer to the questions that we ask. While we can sometimes make a conjecture based on patterns we see, a proof of our theory needs to be sought by other means.

(A version of this activity appears in the *Chance: Year 10/10A* narrative – 'Example 5: Is this a fair game?')

Supporting writing in mathematics

When inferences are drawn from data about the connections we identify, we need to use modalities in our language that indicate the level of confidence we have in the findings.

As teachers, we need to model the use of this language, supply word lists, and discuss both good and bad examples of generalised statements arising in the use of statistics.

Encourage: Based on this sample ..., The data suggests ..., For students in this group ..., Mostly, Rarely, Often, Generally, You might expect ..., Probable, Possible, Estimated to be between ..., Is not likely to be less than ...

Challenge: Impossible, Always, Proves, Never, Certainty, Is, Will be, Must be.

The teacher modules for 'Data investigation and interpretation' from the **Australian Mathematical Sciences Institute (AMSI) TIMES Project** written for all years from Foundation to Year 10, are a great knowledge source for teachers and model good analysis and inference reports.

The Year 8 module can be accessed at:

http://amsi.org.au/teacher_modules/Data_Investigation_and_interpretation8.html

Example 10: Random or not – analyse numbers of jubes (1.1.1)



ACMSP206 ◆

Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes.

ACMSP293 ◆

Students explore the variation of means and proportions of random samples drawn from the same population.



Questions from the BitL tool

Problem-solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?



Instead of **telling** students the information they'll need and the steps they should take, we can challenge them to identify the information they'll need and the steps they could take by **asking** questions.

This activity is from the Scootle website.

In this activity, students are presented with a fruit jube machine, which randomly generates packets of 12 jubes; each containing a combination of penguins, fish and frogs. Each jube type is equally likely to be produced. By analysing the data, the students are tasked with finding the percentage of the most common type of jube, likely to be found in a single packet.

There is a structured digital activity investigation that students can undertake, but it is essential that teachers require students to reflect and report on what they experience, to ensure they connect intellectually with the digital activity and the learning opportunity it presents.

When the thinking has been done for them

In some tasks, the information is already in tables, or statistics have been calculated, or graphs drawn. To ensure that students are equipped to make informed choices, teachers need to be explicit about the features of these tables, graphs and statistics that make them appropriate for different types of data and the context of the statistical questions.

Discuss the following with students:

- *Was there an odd result? Are the overall results what you expected?*
- *What do you notice about how the data is recorded and represented? Is there another way to do this?*
- *What language do they use to generalise?*
- *What does this make you wonder about the packaging of jubes by this machine?*

These types of questions are about the students experiencing the fact that their ideas can change as new information is discovered or revealed (as examined in 'Example 5: Mystery bag'). Students can design their own 'Is it possible ...?' and 'Is it probable ...?' questions.

Alternatively, this activity can be changed to a questioning activity, so the students can ask questions of the person who has made an inference based on their investigation.

Use this digital activity to answer your own question:

- *What do you wonder?*

There are many opportunities throughout this narrative, for teachers to challenge students to communicate their thinking in different mediums. Students at this level have a range of standardised statistics (mean, median, mode) and representations (lists, tables, graphs) to communicate the patterns and connections they have identified.

Teachers can encourage students to choose which suit their context by asking questions of the form:

- *Would the median/mode of the data tell us something about the data?*
- *Why draw a column graph and not a pie chart?*
- *Why would you draw a side-by-side graph, rather than two different graphs?*

This is also a good way to understand a student's reasoning.

After exploring this activity, teachers can stretch students' thinking by allowing them to use the machine to conduct their own open investigations, and answer questions that they wonder about.



The **Random or not** activity can be accessed at: <http://www.scootle.edu.au/ec/viewing/L3655/index.html>

Connections between ‘Data representation and interpretation’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use Data representation and interpretation as a starting point.

Statistical investigations provide students with the opportunity to demonstrate all four proficiencies, in particular problem-solving and reasoning. Find out where students will be dealing with data in their other subjects and collaborate with other teachers to connect and extend their learning across the curriculum.

Here are just some of the possible connections that can be made:

Mathematics: Year 7–8	
Whilst working with Data representation and interpretation, connections can be made to:	How the connection might be made:
Assign probabilities to the outcomes of events and determine probabilities for events. (Year 7) ACMSP168	Refer to: Example 8: What sort of detective are you? Can you pick a fake?
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies. ACMNA183	Refer to: Example 5: Mystery bag Example 6: Mark and recapture Example 7: Dot painting Example 8: What sort of detective are you? Can you pick a fake?
Solve a range of problems involving rates and ratios, with and without digital technologies. ACMNA188	Refer to: Example 5: Mystery bag Example 6: Mark and recapture Example 7: Dot painting
Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies. ACMNA187	Refer to: Example 5: Mystery bag Example 8: What sort of detective are you? Can you pick a fake?

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility to help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Data representation and interpretation’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Data representation and interpretation:

Posing a question ◆

In Foundation to Year 3 students pose and answer questions.

Collecting and organising data ◆

In Foundation to Year 3 students collect information. In Year 6 to Year 8 students collect information.

Summarise and represent data ◆

In Years 4 to Year 10/10A students are mostly summarising and representing data.

Interpretation and inference ◆

In Year 9 and Year 10/10A students are interpreting and inferring.

Year level	‘Data representation and interpretation’ content descriptions from the AC: Mathematics
Foundation ◆ ◆	Students answer yes/no questions to collect information. ACMSP011
Year 1 ◆ ◆	Students choose simple questions and gather responses. ACMSP262
Year 1 ◆ ◆	Students represent data with objects and drawings where one object or drawing represents one data value. Students describe the displays. ACMSP263
Year 2 ◆ ◆	Students identify a question of interest based on one categorical variable. Students gather data relevant to the question. ACMSP048
Year 2 ◆	Students collect, check and classify data. ACMSP049
Year 2 ◆ ◆	Students create displays of data using lists, table and picture graphs and interpret them. ACMSP050
Year 3 ◆ ◆	Students identify questions or issues for categorical variables. Students identify data sources and plan methods of data collection and recording. ACMSP068
Year 3 ◆ ◆	Students collect data, organise into categories and create displays using lists, tables, picture graphs and simple column graphs, with and without the use of digital technologies. ACMSP069
Year 3 ◆	Students interpret and compare data displays. ACMSP070
Year 4 ◆	Students select and trial methods for data collection, including survey questions and recording sheets. ACMSP095
Year 4 ◆	Students construct suitable data displays, with and without the use of digital technologies, from given or collected data. Students include tables, column graphs and picture graphs where one picture can represent many data values. ACMSP096
Year 4 ◆	Students evaluate the effectiveness of different displays in illustrating data features including variability. ACMSP097
Year 5 ◆ ◆	Students pose questions and collect categorical or numerical data by observation or survey. ACMSP118
Year 5 ◆	Students construct displays, including column graphs, dot plots and tables, appropriate for data type, with and without the use of digital technologies. ACMSP119
Year 5 ◆	Students describe and interpret different data sets in context. ACMSP120
Year 6 ◆	Students interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables. ACMSP147
Year 6 ◆	Students interpret secondary data presented in digital media and elsewhere. ACMSP148
Year 7 ◆ ◆	Students identify and investigate issues involving numerical data collected from primary and secondary sources. ACMSP169

Year 7 ♦	Students construct and compare a range of data displays including stem-and-leaf plots and dot plots. ACMSP170
Year 7 ♦ ♦	Students calculate mean, median, mode and range for sets of data. Students interpret these statistics in the context of data. ACMSP171
Year 7 ♦	Students describe and interpret data displays using median, mean and range. ACMSP172
Year 8 ♦	Students investigate techniques for collecting data, including census, sampling and observation. ACMSP284
Year 8 ♦	Students explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes. ACMSP206
Year 8 ♦	Students explore the variation of means and proportions of random samples drawn from the same population. ACMSP293
Year 8 ♦	Students investigate the effect of individual data values, including outliers, on the mean and median. ACMSP207
Year 9 ♦ ♦	Students identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources. ACMSP228
Year 9 ♦	Students construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi modal'. ACMSP282
Year 9 ♦ ♦	Students compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread. ACMSP283
Year 10 ♦	Students determine quartiles and interquartile range. ACMSP248
Year 10 ♦ ♦	Students construct and interpret box plots and use them to compare data sets. ACMSP249
Year 10 ♦	Students compare shapes of box plots to corresponding histograms and dot plots. ACMSP250
Year 10 ♦ ♦	Students use scatter plots to investigate and comment on relationships between two numerical variables. ACMSP251
Year 10 ♦	Students investigate and describe bivariate numerical data where the independent variable is time. ACMSP252
Year 10 ♦	Students evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data. ACMSP253
Year 10A ♦ ♦	Students calculate and interpret the mean and standard deviation of data and use these to compare data sets. ACMSP278
Year 10A ♦ ♦	Students use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation. ACMSP279

Numeracy continuum: Interpreting statistical information	
End Foundation	Recognise how to ask and answer simple data questions and interpret data in drawings or picture graphs.
End Year 2	Collect and describe data on a relevant issue based on one variable and display as lists, tables or picture graphs.
End Year 4	Collect record and display data as tables, diagrams, picture graphs and column graphs.
End Year 6	Collect, compare, describe and interpret data as 2-way tables, double column graphs and sector graphs, including from digital media.
End Year 8	Compare, interpret and assess the effectiveness of different data displays of the same information.
End Year 10	Evaluate media statistics and trends by linking claims to data displays, statistics and representative data.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

<http://nrich.maths.org>

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.



The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

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Dan Meyer's blog: 101 questions

<http://www.101qs.com>

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*



A spreadsheet of *Dan Meyer's Three-Act Maths Tasks* can be accessed at <http://bit.ly/DM3ActMathTasks>.

Scootle

<https://www.scootle.edu.au/ec/p/home>

This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.



Estimation 180

<http://www.estimated180.com>

Estimation 180 is a website with a bank of daily estimation challenges to help students to improve both their number sense and problem-solving skills.



reSolve: maths by inquiry

<https://www.resolve.edu.au>

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.



Plus Magazine

<https://plus.maths.org>

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.



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Numeracy in the News

<http://www.mercurynie.com.au/mathguys/mercury.htm>

Numeracy in the News is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.



TIMES modules

<http://schools.amsi.org.au/times-modules/>

TIMES modules are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.



Top drawer teachers – resources for teachers of mathematics (statistics)

<http://topdrawer.aamt.edu.au/Statistics>

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.



Double Helix Extra

<https://blog.doublehelix.csiro.au/>

This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.



CensusAtSchool NZ

<http://new.censusatschool.org.nz/tools/random-sampler/>

CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics.

It aims to:

- 'foster a positive attitude to statistics through using data that is both relevant and real
- improve understanding of a data gathering process, its purposes and benefits to society
- provide access to large and meaningful multivariate data sets
- encourage effective IT teaching and learning
- enhance the process of statistical enquiry across the curriculum.'



Do you want to feel more confident about the maths you are teaching?
Do you want activities that support you to embed the proficiencies?
Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

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Excluded from NEALS

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