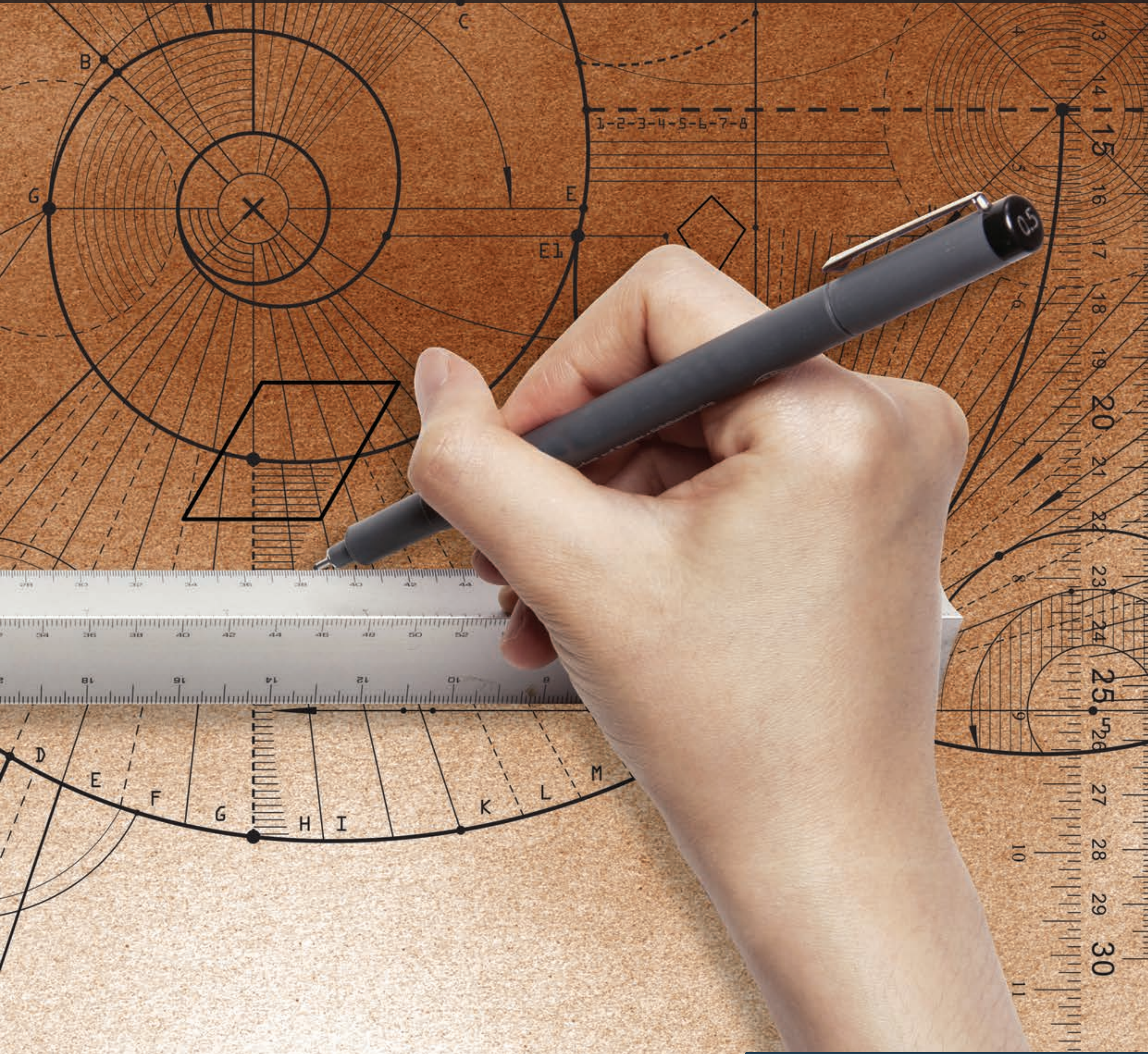
Using units of measurement: Year 8

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education and Child Development

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Using units of measurement' from Foundation to Year 10A** (see page 25)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with units of measurement:

- Using informal units for direct or indirect comparisons
- Using standard metric units
- Establishing and applying formulae
- Estimating.

What the Australian Curriculum says about ‘using units of measurement’

Content descriptions

Strand | Measurement and geometry.

Sub-strand | Using units of measurement.

Year 8 ◆ | ACMMG195
Students choose appropriate units of measurement for area and volume and convert from one unit to another.

Year 8 ◆ | ACMMG196
Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Year 8 ◆ | ACMMG197
Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area.

Year 8 ◆ | ACMMG198
Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.

Year level descriptions

Year 8 ◆ | Students explain measurements of perimeter and area.

Year 8 ◆ | Students evaluate perimeters, areas of common shapes and their volumes and three dimensional objects.

Year 8 ◆ | Students formulate and model practical situations involving areas and perimeters of common shapes.

Year 8 ◆ | Students justify the result of a calculation or estimation as reasonable.

Achievement standards

Year 8 ◆ | Solve problems relating to the volume of prisms.

Year 8 ◆ | Students convert between units of measurement for area and volume.

Year 8 ◆ | Students perform calculations to determine perimeter and area of parallelograms, rhombuses and kites.

Year 8 ◆ | Students name the features of circles and calculate the areas and circumferences of circles.

Numeracy continuum

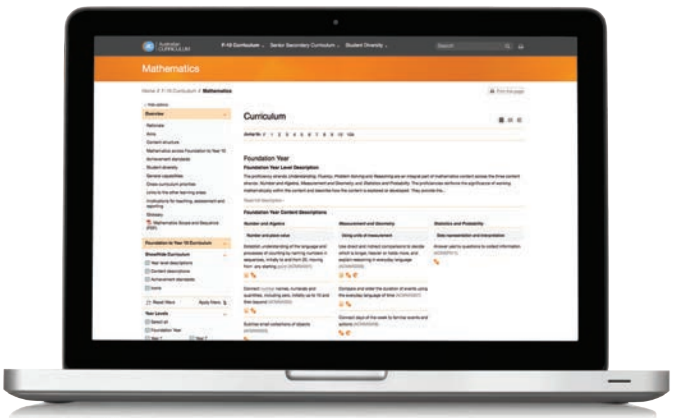
Estimating and calculating with whole numbers | Using measurement

End of Year 8 ◆ | Students solve complex problems by estimating and calculating using efficient mental, written and digital strategies (Estimating and calculating with whole numbers).

End of Year 8 ◆ | Students convert between common metric units for volume and capacity.

End of Year 8 ◆ | Students use perimeter, area and volume formulas to solve authentic problems.

Note: In the Australian Curriculum: Mathematics, the concept of ‘time’ is addressed in the sub-strand ‘Using units of measurement’, but in this resource, ‘time’ has its own narrative.



Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Working with units of measurement

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 8 'using units of measurement'

In Year 7 students *establish* formulas for the area of rectangles, triangles and parallelograms. This is an application of the use of variables, introduced in Year 7, described in the patterns and algebra sub-strand. Students in Year 7 also calculate volumes of rectangular prisms.

In Year 8 area and perimeter calculations include trapezium and kites. Students *investigate* the area and circumference of circles and *they develop formulas* for calculating the volume of prisms. At this stage students convert between units of area and volume, such as changing from millimetres squared to metres squared.

In Year 9 the focus is on calculating areas and perimeters of composite shapes and surface areas of prisms. Students establish the formula for the volume of a cylinder and solve problems involving the volume of cylinders and prisms.

- **There is an expectation that, during Year 7, students will have *established* formulas for the area of rectangles, triangles and parallelograms.** If students have *been told* the relevant formulae rather than supported to *establish* the formulae, they may not have strong enough grounding to be able to establish formulas relating to prisms, as expected in the Year 8 curriculum. Hence, Year 8 teachers may choose to challenge students to explain why the formulas (that they already know) work.
- **Measurement is the use of number applied to a spatial context** but we do not need to develop the relevant number skills before we start. Measurement provides a context for developing and consolidating understanding of concepts within number, eg fractions, decimals, percentages, ratio and proportion. Measurement is also a great context for working with scale, enlargement, shape, angle and statistics. Connections made in this narrative can be found on page 24.
- **Measurement skills are frequently used in the context of estimating.** Although the AC: Mathematics content descriptions do not state that students should estimate measurements, the numeracy continuum does acknowledge the importance of developing a capacity to estimate measurements and make sensible judgements. The numeracy continuum states that by the end of Year 8, students solve complex problems by estimating and calculating using efficient mental, written and digital strategies.
- **We understand that estimating is reasoning, not guessing** and we can support students to know this and to notice their own reasoning. Reasoning may be based on applying a known fact or a prior experience to a new situation. For further details about methods students normally use to estimate distances, refer to page 6 'Developing an ability to estimate' in the *Using units of measurement: Years 5–7 – Mathematics Conceptual Narrative*.

  To be able to estimate at Year 8 level, students must be able to:

  – Make an estimate of their numerical calculation, so that they are confident in the value(s) that they produce.

  – Have a reasonable appreciation if their result is of the correct order of magnitude. To do this, they need to have an appreciation of actual size.

Engaging learners

Classroom techniques for teaching units of measurement

Harnessing students' fascination with scale

People are often fascinated with very large or very small items. We are particularly fascinated with large items that should be small, and small items that should be large. For an example of this fascination, follow the link below to the news story about giant marionettes in Perth, WA. An estimated 1.4 million people attended *'The Giants extravaganza'*!

http://www.perthnow.com.au/news/western-australia/giants-in-perth-day-three/story-fnhocxo3-1227220081679

Perth Now
February 2015
Picture: Stewart Allen

To use this story to engage learners we would play one of the news stories, without the audio (at first) and ask students:

What questions do you have?

There are films, such as *'The Borrowers'* and *'Gulliver's Travels'*, that play on our fascination with scale. Such films and images can be used to make connections between measurement, scale, enlargement and fractions.

Images of large amounts of money and movie scenes that involve the transaction of large amounts of cash in small bags or briefcases, provide another engaging context for working with units of measurement.

We can support our students to develop a disposition towards using maths in their lives, ie becoming numerate, not only through the use of 'real world' maths problems, but through fostering a disposition towards asking mathematical questions about everything they see. We develop this disposition in our students when we promote, value and share their curiosity and provide opportunities for them to develop their questions and explore solutions to their questions.

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–8)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students, and what could I challenge and support them to develop an understanding of, for themselves?

For example, no amount of reasoning will lead my students to **create the name 'parallelogram'** for themselves. They need to receive this information in some way. However, it *is* possible for my students to be challenged with a question that will result in them identifying **a formula to calculate the area of a parallelogram**, so I don't need to instruct that information.

When we challenge our students to **establish formulae**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular formula during the current unit of work, then it probably is quicker to tell them the formula and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator* and *user* of mathematics, then telling students the formulae is a false economy of time.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for Us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Establishing the volume of a prism Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume	ACMMG198 ◆
Example 2: Establishing the area of a parallelogram Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites	ACMMG196 ◆
Example 3: Making connections – similarities and differences [kite, parallelogram, trapezium] Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites	ACMMG196 ◆
Example 4: Establishing the area of a kite Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites	ACMMG196 ◆
Example 5: Making trundle wheels Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area	ACMMG 197 ◆
Example 6: Trapezia and kites – connecting area and perimeter Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites	ACMMG196 ◆
Example 7: How far will this … roll? – establishing the circumference of a circle Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area	ACMMG197 ◆
Example 8: The race – establishing the area of a circle Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area	ACMMG197 ◆

Example 1: Establishing the volume of a prism

ACMMG198 ◆
Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general and use formulas to solve problems involving volume.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Reasoning proficiency:
In what ways can you prove it?

Instead of ***telling*** students the formula for the volume of a triangular prism, we can challenge students to adapt the known to the unknown, by ***asking*** questions.

The following information about displacement is an extract from page 21 ‘Example 5: Connecting volume and capacity’ in the *Using units of measurement: Years 5–7 – Mathematics Conceptual Narrative*.

The metric system is beautifully constructed, such that there is a relationship between the different units. For example, 1 cm^3 is the same as 1 ml and for water this quantity has a mass of 1 g. At this stage of development it is only necessary for students to understand the connection between volume and capacity. For example: 1 ml = 1 cm^3.

Submerging MAB blocks in a measuring cylinder (like those shown in Figure 10) containing water and observing the effect on the water level, is a very easy way to establish this connection, without instructing students. We can ask students to submerge different quantities of MAB blocks in a measuring cylinder and ask:

***What connection do you notice** between the number of centimetre cubes that you place in the measuring cylinder and the number of millilitres the water level rises by?*

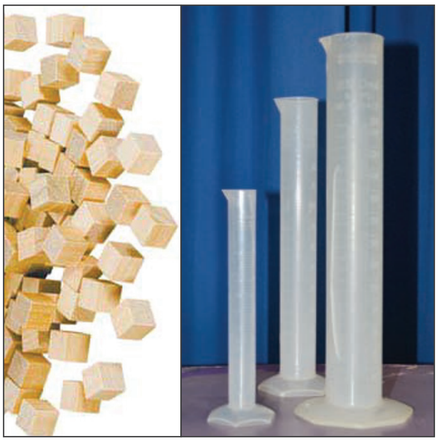


Figure 10

Take care to use MAB cubes/lengths that have been cut to metric measurements. Cubes should be 1 cm x 1 cm x 1 cm and the 10 lengths should be 1 cm x 1 cm x 10 cm. It’s worth checking your MAB blocks before using them in this way.

In this example, we assume that students are familiar with the formula for calculating the volume of a rectangular prism. If your students are not familiar with this, you could refer to page 26 ‘Example 8: Volume of a rectangular prism’ in the *Using units of measurement: Years 5–7* narrative.

Instead of ***telling*** students the formula for the volume of a triangular prism, we can challenge students to **adapt the known to the unknown**, by ***asking*** questions:

- ***How might you** use the formula for the volume of a rectangular prism to suggest a formula for calculating the volume of a triangular prism?*

We can support students to notice connections by asking similarity and difference questions, such as:

- ***What’s the same about** the rectangular prism and the triangular prism?*
- ***What’s different about** the rectangular prism and the triangular prism?*
- ***What connections** can you see between the (rectangular prism) formula and the lengths on the actual prism?*
- ***How could this be adapted** to suit a triangular prism?*
- ***How could you test** your idea?* (Displacement, as described in the extract from the *Using units of measurement: Years 5–7* narrative, is one option.)
- ***How could you convince** someone who thinks differently to you?*

It can help to have a rectangular prism and triangular prism that are the same height as each other. It is common for the boxed sets of 3D objects to contain objects with the same height and cross-sectional area. The same process could be used for predicting and establishing the volume of any prism.

Example 2: Establishing the area of a parallelogram

ACMMG196 ◆

Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can you prove it?

What can you infer?

Instead of ***telling*** students the formula for the area of a parallelogram, we can challenge students to establish the formula, by ***asking*** questions.

Establishing the area of a parallelogram is within the Year 7 content descriptions, together with the area of a triangle and a rectangle.

This example is just one way that we can question students to establish if they are already familiar with the formula and most importantly if they can show that they understand how/why it works. At the same time this question provides an opportunity for students who have not seen the formula, to establish it for themselves.

For shapes shown in Figure 1:

- *Which is bigger, the area of the parallelogram or the area of the rectangle? (For the blue example, the orange example, etc.)*

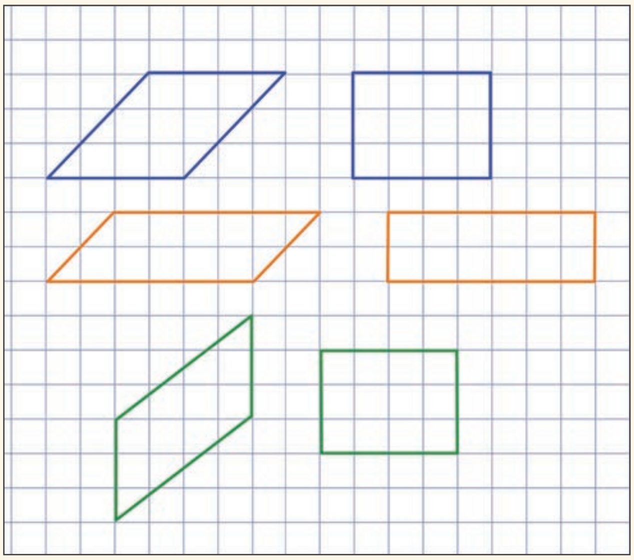


Figure 1

It doesn't matter if the student's initial assessment of the relative areas is accurate or not. We can always say:

- *Prove it to me/convince me/convince someone who thinks differently to you.*

Making the most of mistakes: Create a language shift and a culture shift in your classroom

We can use activities such as this example to actively teach students that sometimes their deeper exploration of an idea will lead to an outcome that they hadn't expected. They have a choice to make when this happens. They can think:

- I was wrong. I'm not good at this, or;
- I've changed my mind. I've learnt something new.

We can promote a growth mindset, by asking the class:

- Who changed their mind/discovered something new to them? Congratulations, you have just changed your brain!

Students can work towards a solution in a number of different ways. They might choose to explore the relationship between the rectangle and the parallelogram:

- numerically: counting squares
- physically: cutting and rearranging the parallelogram, as in Figure 2
- algebraically: applying known formulae for rectangles and triangles.

Figure 2 shows just one way in which this parallelogram can be rearranged to form a rectangle.

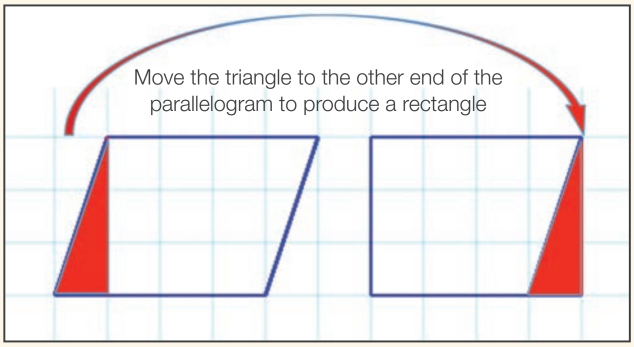


Figure 2

By encouraging students to select their own approach, we can gain insight into their intuitive understanding about area. We can then challenge them to consider other methods and to share their approaches, by asking:

- ***Is there another way*** *that you could show that/check out your thinking?*

We can encourage our students to generalise and to experiment with the use of variables in a meaningful context, by asking:

- ***Is there a rule*** *that you could use to describe a way to work out the area of a parallelogram? Could you explain your rule in words? Could it be explained using symbols?*
- ***What if you change*** *the size of the parallelogram – does your rule work for any size?*
- ***What if you change*** *the angles in your parallelogram – does your rule still work?*
- ***What if*** *all of the sides of the parallelogram are the same length?* (Now students have established the area of a rhombus.)
- ***What other shapes*** *might this rule/formula connect to?*

We can create the opportunity for students to reason about the relationship between area and perimeter:

- ***What can you infer? Now that you know that*** *the area of the parallelogram and the rectangle (shown in these pairs) are the same as each other,* ***what do you think you could you say*** *about the perimeter or the rectangle and the parallelogram? Will they be the same as each other? Remember, this is just your FIRST THINKING. You can change your mind!*

Example 3: Making connections – similarities and differences [kite, parallelogram, trapezium]

ACMMG196 ◆
Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Reasoning proficiency:
In what ways can we communicate?

Instead of ***telling*** students what concept they should comment on, we can challenge students to identify what it is possible to comment on, by ***asking*** questions.

Instead of ***telling*** students what concept they should comment on, we can challenge students to identify what it is possible to comment on. We can then choose to focus in on our desired concept. For example, we can present students with a small selection of shapes and ***ask***:

- ***What's the same** about them all?*
- ***What's different** about them?*

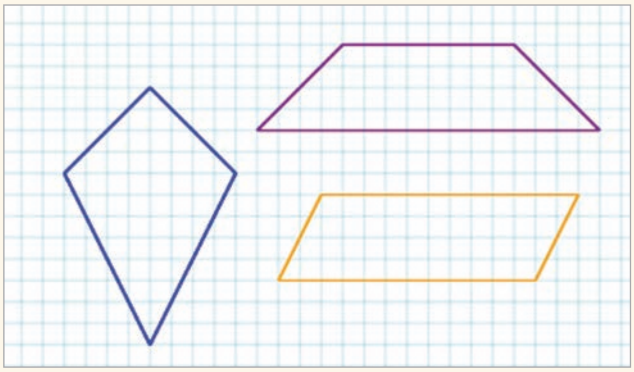


Figure 3

This selection of quadrilaterals (Figure 3) has been designed to have the **same area** as each other, but **different perimeters**. However, asking students questions that focus on similarity and difference between the shapes could elicit responses such as, they all have: 4 sides, 4 vertices or 2 obtuse angles. They are all quadrilaterals, they all tessellate, their internal angles add up to 360°; but only the kite and the trapezium have a line of symmetry. The parallelogram is the only one with rotational symmetry; the kite is the only one with a right angle; the trapezium and the parallelogram are the only ones with two acute angles. The parallelogram and the trapezium can be changed into a rectangle in just one cut and one transformation, but the kite would take two cuts.

These differences lend themselves to the question:

- *Which shape could be the odd one out?*

We can use the question:

- *Is there another similarity/difference? (and another, and another, etc)*

Open questions: Providing multiple entry points and supporting connections to be made

Asking 'odd one out' questions, compared to questions such as: 'Which of these shapes have the same area as each other?' demands different thinking of the student. The 'odd one out' question does not dictate the topic. It requires students to think about the observations/measures that are possible, facilitates different entry levels, facilitates students making connections between different concepts and allows for creative solutions.

Example 4: Establishing the area of a kite

ACMMG196 ◆

Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students the formula for the area of a kite, we can challenge students to establish the formula, by ***asking*** questions.

For the shapes shown in Figure 4, instead of ***telling*** students the formula for the area of a kite, we can challenge students to **establish the formula**, by ***asking***:

- ***Which is bigger****, the area of the kite or the area of the rectangle? (For the purple example, the green example, etc.)*

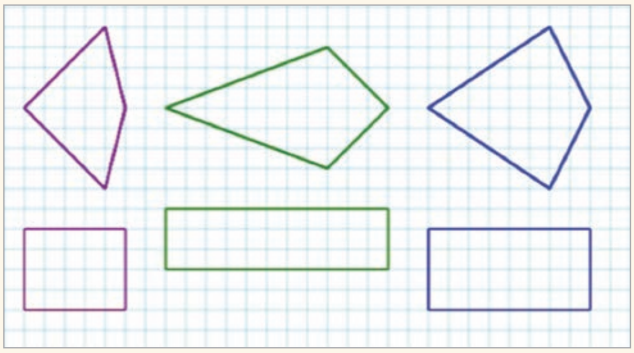


Figure 4

It doesn't matter if the student's initial assessment of the relative areas is accurate or not. We can always say:

- ***Prove it to me/convince me/convince someone who thinks differently to you****.*
- ***Is there another way*** *that you could show that?*

Students can work towards a solution in a number of different ways. They might choose to explore the relationship between the rectangle and the kite:

- physically; cutting and rearranging the kite, as in Figure 5a
- algebraically; applying known formulae for rectangles and triangles.

Value diverse thinking: Utilise peer tuition

Both processes can be valued, even though they use two different levels of mathematics. All students benefit from seeing different ways to explain the same relationship. Seeing the physical rearrangement of the kite is visually convincing, even satisfying, but a physical rearrangement isn't always possible, so we need a technique that can be applied even when we can rearrange the shape. This is a perfect opportunity for peer tutoring to be used.

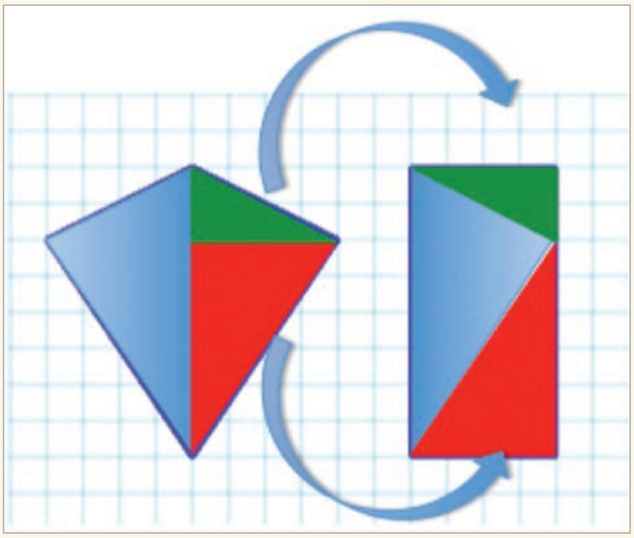


Figure 5a

Figure 5a shows how a kite can be rearranged to form a rectangle. You can see that the length of the kite is the same as the length of the rectangle and the width of the rectangle is half the width of the kite.

Students can highlight the dimensions of the rectangle they have formed to calculate the area, and then reconstruct the original kite. By doing this, they can see that despite the fact they might have cut the kite in different ways, they always use the dimensions of the diagonals (Figure 5b).

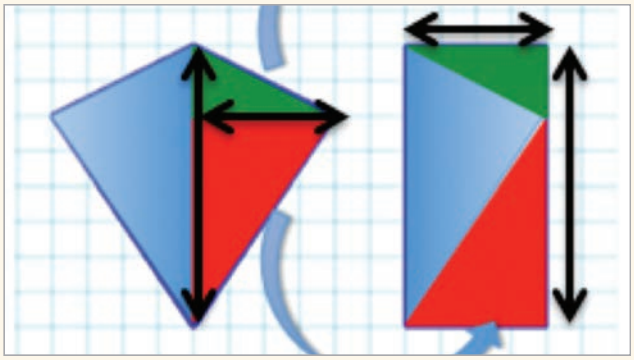


Figure 5b

We can encourage students to generalise, by asking:

- ***Is there a rule** that you could use to describe a way to work out the area of a kite? Could you explain your rule in words? Could it be explained using symbols?*
- ***What if you change** the size of the kite – does your rule work for any size?*
- ***What if you change** the angles in your kite – does your rule still work?*
- ***What if** all of the sides of the kite are the same length?* (Now students have established the area of a rhombus, but in a different way to the one they used when they connected it to a parallelogram.)

At this point, students can research the formula for the area of a kite and relate this to their own thinking.

We can create the opportunity for students to reason about the relationship between area and perimeter:

- ***What can you infer? Now that you know** the area of the kite and the rectangle (shown in these pairs) are the same as each other, **what do you think you could you say** about the perimeter of the rectangle and the kite? Will they be the same as each other? Remember, this is just your FIRST THINKING. You can change your mind!*

Example 5: Making trundle wheels

ACMMG197 ◆
Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area.

Questions from the BitL tool
Understanding proficiency:
Can you answer backwards questions?

Instead of ***telling*** students which circle to calculate the circumference of, we can ***ask*** them to identify a circle which would have a circumference that they might be interested in.

Backwards questions

A 'backwards' question requires a little more thinking about than a standard application question. We ask this type of question to challenge students to work flexibly with a concept. There are many ways that teachers can do this. Two possible options include asking questions where students need to complete missing information and asking questions that start by giving the students an answer.

Trundle wheels fascinate students of all ages. There is something about a trundle wheel that makes you want to run with it!

Most trundle wheels that we see in schools have a circumference of 1m, but why be limited by convention? You could make a trundle wheel that has the circumference of your foot length, your cubit, your hand-span, etc.

When people do not have a tape measure they often 'step off' a distance, sometimes by striding, sometimes putting one foot closely after the other.

- *In what ways are these methods the same? How are they different?*
- *Which do you think is more accurate? Convince me?*

For Year 8 students in an area school or with a buddy primary class, this can be a great shared-activity, with the Year 8 students making trundle wheels that facilitate the younger students measuring in non-standard units such as cubits. (Estimate first.)

- *How many of your foot lengths might fit across this space?*
- *How many of mine might it be? Would it be more or less than yours?*
- *If we were to design our own foot length trundle wheels, how big would the circle have to be?*
- *How might we find that out/calculate that?*

The important point here is that the student is beginning with the circumference and needs to work out the radius to be able to create the circle for the trundle wheel. (Pop-sticks can be used for the handle and a split pin for the turning mechanism.)

This task is an opportunity for students to appreciate the different levels of accuracy that are required in different situations and for different audiences.

Example 6: Trapezia and kites – connecting area and perimeter

ACMMG196 ◆

Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Questions from the BitL tool

Understanding proficiency:

Can you answer backwards questions?

Instead of ***telling*** students the shape to calculate the area of, we can ***ask*** them to identify a set of possibilities.

Instead of ***telling*** students the shape to calculate the area of, we can ***ask*** them to identify a set of possibilities.

Ask students:

- *A trapezium has an area of 24 sq. units* (replace this with mm^2 or cm^2 or m^2 or leave your students to suggest what the units might be and take the opportunity to discuss what those different areas would look like). *How many possible trapezia (plural of trapezium) can you find? What do you notice about the perimeter of the different trapezia?*

To cater for a mixed-ability group students could be invited to choose a shape from a selection, including kites, parallelograms, triangles and rectangles. We can support the development of a growth mindset in our students when we present them with choices such as this.

Growth mindset: Students learning to take responsibility for the level of challenge

When we approach a problem with a growth mindset, we will take on a challenge that we know we may need to struggle and persist with. When we do this we create an opportunity for new learning and new brain growth. In this mindset we are focused on **improving our ability**.

If we approach the problem with a fixed mindset, we will tend to choose a level of difficulty that does not present a challenge to us. In this mindset we are focused on **proving our ability** rather than improving it.

If we always focus on proving what we already know rather than improving, then we reduce our opportunities to learn and to become resilient in learning.

As teachers, we need to be clear about the level of challenge that our students need to aspire to meet, but we also need to support them to choose an appropriate entry point and an appropriate challenge.

Example 7: How far will this … roll? – establishing the circumference of a circle

ACMMG197 ◆

Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area.

Questions from the BitL tool

Understanding proficiency:

Can you answer backwards questions?

What patterns/connections/relationships can you see?

Instead of ***telling*** students the formula for the circumference of a circle, we can challenge students to explore the connection between circumference and diameter and hence establish the formula, by ***asking*** questions.

Choose a large circular object to show students – the bigger the better! Then ask:

- *How far will this … roll in one full turn?*

At first, don't allow any measurements to be taken.

Provide the students with a way of marking their estimate on the floor (ie chalk, a counter, a piece of paper). This is one way of asking, '*What's about the right answer/what feels right?*'

Now roll the circle.

Then ask:

- *How close were you?*
- *What could you learn from this to help you to make a closer estimate for a different circle?*
- *If you were able to take measurements, but not the circumference of the circle, what else could you measure?* (The options are limited with a circle, so students will describe the diameter, although probably not using the term 'diameter'. This is a perfect time to introduce the language of 'diameter' and maybe even 'radius', because now having this language makes the conversation easier and hence there is a purpose to the language and an opportunity to use it in communicating thinking.)

Still without using measuring tools, we can ask:

- *What connection can you see between the diameter and the circumference for this circle?* (Challenge them to record their thinking in words, using symbols or both. At this stage, most students will be able to say that the circumference is about three times the diameter, or a little more than three times the diameter.)

Develop the concept before the detail

Persisting with estimating, rather than measuring, can help to keep students focused on the relationship rather than the calculation, which will probably involve decimal values that may distract some learners.

Example 8: The race – establishing the formula for circumference and area of a circle

ACMMG197 ◆

Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/ relationships can you see?

Instead of ***telling*** students the formula for the circumference of a circle, we can challenge students to explore the connection between circumference and diameter and hence establish the formula, by ***asking*** questions.

Consider a race from one red post to another then back, as represented in Figure 6. You can choose to run around the sides of a square (as shown in orange), or take a circular path (blue) or take a straight path, over and back (green).

- *Which path would you take and why?*

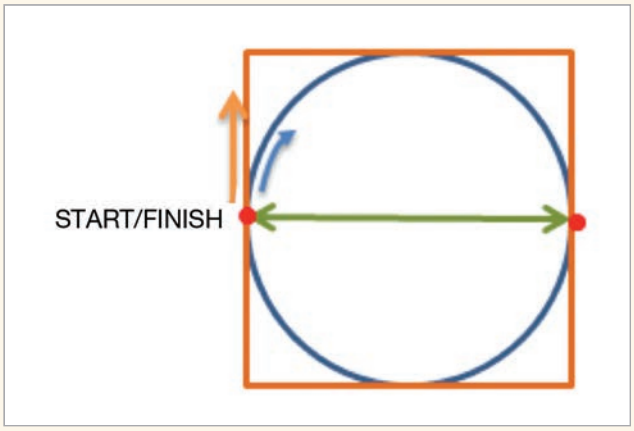


Figure 6

Consider the length of the different paths:

- *Which is the shortest/the longest path?*
- *If the distance between the two posts is 100 metres, can you determine the lengths of any of the paths accurately? For any of the paths you can't, can you make any statement about their length?* (Having a dimension for the circle provides a more concrete way of thinking about this problem. It also allows students to make an estimate for the circumference of the circle before they are asked to generalise for any diameter.)
- *If the distance between the two posts is d metres, can you determine an expression for the lengths of any of the paths accurately? For any of the paths you can't, can you make any statement about their length?* (We are looking for students to be able to make a statement such as, *'The square is 4 times the diameter, 4d, the straight path is 2d and the circular path is less than 4d but more than 2d'*.)

Some students may believe that the relationship will change depending on the size of the circle. Ask them:

- *What if we use a bigger/smaller circle? What do you think the relationship will be then?*

Students could begin to explore this problem using a range of different circles and share their findings. They can change the method for measuring circumference and increase the accuracy of their measurements. Software could be used for a high degree of accuracy.

The relationship between diameter and circumference could be plotted on a graph and the equation of the line could be investigated. This can be done manually or using Excel and 'Fitting a trendline' (see Figure 7).

http://office.microsoft.com/en-au/training/results.aspx?qu=trendlines&ex=1&origin=RZ006107930

	A	B
1	Diameter	Circumference
2	2.5	7.9
3	35	110
4	18	57
5	1.6	5
6	13	40
7	27	84
8	10.5	33
9	4.2	13
10	6.5	20
11	30	95
12	42	132
13	17	52

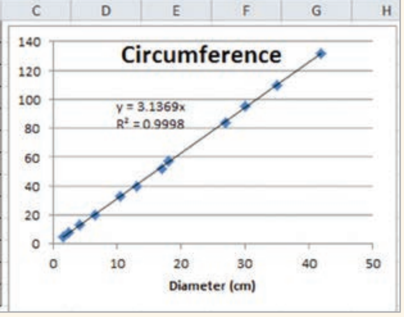


Figure 7

A similar investigative approach could be taken to establish the relationship between radius and area. Students can draw upon their understanding of calculating the areas of other shapes in their estimate of the areas of a circle.

We can also take this opportunity to consolidate understanding within the Statistics strand. When discussing errors that occur when measuring physical objects, we are exploring the practicalities and implications of collecting data.

We can ask:

- *How accurate are the measurements taken from smaller circles? How accurate are the measurements taken from larger circles?*
- *How can you determine which of these two sets of measurements are more accurate?*

Often the data from the larger circles is more accurate as it is physically easier to collect. Even if this is not the case, if you find the errors as a percentage of the measurement being taken, the percentage error of large circles is less than the percentage error for the small circles (all other factors being the same). When investigating percentage differences in this context, we are connecting our understanding in number, statistics and measurement. In this way, we are giving students the opportunity to **work mathematically** rather than just **do mathematics**.

Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 9–12)

Problems are described in the AC: Mathematics as ‘meaningful’ or ‘unfamiliar’. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as ‘fun’ nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, ‘I don’t know’, or ‘I’m not sure’. Students respond differently to this feeling; some shut down, others begin to ask, ‘But how could I work that out?’

In developing powerful learners we are aiming for all of our students to learn that ‘not knowing’ is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, ‘What could I do to work this out?’

Engaging in problem solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they’ll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they’ll need
- a possible process to use.

Proficiency: Problem Solving examples	
Example 9: Car caravan – cars parked in a circle Students investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.	ACMMG197 ◆
Example 10: Packaging problems – redesigning a rectangular prism Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.	ACMMG198 ◆
Example 11: Areas of parallelograms and kites Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.	ACMMG196 ◆
Example 12: Areas of a trapezium Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.	ACMMG196 ◆

Example 9: Car caravan – cars parked in a circle

ACMMG197 ◆

Students investigate the relationship between features of circles such as circumference, area, radius and diameter and use formulas to solve problems involving circumference and area.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

This activity could be considered a ***Three-Act Maths Task*** like those on Dan Meyer's blog: 101 questions. It can be presented to students along with the question, *What's the first question that comes to mind?*

Identifying the question to solve

The group can share questions and sort them into mathematical and non-mathematical questions. Then, of the mathematical questions, students can sort their questions into those that cannot be answered with the given information and those that could be answered using the given information or additional information that could be inferred.

Dan Meyer has a technique, that we have seen many teachers adopt when generating and collecting questions from students. First, he asks students to individually write down questions that come to mind. Then, as he invites students to share their questions, he writes students' names next to the questions. He also asks if anyone else likes that question. *'Did you write it down, or if you didn't perhaps you still think that it's a good question.'* Through doing this, both Dan and his class get a sense of the questions that are of interest to the students.

Figure 8 | Source: Dan Meyer 101 questions http://www.101qs.com

Ask students questions related to the image in Figure 8, which could include:

- *How many cars are there?* (The class could work as a team to solve this.)
- *How many cars are in the final lap?*
- *How big is this display?*
- *Which colour car is there more of?*
- *How much would that collection cost?*
- *How long would it take to create this?*
- *How big would that be if real cars were used?*
- *How much would that cost in toy cars/in real cars?*

Interpret

What question have you selected? What's an answer that's too big? What's an answer that's too small? What's a bit closer to the answer that you think you'll find? What do you need to show to answer that question? What information is helpful? What information is not useful? What extra information do you want to collect? What information will you need/can you reasonably infer? (Remember that telling students or prompting heavily is STEALING the opportunity for them to realise for themselves later on in the task.)

Model and plan

Do you have an idea? How might you start? What equipment will be helpful? Would it help to start by thinking about a smaller version of this pattern? (Ask students to speak to someone who they think is being a good problem solver today and ask them to show them what they are trying.)

Solve and check

Questions to be used only after students have grappled with the problem for a few minutes:

Could you think of this as a series of circles? If you had a line 1m long, how could you work out the number of cars in the line? What could you do to estimate/calculate the radius of the outer circle? What formulas do you know that may help? Does that seem right to you? Do other people think that too?

Reflect

Ask students to pair up with someone who did it differently to discuss:

How do your methods compare? What do you like about each other's strategy? How could you help each other to improve? Have you reached the same/a similar conclusion? How efficient was your strategy? Is there something that you would do differently next time? Is there a more efficient way?

Keeping control of the question

If students' questions sit outside of the area that we want them to work in, we have some choices to make. We can always add our own question to the list and ask for that question to be solved. But we probably want to minimise that, as students may lose interest in generating possible questions if they know that we'll always replace their questions with our own.

Students may have generated variations on the question that we had intended and if we can see that they will still use the concepts that we had intended, we could either let them answer their own question or ask them to answer our question and then reflect on whether or not their question was also answered in the process.

Example 10: Packaging problems – redesigning a rectangular prism

ACMMG198 ◆

Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general and use formulas to solve problems involving volume.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the problem to solve, the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

Redesign the milk carton in Figure 9 to be a different rectangular prism (or perhaps a triangular prism).

This can be developed into a 'multiple representations' question:

- *Is there another solution?*

It can be extended to require reasoning (about surface area):

- *Why might the actual dimensions have been chosen? Are there benefits to changing the dimensions to your design?* (The width of the carton is usually designed based on the comfortable grip of the user. Compare the width of a child's fruit box to an iced coffee carton. Other considerations might be strength of the container, its ease for stacking and storing, practicality of the pouring spout, etc.)

This type of question could easily incorporate percentage increases and decreases, by asking students to create a container for when the company wants to increase/decrease the amount of milk by a particular percentage.

Figure 9

Example 11: Area of parallelograms and kites

ACMMG196 ◆

Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Questions from the BitL tool

Problem solving proficiency:

Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:

What can you infer?

Instead of ***telling*** students the information they'll need and the steps they should take, we can ***ask*** them to identify the problem to solve, the information they'll need and the steps they could take.

We can provide opportunities for students to identify the value in having formulae for areas of shapes other than rectangles and triangles.

Not having the formula for the area of a parallelogram or kite only becomes a problem when students don't have access to the measurements for their internal rectangles and triangles. So one way to challenge students to think differently is to restrict the measurements available to them and provide sketches that aren't drawn to scale, like the examples in Figure 10.

The value proposition for students is that the area of a rhombus, trapezium, kite and parallelogram can be calculated using fewer steps and hence less work if you can just figure out a formula to use.

For students who have already established the formulae for the area of a kite and a parallelogram, this question would be a fluency question, as it would be a basic application problem.

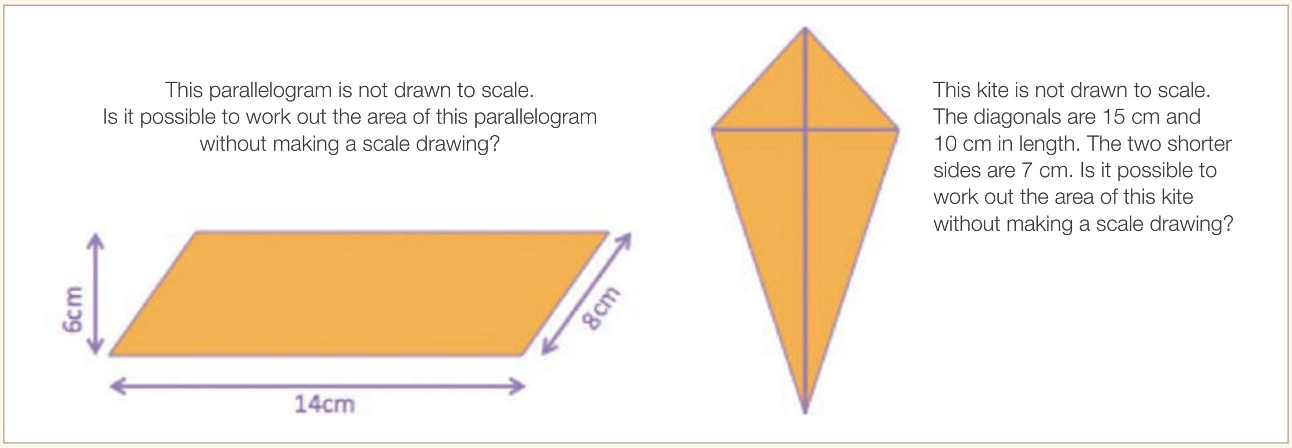


Figure 10

Example 12: Area of a trapezium

ACMMG196 ◆
Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.

Questions from the BitL tool
Problem solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.
Reasoning proficiency:
What can you infer?

Instead of ***telling*** students the relationship between a, b and h and modelling how this can be shown, we can ***ask*** them to identify the relationship and design a convincing argument about how they know.

Instead of ***telling*** students the relationship between a, b and h in Figure 11 and modelling how this can be shown, we can ***ask*** them to identify the relationship and design a convincing argument about how they know.

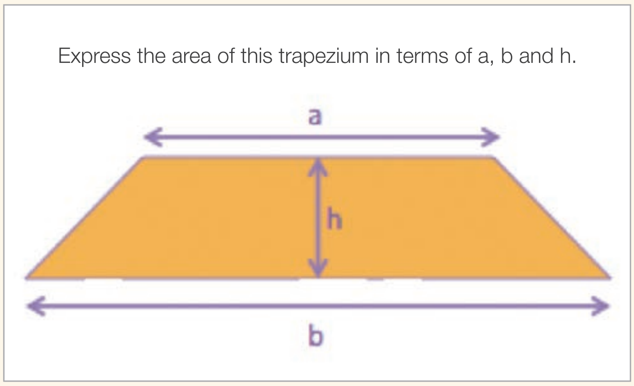


Figure 11

Building resilience and positive learner identity: Using high challenge problems

We know that:

'Positive learner identity – is more readily built through succeeding at challenging tasks than experiencing 'dumbed down' activities.'

(David Price, Learning Futures, UK)

When presenting challenging tasks it is important for us to have thought about 'enabling prompts' that we can release to students as necessary. Such prompts could include:

- *How might you put more than one of these shapes together to form a shape you could find the area of?* (See suggestions in Figures 12a and 12b.)
- *Might it be helpful to cut the shape up?*

This arrangement is limited due to the need for the angles to be 45° and 135°, but students can be challenged to identify those limitations for themselves rather than being prevented from trying the idea.

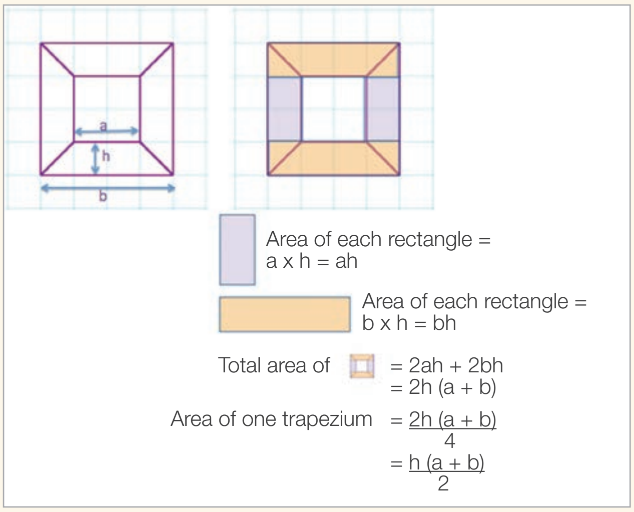


Figure 12a

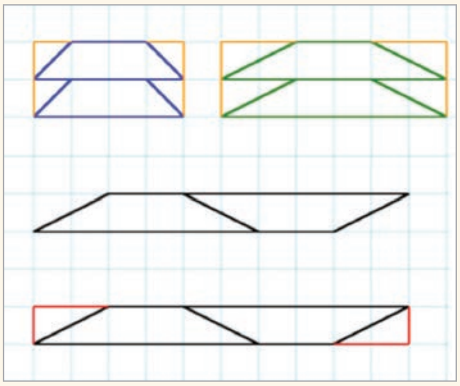


Figure 12b

Connections between ‘using units of measurement’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use units of measurement as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 8	
Whilst working with units of measurement, connections can be made to:	**How the connection might be made:**
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies. ACMNA183	Processing any calculations in relation to area or volume.
Investigate the concept of irrational numbers, including π. ACMNA186	Refer to learning about area and circumference of a circle: **Example 7: How far will this … roll? – establishing the circumference of a circle** **Example 8: The race – establishing the area of a circle**.
Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies. ACMNA187	Refer to learning about increasing or decreasing perimeter, area or volume by any given percentage: **Example 5: Making trundle wheels** **Example 8: The race – establishing the area of a circle** **Example 10: Packaging problems – redesigning a rectangular prism**.
Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes. ACMSP206	Refer to: **Example 7: How far will this … roll? – establishing the circumference of a circle**.
Plot linear relationships on the Cartesian plane with and without the use of digital technologies. ACMNA193	Refer to learning about plotting the relationship between diameter and circumference of a circle: **Example 7: How far will this … roll? – establishing the circumference of a circle** **Example 8: The race – establishing the area of a circle**.
Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes. ACMSP206	Refer to: **Example 8: The race – establishing the area of a circle**.

Making connections to other learning areas

There is great potential to make connections to history when working with units of measurement. Many of the imperial units that students will hear being used, particularly by older relatives, will have stories that explain the use of a word. For example: pints and gallons. At this stage, when students are learning to convert between metric units of measurement, it can be useful to look at the alternative (the imperial system) and the challenges involved in changing between units in that system.

There are also many connections to science. One connection that can be used to remind students of the importance of recording units of measure, is this story about NASA losing a spacecraft because they had a miscommunication about units:

‘Mars Climate Orbiter team finds likely cause of loss’: http://mars.jpl.nasa.gov/msp98/news/mco990930.html

‘Using units of measurement’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with measurement:

Using informal units for direct or indirect comparisons ◆
From **Foundation to Year 2** students focus on **informal units of measurement**.

Using standard metric units ◆
From **Year 3 to Year 8** students develop their **understanding of metric units of measure**. This begins with the use of familiar metric units and extends to include a greater range of metric units and the flexibility to convert between different units.

Establishing and applying formulae ◆
From **Year 5 to Year 10** students **establish and use formulae** of increasing complexity relating to perimeter, area and volume.

Estimating ◆
Australian Curriculum references to **estimation** in relation to measurement lie entirely in the **Numeracy Continuum**.

Year level	‘Using units of measurement’ content descriptions from the AC: Mathematics
Foundation ◆	Students use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.
Year 1 ◆	Students measure and compare the lengths and capacities of pairs of objects using uniform informal units.
Year 2 ◆	Students compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units.
Year 2 ◆	Students compare masses of objects using balance scales.
Year 3 ◆	Students measure, order and compare objects using familiar metric units of length, mass and capacity.
Year 3 ◆	Students use scaled instruments to measure and compare lengths, masses, capacities and temperatures.
Year 4 ◆	Students compare objects using familiar metric units of area and volume.
Year 5 ◆	Students choose appropriate units of measurement for length, area, volume, capacity and mass.
Year 5 ◆	Students calculate the perimeter and area of rectangles using familiar metric units.
Year 6 ◆	Students connect decimal representations to the metric system.
Year 6 ◆	Students convert between common metric units of length, mass and capacity.
Year 6 ◆	Students solve problems involving the comparison of lengths and areas using appropriate units.
Year 6 ◆	Students connect volume and capacity and their units of measurement.
Year 7 ◆	Students establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
Year 7 ◆	Students calculate volumes of rectangular prisms.
Year 8 ◆	Students choose appropriate units of measurement for area and volume and convert from one unit to another.
Year 8 ◆	Students find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.
Year 8 ◆	Students investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Year level	'Using units of measurement' content descriptions from the AC: Mathematics *continued*
Year 8 ◆	Students develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
Year 9 ◆	Students calculate the areas of composite shapes.
Year 9 ◆	Students calculate the surface area and volume of cylinders and solve related problems.
Year 9 ◆	Students solve problems involving the surface area and volume of right prisms.
Year 10 ◆	Students solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.
Year 10A ◆	Students solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Numeracy continuum: Using measurement	
End Year 2 ◆	Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects.
End Year 4 ◆	Estimate and measure with metric units: estimate, measure and compare the length, temperature, volume, capacity and mass of everyday objects using metric units and scaled instruments.
End Year 6 ◆	Estimate and measure with metric units: choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems.
End Year 8 ◆	Estimate and measure with metric units: convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems.
End Year 10 ◆	Estimate and measure with metric units: solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.1

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by Office for Education and Early Childhood

Excluded from NEALS